

# Algebra and Trigonometry OpenStax



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## OpenStax

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PART I

# PREREQUISITES





# I. Real Numbers: Algebra Essentials

## *Learning Objectives*

In this section students will:

- Classify a real number as a natural, whole, integer, rational, or irrational number.
- Perform calculations using order of operations.
- Use the following properties of real numbers: commutative, associative, distributive, inverse, and identity.
- Evaluate algebraic expressions.
- Simplify algebraic expressions.

It is often said that mathematics is the language of science. If this is true, then an essential part of the language of mathematics is numbers. The earliest use of numbers occurred 100 centuries ago in the Middle East to count, or enumerate items. Farmers, cattlemen, and tradesmen used tokens, stones, or markers to signify a single quantity—a sheaf of grain, a head of livestock, or a fixed length of cloth, for example. Doing so made commerce possible, leading to improved communications and the spread of civilization.

Three to four thousand years ago, Egyptians introduced fractions. They first used them to show reciprocals. Later, they used them to represent the amount when a quantity was divided into equal parts.

But what if there were no cattle to trade or an entire crop of grain was lost in a flood? How could someone indicate the existence

of nothing? From earliest times, people had thought of a “base state” while counting and used various symbols to represent this null condition. However, it was not until about the fifth century A.D. in India that zero was added to the number system and used as a numeral in calculations.

Clearly, there was also a need for numbers to represent loss or debt. In India, in the seventh century A.D., negative numbers were used as solutions to mathematical equations and commercial debts. The opposites of the counting numbers expanded the number system even further.

Because of the evolution of the number system, we can now perform complex calculations using these and other categories of real numbers. In this section, we will explore sets of numbers, calculations with different kinds of numbers, and the use of numbers in expressions.

## Classifying a Real Number

The numbers we use for counting, or enumerating items, are the natural numbers: 1, 2, 3, 4, 5, and so on. We describe them in set notation as  $\{1, 2, 3, \dots\}$  where the ellipsis (...) indicates that the numbers continue to infinity. The natural numbers are, of course, also called the *counting numbers*. Any time we enumerate the members of a team, count the coins in a collection, or tally the trees in a grove, we are using the set of natural numbers. The set of whole numbers is the set of natural numbers plus zero:  $\{0, 1, 2, 3, \dots\}$ .

The set of integers adds the opposites of the natural numbers to the set of whole numbers:  $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$ . It is useful to note that the set of integers is made up of three distinct subsets: negative integers, zero, and positive integers. In this sense, the positive

integers are just the natural numbers. Another way to think about it is that the natural numbers are a subset of the integers.

negative integers
zero
positive integers

$\dots, -3, -2, -1,$ 
 $0,$ 
 $1, 2, 3, \dots$

The set of rational numbers is written as  $\left\{ \frac{m}{n} \mid m \text{ and } n \text{ are integers and } n \neq 0 \right\}$ . Notice

from the definition that rational numbers are fractions (or quotients) containing integers in both the numerator and the denominator, and the denominator is never 0. We can also see that every natural number, whole number, and integer is a rational number with a denominator of 1.

Because they are fractions, any rational number can also be expressed in decimal form. Any rational number can be represented as either:

1. a terminating decimal:  $\frac{15}{8} = 1.875$ , or
2. a repeating decimal:  $\frac{4}{11} = 0.36363636 \dots = 0.\overline{36}$

We use a line drawn over the repeating block of numbers instead of writing the group multiple times.

### *Writing Integers as Rational Numbers*

Write each of the following as a rational number.

- a. 7
- b. 0

c.  $-8$

[reveal-answer q="fs-id2196956"]Show

Solution[/reveal-answer]

[hidden-answer a="fs-id2196956"]

Write a fraction with the integer in the numerator and 1 in the denominator.

a.  $7 = \frac{7}{1}$

b.  $0 = \frac{0}{1}$

c.  $-8 = -\frac{8}{1}$

[/hidden-answer]

## Try It

Write each of the following as a rational number.

a.  $11$

- b. 3
- c. -4

[reveal-answer q="fs-id1460065"]Show  
Solution[/reveal-answer]  
[hidden-answer a="fs-id1460065"]

- a.  $\frac{11}{1}$
- b.  $\frac{3}{1}$
- c.  $-\frac{4}{1}$

[/hidden-answer]

## Identifying Rational Numbers

Write each of the following rational numbers as either a terminating or repeating decimal.

- a.  $-\frac{5}{7}$

b.  $\frac{15}{5}$

c.  $\frac{13}{25}$

[reveal-answer q="fs-id1385693"]Show

Solution[/reveal-answer]

[hidden-answer a="fs-id1385693"]

Write each fraction as a decimal by dividing the numerator by the denominator.

a.  $-\frac{5}{7} = -0.\overline{714285}$ , a repeating decimal

b.  $\frac{15}{5} = 3$  (or 3.0), a terminating decimal

c.  $\frac{13}{25} = 0.52$ ,  
a terminating decimal[/hidden-answer]

## Try It

Write each of the following rational numbers as either a terminating or repeating decimal.

a.  $\frac{68}{17}$

b.  $\frac{8}{13}$

c.  $-\frac{17}{20}$

[reveal-answer q="fs-id2146703"]Show  
Solution[/reveal-answer]  
[hidden-answer a="fs-id2146703"]

- a. 4 (or 4.0), terminating;  
b.  $0.\overline{615384}$ , repeating;  
c. -0.85, terminating

[/hidden-answer]

## *Irrational Numbers*

At some point in the ancient past, someone discovered that not all numbers are rational numbers. A builder, for instance, may have found that the diagonal of a square with unit sides was not 2 or even  $\frac{3}{2}$ , but was something else. Or a garment maker might have

observed that the ratio of the circumference to the diameter of a roll of cloth was a little bit more than 3, but still not a rational number. Such numbers are said to be *irrational* because they cannot be written as fractions. These numbers make up the set of irrational numbers. Irrational numbers cannot be expressed as a fraction of two integers. It is impossible to describe this set of numbers by a single rule except to say that a number is irrational if it is not rational. So we write this as shown.

$\{h|h \text{ is not a rational number}\}$

### *Differentiating Rational and Irrational Numbers*

Determine whether each of the following numbers is rational or irrational. If it is rational, determine whether it is a terminating or repeating decimal.

a.  $\sqrt{25}$

b.  $\frac{33}{9}$

c.  $\sqrt{11}$

d.  $\frac{17}{34}$



e. 0.303303330333...

[reveal-answer q="fs-id2097644"]Show

Solution[/reveal-answer]

[hidden-answer a="fs-id2097644"]

a.  $\sqrt{25}$  : This can be simplified as  $\sqrt{25} = 5$ . Therefore,  $\sqrt{25}$  is rational.

b.  $\frac{33}{9}$  : Because it is a fraction,  $\frac{33}{9}$  is a rational number. Next, simplify and divide.

$$\frac{33}{9} = \frac{\overset{11}{\cancel{33}}}{\underset{3}{\cancel{9}}} = \frac{11}{3} = 3.\bar{6}$$

So,  $\frac{33}{9}$  is rational and a repeating decimal.

c.  $\sqrt{11}$  : This cannot be simplified any further. Therefore,  $\sqrt{11}$  is an irrational number.

d.  $\frac{17}{34}$  : Because it is a fraction,  $\frac{17}{34}$  is a rational number. Simplify and divide.

$$\frac{17}{34} = \frac{\overset{1}{\overline{)17}}}{\underset{2}{\overline{)34}}} = \frac{1}{2} = 0.5$$

So,  $\frac{17}{34}$  is rational and a terminating decimal.

- e. **0.3033033303333 . . .** is not a terminating decimal. Also note that there is no repeating pattern because the group of 3s increases each time. Therefore it is neither a terminating nor a repeating decimal and, hence, not a rational number. It is an irrational number. [/hidden-answer]

## Try It

Determine whether each of the following numbers is

rational or irrational. If it is rational, determine whether it is a terminating or repeating decimal.

a.  $\frac{7}{77}$

b.  $\sqrt{81}$

c.  $4.27027002700027 \dots$

d.  $\frac{91}{13}$

e.  $\sqrt{39}$

[reveal-answer q="fs-id1842498"]Show  
Solution[/reveal-answer]  
[hidden-answer a="fs-id1842498"]

- a. rational and repeating;
- b. rational and terminating;
- c. irrational;
- d. rational and repeating;
- e. irrational

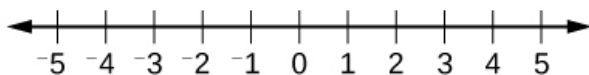
[/hidden-answer]

## Real Numbers

Given any number  $n$ , we know that  $n$  is either rational or irrational.

It cannot be both. The sets of rational and irrational numbers together make up the set of real numbers. As we saw with integers, the real numbers can be divided into three subsets: negative real numbers, zero, and positive real numbers. Each subset includes fractions, decimals, and irrational numbers according to their algebraic sign (+ or -). Zero is considered neither positive nor negative.

The real numbers can be visualized on a horizontal number line with an arbitrary point chosen as 0, with negative numbers to the left of 0 and positive numbers to the right of 0. A fixed unit distance is then used to mark off each integer (or other basic value) on either side of 0. Any real number corresponds to a unique position on the number line. The converse is also true: Each location on the number line corresponds to exactly one real number. This is known as a one-to-one correspondence. We refer to this as the real number line as shown in (Figure).



**Figure 1.** The real number line

## *Classifying Real Numbers*

Classify each number as either positive or negative and as either rational or irrational. Does the number lie to the left or the right of 0 on the number line?

a.  $-\frac{10}{3}$

- b.  $\sqrt{5}$
- c.  $-\sqrt{289}$
- d.  $-6\pi$
- e.  $0.615384615384 \dots$

[reveal-answer q="fs-id1613107"]Show  
Solution[/reveal-answer]  
[hidden-answer a="fs-id1613107"]

- a.  $-\frac{10}{3}$  is negative and rational. It lies to the left of 0 on the number line.
- b.  $\sqrt{5}$  is positive and irrational. It lies to the right of 0.
- c.  $-\sqrt{289} = -\sqrt{17^2} = -17$  is negative and rational. It lies to the left of 0.
- d.  $-6\pi$  is negative and irrational. It lies to the left of 0.
- e.  $0.615384615384 \dots$  is a repeating decimal so it is rational and positive. It lies to the right of 0.

[/hidden-answer]

## Try It

Classify each number as either positive or negative and as either rational or irrational. Does the number lie to the left or the right of 0 on the number line?

- a.  $\sqrt{73}$
- b.  $-11.411411411 \dots$
- c.  $\frac{47}{19}$
- d.  $-\frac{\sqrt{5}}{2}$
- e.  $6.210735$

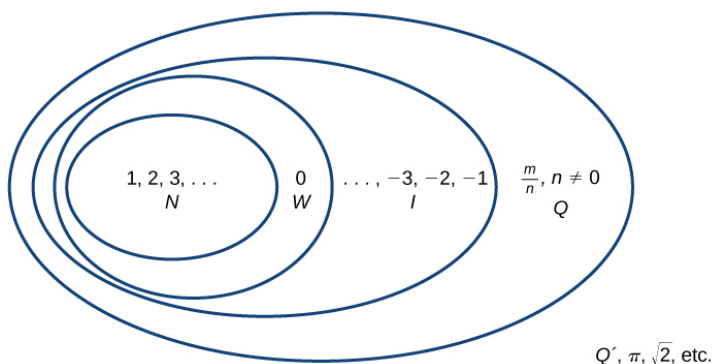
[reveal-answer q="fs-id1692671"]Show  
Solution[/reveal-answer]  
[hidden-answer a="fs-id1692671"]

- a. positive, irrational; right
- b. negative, rational; left
- c. positive, rational; right
- d. negative, irrational; left
- e. positive, rational; right

[/hidden-answer]

## *Sets of Numbers as Subsets*

Beginning with the natural numbers, we have expanded each set to form a larger set, meaning that there is a subset relationship between the sets of numbers we have encountered so far. These relationships become more obvious when seen as a diagram, such as (Figure).



**Figure 2.** Sets of numbers<math>N</math>: the set of natural numbers<math>W</math>: the set of whole numbers<math>I</math>: the set of integers<math>Q</math>: the set of rational numbers<math>Q'</math>: the set of irrational numbers

## Sets of Numbers

The set of natural numbers includes the numbers used for counting:  $\{1, 2, 3, \dots\}$ .

The set of whole numbers is the set of natural numbers plus zero:  $\{0, 1, 2, 3, \dots\}$ .

The set of integers adds the negative natural numbers to the set of whole numbers:

$$\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}.$$

The set of rational numbers includes fractions written as

$$\left\{\frac{m}{n} \mid m \text{ and } n \text{ are integers and } n \neq 0\right\}.$$

The set of irrational numbers is the set of numbers that



are not rational, are nonrepeating, and are nonterminating:  
 $\{h|h \text{ is not a rational number}\}.$

### *Differentiating the Sets of Numbers*

Classify each number as being a natural number (N), whole number (W), integer (I), rational number (Q), and/or irrational number (Q').

a.  $\sqrt{36}$

b.  $\frac{8}{3}$

c.  $\sqrt{73}$

d.  $-6$

e.  $3.2121121112\dots$

[reveal-answer q="267724"]Show

Solution[/reveal-answer]

[hidden-answer a="267724"]

	N	W	I	Q	Q'
a. $\sqrt{36} = 6$	X	X	X	X	
b. $\frac{8}{3} = 2.\bar{6}$				X	
c. $\sqrt{73}$					X
d. -6			X	X	
e. 3.2121121112...					X

[/hidden-answer]

## Try It

Classify each number as being a natural number (N), whole number (W), integer (I), rational number (Q), and/or irrational number (Q').

a.  $-\frac{35}{7}$

- b. 0
- c.  $\sqrt{169}$
- d.  $\sqrt{24}$
- e. 4.763763763...

[reveal-answer q="78713"]Show  
Solution[/reveal-answer]  
[hidden-answer a="78713"]

	N	W	I	Q , Q
a. $-\frac{35}{7}$			X	X
b. 0		X	X	X
c. $\sqrt{169}$	X	X	X	X
d. $\sqrt{24}$				X
e. 4.763763763...				X

[/hidden-answer]

## Performing Calculations Using the Order of

## Operations

When we multiply a number by itself, we square it or raise it to a power of 2. For example,  $4^2 = 4 \cdot 4 = 16$ . We can raise any number to any power. In general, the exponential notation  $a^n$  means that the number or variable  $a$  is used as a factor  $n$  times.

$$a^n = \overset{n \text{ factors}}{a \cdot a \cdot a \cdot \cdots \cdot a}$$

In this notation,  $a^n$  is read as the  $n$ th power of  $a$ , where  $a$  is called the base and  $n$  is called the exponent. A term in exponential notation may be part of a mathematical expression, which is a combination of numbers and operations. For example,

$$24 + 6 \cdot \frac{2}{3} - 4^2 \text{ is a mathematical expression.}$$

To evaluate a mathematical expression, we perform the various operations. However, we do not perform them in any random order. We use the order of operations. This is a sequence of rules for evaluating such expressions.

Recall that in mathematics we use parentheses ( ), brackets [ ], and braces { } to group numbers and expressions so that anything appearing within the symbols is treated as a unit. Additionally, fraction bars, radicals, and absolute value bars are treated as grouping symbols. When evaluating a mathematical expression, begin by simplifying expressions within grouping symbols.

The next step is to address any exponents or radicals. Afterward, perform multiplication and division from left to right and finally addition and subtraction from left to right.

Let's take a look at the expression provided.

$$24 + 6 \cdot \frac{2}{3} - 4^2$$

There are no grouping symbols, so we move on to exponents or

radicals. The number 4 is raised to a power of 2, so simplify  $4^2$  as 16.

$$24 + 6 \cdot \frac{2}{3} - 4^2$$

$$24 + 6 \cdot \frac{2}{3} - 16$$

Next, perform multiplication or division, left to right.

$$24 + 6 \cdot \frac{2}{3} - 16$$

$$24 + 4 - 16$$

Lastly, perform addition or subtraction, left to right.

$$24 + 4 - 16$$

$$28 - 16$$

$$12$$

Therefore,  $24 + 6 \cdot \frac{2}{3} - 4^2 = 12$ .

For some complicated expressions, several passes through the order of operations will be needed. For instance, there may be a radical expression inside parentheses that must be simplified before the parentheses are evaluated. Following the order of operations ensures that anyone simplifying the same mathematical expression will get the same result.

## *Order of Operations*

Operations in mathematical expressions must be evaluated in a systematic order, which can be simplified using the acronym **PEMDAS**:

**P**(arentheses)

**E**xponents)

**M**(ultiplication) and **D**(ivision)

**A**(ddition) and **S**(ubtraction)

### How To

**Given a mathematical expression, simplify it using the order of operations.**

1. Simplify any expressions within grouping symbols.
2. Simplify any expressions containing exponents or radicals.
3. Perform any multiplication and division in order, from left to right.
4. Perform any addition and subtraction in order, from left to right.

## Using the Order of Operations

Use the order of operations to evaluate each of the following expressions.

a.  $(3 \cdot 2)^2 - 4(6 + 2)$

b.  $\frac{5^2 - 4}{7} - \sqrt{11 - 2}$

c.  $6 - |5 - 8| + 3(4 - 1)$

d.  $\frac{14 - 3 \cdot 2}{2 \cdot 5 - 3^2}$

e.  $7(5 \cdot 3) - 2[(6 - 3) - 4^2] + 1$

[reveal-answer q="fs-id2553631"]Show

Solution[/reveal-answer]

[hidden-answer a="fs-id2553631"]

a. <

$$\begin{aligned}(3 \cdot 2)^2 - 4(6 + 2) &= (6)^2 - 4(8) && \text{Simplify parentheses} \\ &= 36 - 4(8) && \text{Simplify exponent} \\ &= 36 - 32 && \text{Simplify multiplication} \\ &= 4 && \text{Simplify subtraction}\end{aligned}$$

</li>

b. Note that in the first step, the radical is treated as a grouping symbol, like parentheses. Also, in the third step, the fraction bar is considered a grouping

symbol so the numerator is considered to be grouped.

c. 
$$\begin{aligned} 6 - |5 - 8| + 3(4 - 1) &= 6 - |-3| + 3(3) && \text{Simplify inside grouping symbols} \\ &= 6 - 3 + 3(3) && \text{Simplify absolute value} \\ &= 6 - 3 + 9 && \text{Simplify multiplication} \\ &= 3 + 9 && \text{Simplify subtraction} \\ &= 12 && \text{Simplify addition} \end{aligned}$$

d. In this example, the fraction bar separates the numerator and denominator, which we simplify separately until the last step.

e. 
$$\begin{aligned} 7(5 \cdot 3) - 2[(6 - 3) - 4^2] + 1 &= 7(15) - 2[(3) - 4^2] + 1 && \text{Simplify inside parentheses} \\ &= 7(15) - 2(3 - 16) + 1 && \text{Simplify exponent} \\ &= 7(15) - 2(-13) + 1 && \text{Subtract} \\ &= 105 + 26 + 1 && \text{Multiply} \\ &= 132 && \text{Add} \end{aligned}$$

[/hidden-answer]



## Try It

Use the order of operations to evaluate each of the following expressions.

- a.  $\sqrt{5^2 - 4^2} + 7(5 - 4)^2$
- b.  $1 + \frac{7 \cdot 5 - 8 \cdot 4}{9 - 6}$
- c.  $|1.8 - 4.3| + 0.4\sqrt{15 + 10}$
- d.  $\frac{1}{2}[5 \cdot 3^2 - 7^2] + \frac{1}{3} \cdot 9^2$
- e.  $[(3 - 8)^2 - 4] - (3 - 8)$

[reveal-answer q="fs-id4295970"]Show  
Solution[/reveal-answer]  
[hidden-answer a="fs-id4295970"]

- a. 10
- b. 2
- c. 4.5
- d. 25
- e. 26

[/hidden-answer]

## Using Properties of Real Numbers

For some activities we perform, the order of certain operations does not matter, but the order of other operations does. For example, it does not make a difference if we put on the right shoe before the left or vice-versa. However, it does matter whether we put on shoes or socks first. The same thing is true for operations in mathematics.

### *Commutative Properties*

The commutative property of addition states that numbers may be added in any order without affecting the sum.

$$a + b = b + a$$

We can better see this relationship when using real numbers.

$$(-2) + 7 = 5 \quad \text{and} \quad 7 + (-2) = 5$$

Similarly, the commutative property of multiplication states that numbers may be multiplied in any order without affecting the product.

$$a \cdot b = b \cdot a$$

Again, consider an example with real numbers.

$$(-11) \cdot (-4) = 44 \quad \text{and} \quad (-4) \cdot (-11) = 44$$

It is important to note that neither subtraction nor division is commutative. For example,  $17 - 5$  is not the same as  $5 - 17$ . Similarly,  $20 \div 5 \neq 5 \div 20$ .

### *Associative Properties*

The associative property of multiplication tells us that it does not

matter how we group numbers when multiplying. We can move the grouping symbols to make the calculation easier, and the product remains the same.

$$a(bc) = (ab)c$$

Consider this example.

$$(3 \cdot 4) \cdot 5 = 60 \quad \text{and} \quad 3 \cdot (4 \cdot 5) = 60$$

The associative property of addition tells us that numbers may be grouped differently without affecting the sum.

$$a + (b + c) = (a + b) + c$$

This property can be especially helpful when dealing with negative integers. Consider this example.

$$[15 + (-9)] + 23 = 29 \quad \text{and} \quad 15 + [(-9) + 23] = 29$$

Are subtraction and division associative? Review these examples.

$$\begin{array}{rclcl} 8 - (3 - 15) & \stackrel{?}{=} & (8 - 3) - 15 & 64 \div (8 \div 4) & \stackrel{?}{=} & (64 \div 8) \div 4 \\ 8 - (-12) & = & 5 - 15 & 64 \div 2 & \stackrel{?}{=} & 8 \div 4 \\ 20 & \neq & 20 - 10 & 32 & \neq & 2 \end{array}$$

As we can see, neither subtraction nor division is associative.

### *Distributive Property*

The distributive property states that the product of a factor times a sum is the sum of the factor times each term in the sum.

$$a \cdot (b + c) = a \cdot b + a \cdot c$$

This property combines both addition and multiplication (and is the only property to do so). Let us consider an example.

$$\begin{array}{rcl} 4 \cdot [12 + (-7)] & = & 4 \cdot 12 + 4 \cdot (-7) \\ & = & 48 + (-28) \\ & = & 20 \end{array}$$

Note that 4 is outside the grouping symbols, so we distribute the 4 by multiplying it by 12, multiplying it by  $-7$ , and adding the products.

To be more precise when describing this property, we say that multiplication distributes over addition. The reverse is not true, as we can see in this example.

$$6 + (3 \cdot 5) \stackrel{?}{=} (6 + 3) \cdot (6 + 5)$$

$$6 + (15) \stackrel{?}{=} (9) \cdot (11)$$

$$21 \neq 99$$

A special case of the distributive property occurs when a sum of terms is subtracted.

$$a - b = a + (-b)$$

For example, consider the difference  $12 - (5 + 3)$ . We can rewrite the difference of the two terms 12 and  $(5 + 3)$  by turning the subtraction expression into addition of the opposite. So instead of subtracting  $(5 + 3)$ , we add the opposite.

$$12 + (-1) \cdot (5 + 3)$$

Now, distribute  $-1$  and simplify the result.

$$\begin{aligned} 12 - (5 + 3) &= 12 + (-1) \cdot (5 + 3) \\ &= 12 + [(-1) \cdot 5 + (-1) \cdot 3] \\ &= 12 + (-8) \\ &= 4 \end{aligned}$$

This seems like a lot of trouble for a simple sum, but it illustrates a powerful result that will be useful once we introduce algebraic terms. To subtract a sum of terms, change the sign of each term and add the results. With this in mind, we can rewrite the last example.

$$\begin{aligned}
 12 - (5 + 3) &= 12 + (-5 - 3) \\
 &= 12 + (-8) \\
 &= 4
 \end{aligned}$$

### *Identity Properties*

The identity property of addition states that there is a unique number, called the additive identity (0) that, when added to a number, results in the original number.

$$a + 0 = a$$

The identity property of multiplication states that there is a unique number, called the multiplicative identity (1) that, when multiplied by a number, results in the original number.

$$a \cdot 1 = a$$

For example, we have  $(-6) + 0 = -6$  and  $23 \cdot 1 = 23$ .

There are no exceptions for these properties; they work for every real number, including 0 and 1.

### *Inverse Properties*

The inverse property of addition states that, for every real number  $a$ , there is a unique number, called the additive inverse (or opposite), denoted  $-a$ , that, when added to the original number, results in the additive identity, 0.

$$a + (-a) = 0$$

For example, if  $a = -8$ , the additive inverse is 8, since  $(-8) + 8 = 0$ .

The inverse property of multiplication holds for all real numbers except 0 because the reciprocal of 0 is not defined. The property

states that, for every real number  $a$ , there is a unique number, called the multiplicative inverse (or reciprocal), denoted  $\frac{1}{a}$ , that, when multiplied by the original number, results in the multiplicative identity, 1.

$$a \cdot \frac{1}{a} = 1$$

For example, if  $a = -\frac{2}{3}$ , the reciprocal, denoted  $\frac{1}{a}$ , is  $-\frac{3}{2}$

because

$$a \cdot \frac{1}{a} = \left(-\frac{2}{3}\right) \cdot \left(-\frac{3}{2}\right) = 1$$

### *Properties of Real Numbers*

The following properties hold for real numbers  $a$ ,  $b$ , and  $c$ .

	Addition	Multiplication
<b>Commutative Property</b>	$a + b = b + a$	$a \cdot b = b \cdot a$
<b>Associative Property</b>	$a + (b + c) = (a + b) + c$	$a(bc) = (ab)c$
<b>Distributive Property</b>	$a \cdot (b + c) = a \cdot b + a \cdot c$	
<b>Identity Property</b>	<p>There exists a unique real number called the additive identity, 0, such that, for any real number <math>a</math></p> $a + 0 = a$	<p>There exists a unique real number called the multiplicative identity, 1, such that, for any real number <math>a</math></p> $a \cdot 1 = a$
<b>Inverse Property</b>	<p>Every real number <math>a</math> has an additive inverse, or opposite, denoted <math>-a</math>, such that</p> $a + (-a) = 0$	<p>Every nonzero real number <math>a</math> has a multiplicative inverse, or reciprocal, denoted <math>\frac{1}{a}</math>, such that</p> $a \cdot \left(\frac{1}{a}\right) = 1$

## Using Properties of Real Numbers

Use the properties of real numbers to rewrite and simplify each expression. State which properties apply.

a.  $3 \cdot 6 + 3 \cdot 4$

- b.  $(5 + 8) + (-8)$   
 c.  $6 - (15 + 9)$   
 d.  $\frac{4}{7} \cdot \left(\frac{2}{3} \cdot \frac{7}{4}\right)$   
 e.  $100 \cdot [0.75 + (-2.38)]$

[reveal-answer q="2286347"]Show  
 Solution[/reveal-answer][hidden-answer  
 a="2286347"]

a.	$3 \cdot 6 + 3 \cdot 4$	=	$3 \cdot (6 + 4)$	Distributive property
		=	$3 \cdot 10$	Simplify
		=	30	Simplify
b.	$(5 + 8) + (-8)$	=	$5 + [8 + (-8)]$	Associative property of addition
		=	$5 + 0$	Inverse property of addition
		=	5	Identity property of addition
c.	$6 - (15 + 9)$	=	$6 + [(-15) + (-9)]$	Distributive property
		=	$6 + (-24)$	Simplify
		=	-18	Simplify
d.	$\frac{4}{7} \cdot \left(\frac{2}{3} \cdot \frac{7}{4}\right)$	=	$\frac{4}{7} \cdot \left(\frac{7}{4} \cdot \frac{2}{3}\right)$	Commutative property of multiplication
		=	$\left(\frac{4}{7} \cdot \frac{7}{4}\right) \cdot \frac{2}{3}$	Associative property of multiplication
		=	$1 \cdot \frac{2}{3}$	Inverse property of multiplication
		=	$\frac{2}{3}$	Identity property of multiplication
e.	$100 \cdot [0.75 + (-2.38)]$	=	$100 \cdot 0.75 + 100 \cdot (-2.38)$	Distributive property
		=	$75 + (-238)$	Simplify
		=	-163	Simplify

[/hidden-answer]



## Try It

Use the properties of real numbers to rewrite and simplify each expression. State which properties apply.

a.  $\left(-\frac{23}{5}\right) \cdot \left[11 \cdot \left(-\frac{5}{23}\right)\right]$

b.  $5 \cdot (6.2 + 0.4)$

c.  $18 - (7 - 15)$

d.  $\frac{17}{18} + \left[\frac{4}{9} + \left(-\frac{17}{18}\right)\right]$

e.  $6 \cdot (-3) + 6 \cdot 3$

[reveal-answer q="2821090"]Show  
Solution[/reveal-answer][hidden-answer  
a="2821090"]

- a. 11, commutative property of multiplication, associative property of multiplication, inverse property of multiplication, identity property of multiplication;
- b. 33, distributive property;

- c. 26, distributive property;
  - d.  $\frac{4}{9}$ , commutative property of addition, associative property of addition, inverse property of addition, identity property of addition;
  - e. 0, distributive property, inverse property of addition, identity property of addition
- [/hidden-answer]

## *Evaluating Algebraic Expressions*

So far, the mathematical expressions we have seen have involved real numbers only. In mathematics, we may see expressions such as  $x + 5$ ,  $\frac{4}{3}\pi r^3$ , or  $\sqrt{2m^3n^2}$ . In the expression  $x + 5$ , 5 is called a constant because it does not vary and  $x$  is called a variable because it does. (In naming the variable, ignore any exponents or radicals containing the variable.) An algebraic expression is a collection of constants and variables joined together by the algebraic operations of addition, subtraction, multiplication, and division.

We have already seen some real number examples of exponential

notation, a shorthand method of writing products of the same factor. When variables are used, the constants and variables are treated the same way.

$$\begin{array}{ll} (-3)^5 = (-3) \cdot (-3) \cdot (-3) \cdot (-3) \cdot (-3) & x^5 = x \cdot x \cdot x \cdot x \cdot x \\ (2 \cdot 7)^3 = (2 \cdot 7) \cdot (2 \cdot 7) \cdot (2 \cdot 7) & (yz)^3 = (yz) \cdot (yz) \cdot (yz) \end{array}$$

In each case, the exponent tells us how many factors of the base to use, whether the base consists of constants or variables.

Any variable in an algebraic expression may take on or be assigned different values. When that happens, the value of the algebraic expression changes. To evaluate an algebraic expression means to determine the value of the expression for a given value of each variable in the expression. Replace each variable in the expression with the given value, then simplify the resulting expression using the order of operations. If the algebraic expression contains more than one variable, replace each variable with its assigned value and simplify the expression as before.

### *Describing Algebraic Expressions*

List the constants and variables for each algebraic expression.

a.  $x + 5$

b.  $\frac{4}{3}\pi r^3$

c.  $\sqrt{2m^3n^2}$

[reveal-answer q="85354"]Show  
Solution[/reveal-answer]  
[hidden-answer a="85354"]

	Constants	Variables
a. $x + 5$	5	$x$
b. $\frac{4}{3}\pi r^3$	$\frac{4}{3}, \pi$	$r$
c. $\sqrt{2m^3n^2}$	2	$m, n$

[/hidden-answer]

## Try It

List the constants and variables for each algebraic expression.

- $2\pi r(r + h)$
- $2(L + W)$
- $4y^3 + y$

[reveal-answer q="779207"]Show  
Solution[/reveal-answer]  
[hidden-answer a="779207"]

	Constant s	Variable s
a. $2\pi r(r + h)$	$2, \pi$	$r, h$
b. $2(L + W)$	2	L, W
c. $4y^3 + y$	4	$y$

[/hidden-answer]

## Evaluating an Algebraic Expression at Different Values

Evaluate the expression  $2x - 7$  for each value for  $x$ .

- a.  $x = 0$
- b.  $x = 1$

c.  $x = \frac{1}{2}$

d.  $x = -4$

[reveal-answer q="702997"]Show

Solution[/reveal-answer]

[hidden-answer a="702997"]

a. Substitute 0 for  $x$ .

$$\begin{aligned} 2x - 7 &= 2(0) - 7 \\ &= 0 - 7 \\ &= -7 \end{aligned}$$

b. Substitute 1 for  $x$ .

$$\begin{aligned} 2x - 7 &= 2(1) - 7 \\ &= 2 - 7 \\ &= -5 \end{aligned}$$

c. Substitute  $\frac{1}{2}$  for  $x$ .

$$\begin{aligned} 2x - 7 &= 2\left(\frac{1}{2}\right) - 7 \\ &= 1 - 7 \\ &= -6 \end{aligned}$$

d. Substitute  $-4$  for  $x$ .

$$\begin{aligned}
 2x - 7 &= 2(-4) - 7 \\
 &= -8 - 7 \\
 &= -15
 \end{aligned}$$

[/hidden-answer]

Try It

Evaluate the expression  $11 - 3y$  for each value for  $y$ .

- a.  $y = 2$
- b.  $y = 0$
- c.  $y = \frac{2}{3}$
- d.  $y = -5$

[reveal-answer q="2261567"]Show  
Solution[/reveal-answer][hidden-answer  
a="2261567"]

- a. 5;
- b. 11;
- c. 9;
- d. 26

[/hidden-answer]

## *Evaluating Algebraic Expressions*

Evaluate each expression for the given values.

- a.  $x + 5$  for  $x = -5$
- b.  $\frac{t}{2t - 1}$  for  $t = 10$
- c.  $\frac{4}{3}\pi r^3$  for  $r = 5$
- d.  $a + ab + b$  for  $a = 11, b = -8$
- e.  $\sqrt{2m^3n^2}$  for  $m = 2, n = 3$



[reveal-answer q="60618"]Show  
 Solution[/reveal-answer]  
 [hidden-answer a="60618"]

- a. Substitute  $-5$  for  $x$ .

$$\begin{aligned} x + 5 &= (-5) + 5 \\ &= 0 \end{aligned}$$

- b. Substitute 10 for  $t$ .

$$\begin{aligned} \frac{t}{2t-1} &= \frac{(10)}{2(10)-1} \\ &= \frac{10}{20-1} \\ &= \frac{10}{19} \end{aligned}$$

- c. Substitute 5 for  $r$ .

$$\begin{aligned} \frac{4}{3}\pi r^3 &= \frac{4}{3}\pi(5)^3 \\ &= \frac{4}{3}\pi(125) \\ &= \frac{500}{3}\pi \end{aligned}$$

- d. Substitute 11 for  $a$  and  $-8$  for  $b$ .

$$\begin{aligned} a + ab + b &= (11) + (11)(-8) + (-8) \\ &= 11 - 88 - 8 \\ &= -85 \end{aligned}$$

- e. Substitute 2 for  $m$  and 3 for  $n$ .

$$\begin{aligned}
 \sqrt{2m^3n^2} &= \sqrt{2(2)^3(3)^2} \\
 &= \sqrt{2(8)(9)} \\
 &= \sqrt{144} \\
 &= 12
 \end{aligned}$$

[/hidden-answer]

### Try It

Evaluate each expression for the given values.

- $\frac{y+3}{y-3}$  for  $y = 5$
- $7 - 2t$  for  $t = -2$
- $\frac{1}{3}\pi r^2$  for  $r = 11$
- $(p^2q)^3$  for  $p = -2, q = 3$

e.  $4(m - n) - 5(n - m)$  for  
 $m = \frac{2}{3}, n = \frac{1}{3}$

[reveal-answer q="2808478"]Show  
 Solution[/reveal-answer][hidden-answer  
 a="2808478"]

- a. 4;
- b. 11;
- c.  $\frac{121}{3}\pi$ ;
- d. 1728;
- e. 3

[/hidden-answer]

## Formulas

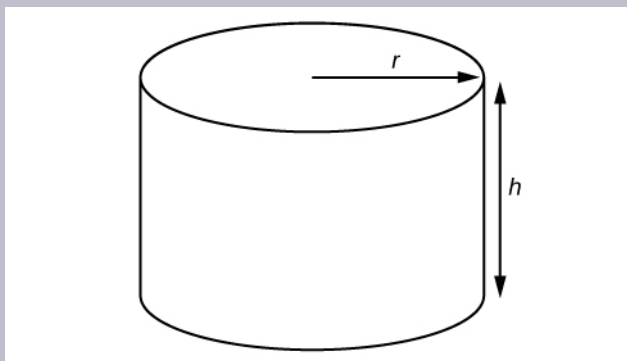
An equation is a mathematical statement indicating that two expressions are equal. The expressions can be numerical or algebraic. The equation is not inherently true or false, but only a proposition. The values that make the equation true, the solutions, are found using the properties of real numbers and other results. For example, the equation  $2x + 1 = 7$  has the unique solution

of 3 **Formula does not parse** because when we substitute 3 for  $x$  in the equation, we obtain the true statement  $2(3) + 1 = 7$ .

A formula is an equation expressing a relationship between constant and variable quantities. Very often, the equation is a means of finding the value of one quantity (often a single variable) in terms of another or other quantities. One of the most common examples is the formula for finding the area  $A$  of a circle in terms of the radius  $r$  of the circle:  $A = \pi r^2$ . For any value of  $r$ , the area  $A$  can be found by evaluating the expression  $\pi r^2$ .

### *Using a Formula*

A right circular cylinder with radius  $r$  and height  $h$  has the surface area  $S$  (in square units) given by the formula  $S = 2\pi r(r + h)$ . See (Figure). Find the surface area of a cylinder with radius 6 in. and height 9 in. Leave the answer in terms of  $\pi$ .



Right circular cylinder

[reveal-answer q="fs-id2182246"]Show  
Solution[/reveal-answer]

[hidden-answer a="fs-id2182246"]

Evaluate the expression  $2\pi r(r + h)$  for  
 $r = 6$  and  $h = 9$ .

$$\begin{aligned} S &= 2\pi r(r + h) \\ &= 2\pi(6)[(6) + (9)] \\ &= 2\pi(6)(15) \\ &= 180\pi \end{aligned}$$

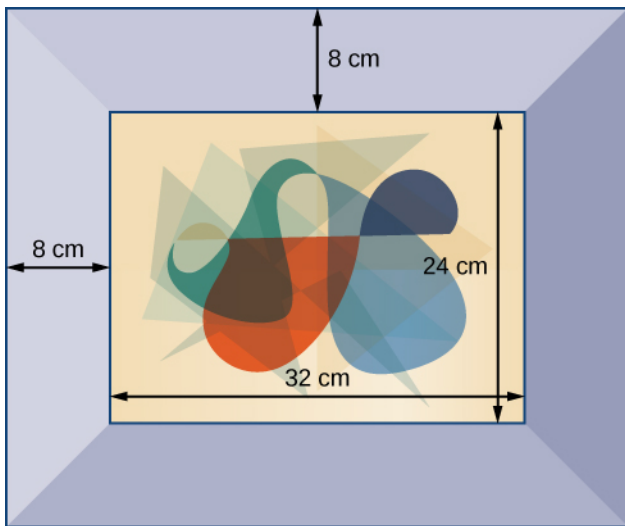
The surface area is  $180\pi$  square  
inches.[/hidden-answer]

## Try It

A photograph with length  $L$  and width  $W$  is placed in a matte of width 8 centimeters (cm). The area of the matte (in square centimeters, or  $\text{cm}^2$ ) is found to be

$$A = (L + 16)(W + 16) - L \cdot W.$$

See (Figure). Find the area of a matte for a photograph with length 32 cm and width 24 cm.



[reveal-answer q="fs-id2736076"]Show  
Solution[/reveal-answer]  
[hidden-answer a="fs-id2736076"]  
1,152 cm<sup>2</sup>  
[/hidden-answer]

### *Simplifying Algebraic Expressions*

Sometimes we can simplify an algebraic expression to make it easier to evaluate or to use in some other way. To do so, we use the properties of real numbers. We can use the same properties in formulas because they contain algebraic expressions.

### *Simplifying Algebraic Expressions*

Simplify each algebraic expression.

- a.  $3x - 2y + x - 3y - 7$
- b.  $2r - 5(3 - r) + 4$
- c.  $\left(4t - \frac{5}{4}s\right) - \left(\frac{2}{3}t + 2s\right)$

d.  $2mn - 5m + 3mn + n$

[reveal-answer q="2134630"]Show  
Solution[/reveal-answer][hidden-answer  
a="2134630"]

a.	$3x - 2y + x - 3y - 7 = 4x - 5y - 7$	Commutative property of addition Simplify
b.	$2r - 5(3 - r) + 4 = 2r - 15 + 5r + 4$	Distributive property
	$= 2r + 5r - 15 + 4$	Commutative property of addition
	$= 7r - 11$	Simplify
	$\left(4t - \frac{5}{4}s\right) - \left(\frac{2}{3}t + 2s\right) = 4t - \frac{5}{4}s - \frac{2}{3}t - 2s$	Distributive property
c.	$= 4t - \frac{2}{3}t - \frac{5}{4}s - 2s$	Commutative property of addition
	$= \frac{10}{3}t - \frac{13}{4}s$	Simplify
d.	$2mn - 5m + 3mn + n = 2mn + 3mn - 5m + n$	Commutative property of addition
	$= 5mn - 5m + n$	Simplify

[/hidden-answer]

Try It

Simplify each algebraic expression.

a.  $\frac{2}{3}y - 2\left(\frac{4}{3}y + z\right)$

b.  $\frac{5}{t} - 2 - \frac{3}{t} + 1$



- c.  $4p(q - 1) + q(1 - p)$   
 d.  $9r - (s + 2r) + (6 - s)$

[reveal-answer q="2369007"]Show  
 Solution[/reveal-answer][hidden-answer  
 a="2369007"]

- a.  $-2y - 2z$  or  $-2(y + z)$  ;  
 b.  $\frac{2}{t} - 1$  ;  
 c.  $3pq - 4p + q$  ;  
 d.  $7r - 2s + 6$

[/hidden-answer]

## *Simplifying a Formula*

A rectangle with length  $L$  and width  $W$  has a perimeter  $P$  given by  $P = L + W + L + W$ . Simplify this expression.

[reveal-answer q="1605340"]Show  
 Solution[/reveal-answer][hidden-answer  
 a="1605340"]

$$P = L + W + L + W$$

$$P = L + L + W + W$$

$$P = 2L + 2W$$

$$P = 2(L + W)$$

Commutative property of addition

Simplify

Distributive property

[/hidden-answer]

## Try It

If the amount  $P$  is deposited into an account paying simple interest  $r$  for time  $t$ , the total value of the deposit  $A$  is given by  $A = P + Prt$ . Simplify the expression. (This formula will be explored in more detail later in the course.)

[reveal-answer q="fs-id3166246"]Show  
 Solution[/reveal-answer]

[hidden-answer a="fs-id3166246"]

$$A = P(1 + rt)$$

[/hidden-answer]

Access these online resources for additional instruction and practice with real numbers.

- Simplify an Expression
- Evaluate an Expression1
- Evaluate an Expression2

### *Key Concepts*

- Rational numbers may be written as fractions or terminating or repeating decimals. See (Figure) and (Figure).
- Determine whether a number is rational or irrational by writing it as a decimal. See (Figure).
- The rational numbers and irrational numbers make up the set of real numbers. See (Figure). A number can be classified as natural, whole, integer, rational, or irrational. See (Figure).
- The order of operations is used to evaluate

expressions. See (Figure).

- The real numbers under the operations of addition and multiplication obey basic rules, known as the properties of real numbers. These are the commutative properties, the associative properties, the distributive property, the identity properties, and the inverse properties. See (Figure).
- Algebraic expressions are composed of constants and variables that are combined using addition, subtraction, multiplication, and division. See (Figure). They take on a numerical value when evaluated by replacing variables with constants. See (Figure), (Figure), and (Figure)
- Formulas are equations in which one quantity is represented in terms of other quantities. They may be simplified or evaluated as any mathematical expression. See (Figure) and (Figure).

### *Verbal*

Is  $\sqrt{2}$  an example of a rational terminating, rational repeating, or irrational number? Tell why it fits that category.

[reveal-answer q="fs-id1473935"]Show  
Solution[/reveal-answer]

[hidden-answer a="fs-id1473935"]

irrational number. The square root of two does not terminate, and it does not repeat a pattern. It cannot be written as a quotient of two integers, so it is irrational.

[/hidden-answer]

What is the order of operations? What acronym is used to describe the order of operations, and what does it stand for?

What do the Associative Properties allow us to do when following the order of operations? Explain your answer.

[reveal-answer q="fs-id1571749"]Show  
Solution[/reveal-answer]

[hidden-answer a="fs-id1571749"]

The Associative Properties state that the sum or product of multiple numbers can be grouped differently without affecting the result. This is because the same operation is performed (either addition or subtraction), so the terms can be re-ordered.

[/hidden-answer]

### *Numeric*

For the following exercises, simplify the given expression.

$$10 + 2 \times (5 - 3)$$

$$6 \div 2 - (81 \div 3^2)$$

[reveal-answer q="fs-id1388089"]Show

Solution[/reveal-answer]

[hidden-answer a="fs-id1388089"]

$$-6$$

[/hidden-answer]

$$18 + (6 - 8)^3$$

$$-2 \times \left[ 16 \div (8 - 4)^2 \right]^2$$

[reveal-answer q="fs-id2584631"]Show  
Solution[/reveal-answer]  
[hidden-answer a="fs-id2584631"]  
     $-2$   
[/hidden-answer]

$$4 - 6 + 2 \times 7$$

$$3(5 - 8)$$

[reveal-answer q="fs-id1601064"]Show  
Solution[/reveal-answer]  
[hidden-answer a="fs-id1601064"]  
     $-9$   
[/hidden-answer]

$$4 + 6 - 10 \div 2$$

$$12 \div (36 \div 9) + 6$$

[reveal-answer q="fs-id2555788"]Show  
Solution[/reveal-answer]

[hidden-answer a="fs-id2555788"]

9

[/hidden-answer]

$$(4 + 5)^2 \div 3$$

$$3 - 12 \times 2 + 19$$

[reveal-answer q="fs-id2027958"]Show

Solution[/reveal-answer]

[hidden-answer a="fs-id2027958"]

-2

[/hidden-answer]

$$2 + 8 \times 7 \div 4$$

$$5 + (6 + 4) - 11$$

[reveal-answer q="fs-id1452728"]Show

Solution[/reveal-answer]

[hidden-answer a="fs-id1452728"]



4

[/hidden-answer]

$$9 - 18 \div 3^2$$

$$14 \times 3 \div 7 - 6$$

[reveal-answer q="fs-id3166125"]Show

Solution[/reveal-answer]

[hidden-answer a="fs-id3166125"]

0

[/hidden-answer]

$$9 - (3 + 11) \times 2$$

$$6 + 2 \times 2 - 1$$

[reveal-answer q="fs-id2616438"]Show

Solution[/reveal-answer]

[hidden-answer a="fs-id2616438"]

9

[/hidden-answer]

$$64 \div (8 + 4 \times 2)$$

$$9 + 4(2^2)$$

[reveal-answer q="fs-id1859161"]Show

Solution[/reveal-answer]

[hidden-answer a="fs-id1859161"]

25

[/hidden-answer]

$$(12 \div 3 \times 3)^2$$

$$25 \div 5^2 - 7$$

[reveal-answer q="fs-id1442482"]Show

Solution[/reveal-answer]

[hidden-answer a="fs-id1442482"]

−6

[/hidden-answer]

$$(15 - 7) \times (3 - 7)$$

$$2 \times 4 - 9(-1)$$

[reveal-answer q="fs-id1737156"]Show

Solution[/reveal-answer]

[hidden-answer a="fs-id1737156"]

17

[/hidden-answer]

$$4^2 - 25 \times \frac{1}{5}$$

$$12(3 - 1) \div 6$$

[reveal-answer q="fs-id2135191"]Show

Solution[/reveal-answer]

[hidden-answer a="fs-id2135191"]

4

[/hidden-answer]

## *Algebraic*

For the following exercises, solve for the variable.

$$8(x + 3) = 64$$

$$4y + 8 = 2y$$

[reveal-answer q="fs-id1673608"]Show  
Solution[/reveal-answer]  
[hidden-answer a="fs-id1673608"]  
     $-4$   
[/hidden-answer]

$$(11a + 3) - 18a = -4$$

$$4z - 2z(1 + 4) = 36$$

[reveal-answer q="fs-id1459984"]Show  
Solution[/reveal-answer]  
[hidden-answer a="fs-id1459984"]  
     $-6$   
[/hidden-answer]

$$4y(7 - 2)^2 = -200$$

$$-(2x)^2 + 1 = -3$$

[reveal-answer q="fs-id1673356"]Show  
Solution[/reveal-answer]  
[hidden-answer a="fs-id1673356"]  
 $\pm 1$   
[/hidden-answer]

$$8(2 + 4) - 15b = b$$

$$2(11c - 4) = 36$$

[reveal-answer q="fs-id1453899"]Show  
Solution[/reveal-answer]  
[hidden-answer a="fs-id1453899"]  
2  
[/hidden-answer]

$$4(3 - 1)x = 4$$

$$\frac{1}{4}(8w - 4^2) = 0$$

[reveal-answer q="fs-id2380948"]Show  
Solution[/reveal-answer]  
[hidden-answer a="fs-id2380948"]  
 $2$   
[/hidden-answer]

For the following exercises, simplify the expression.

$$4x + x(13 - 7)$$

$$2y - (4)^2y - 11$$

[reveal-answer q="fs-id2983288"]Show  
Solution[/reveal-answer]  
[hidden-answer a="fs-id2983288"]  
 $-14y - 11$   
[/hidden-answer]

$$\frac{a}{2^3}(64) - 12a \div 6$$

$$8b - 4b(3) + 1$$

[reveal-answer q="fs-id2363657"]Show  
Solution[/reveal-answer]  
[hidden-answer a="fs-id2363657"]  
 $-4b + 1$   
[/hidden-answer]

$$5l \div 3l \times (9 - 6)$$

$$7z - 3 + z \times 6^2$$

[reveal-answer q="fs-id2365286"]Show  
Solution[/reveal-answer]  
[hidden-answer a="fs-id2365286"]  
 $43z - 3$   
[/hidden-answer]

$$4 \times 3 + 18x \div 9 - 12$$

$$9(y + 8) - 27$$

[reveal-answer q="fs-id2278383"]Show  
Solution[/reveal-answer]

[hidden-answer a="fs-id2278383"]

$$9y + 45$$

[/hidden-answer]

$$\left(\frac{9}{6}t - 4\right)^2$$

$$6 + 12b - 3 \times 6b$$

[reveal-answer q="fs-id2555553"]Show

Solution[/reveal-answer]

[hidden-answer a="fs-id2555553"]

$$-6b + 6$$

[/hidden-answer]

$$18y - 2(1 + 7y)$$

$$\left(\frac{4}{9}\right)^2 \times 27x$$

[reveal-answer q="fs-id2750495"]Show



Solution[/reveal-answer]

[hidden-answer a="fs-id2750495"]

$$\frac{16x}{3}$$

[/hidden-answer]

$$8(3 - m) + 1(-8)$$

$$9x + 4x(2 + 3) - 4(2x + 3x)$$

[reveal-answer q="fs-id2515354"]Show

Solution[/reveal-answer]

[hidden-answer a="fs-id2515354"]

$$9x$$

[/hidden-answer]

$$5^2 - 4(3x)$$

### *Real-World Applications*

For the following exercises, consider this scenario: Fred earns \$40 mowing lawns. He spends \$10 on mp3s, puts half

of what is left in a savings account, and gets another \$5 for washing his neighbor's car.

Write the expression that represents the number of dollars Fred keeps (and does not put in his savings account). Remember the order of operations.

[reveal-answer q="fs-id2404933"]Show

Solution[/reveal-answer]

[hidden-answer a="fs-id2404933"]

$$\frac{1}{2}(40 - 10) + 5$$

[/hidden-answer]

How much money does Fred keep?

For the following exercises, solve the given problem.

According to the U.S. Mint, the diameter of a quarter is 0.955 inches. The circumference of the quarter would be the diameter multiplied by  $\pi$ . Is the circumference of a quarter a whole number, a rational number, or an irrational number?

[reveal-answer q="fs-id1690221"]Show

Solution[/reveal-answer]

[hidden-answer a="fs-id1690221"]

irrational number

[/hidden-answer]

Jessica and her roommate, Adriana, have decided to share a change jar for joint expenses. Jessica put her loose change in the jar first, and then Adriana put her change in the jar. We know that it does not matter in which order the change was added to the jar. What property of addition describes this fact?

For the following exercises, consider this scenario: There is a mound of  $g$  pounds of gravel in a quarry. Throughout the day, 400 pounds of gravel is added to the mound. Two orders of 600 pounds are sold and the gravel is removed from the mound. At the end of the day, the mound has 1,200 pounds of gravel.

Write the equation that describes the situation.

[reveal-answer q="fs-id1453382"]Show

Solution[/reveal-answer]

[hidden-answer a="fs-id1453382"]

$$g + 400 - 2(600) = 1200$$

[/hidden-answer]

Solve for  $g$ .

For the following exercise, solve the given problem.

Ramon runs the marketing department at his company. His department gets a budget every year, and every year, he must spend the entire budget without going over. If he spends less than the budget, then his department gets a smaller budget the following year. At the beginning of this year, Ramon got \$2.5 million for the annual marketing budget. He must spend the budget such that  $2,500,000 - x = 0$ . What property of addition tells us what the value of  $x$  must be?

[reveal-answer q="fs-id3581441"]Show  
Solution[/reveal-answer]  
[hidden-answer a="fs-id3581441"]  
inverse property of addition  
[/hidden-answer]

### *Technology*

For the following exercises, use a graphing calculator to solve for  $x$ . Round the answers to the nearest hundredth.

$$0.5(12.3)^2 - 48x = \frac{3}{5}$$

$$(0.25 - 0.75)^2 x - 7.2 = 9.9$$

```
[reveal-answer q="fs-id1720804"]Show  
Solution[/reveal-answer]  
[hidden-answer a="fs-id1720804"]  
68.4  
[/hidden-answer]
```

### *Extensions*

If a whole number is not a natural number, what must the number be?

Determine whether the statement is true or false: The multiplicative inverse of a rational number is also rational.

```
[reveal-answer q="fs-id2806507"]Show  
Solution[/reveal-answer]  
[hidden-answer a="fs-id2806507"]  
true  
[/hidden-answer]
```

Determine whether the statement is true or false: The product of a rational and irrational number is always irrational.

Determine whether the simplified expression is rational or irrational:  $\sqrt{-18 - 4(5)(-1)}$ .

[reveal-answer q="fs-id1440420"]Show  
Solution[/reveal-answer]  
[hidden-answer a="fs-id1440420"]  
irrational  
[/hidden-answer]

Determine whether the simplified expression is rational or irrational:  $\sqrt{-16 + 4(5) + 5}$ .

The division of two whole numbers will always result in what type of number?

[reveal-answer q="fs-id2608461"]Show  
Solution[/reveal-answer]  
[hidden-answer a="fs-id2608461"]  
rational  
[/hidden-answer]

What property of real numbers would simplify the following expression:  $4 + 7(x - 1)$ ?

## Glossary

### **algebraic expression**

constants and variables combined using addition, subtraction, multiplication, and division

### **associative property of addition**

the sum of three numbers may be grouped differently without affecting the result; in symbols,

$$a + (b + c) = (a + b) + c$$

### **associative property of multiplication**

the product of three numbers may be grouped differently without affecting the result; in symbols,

$$a \cdot (b \cdot c) = (a \cdot b) \cdot c$$

### **base**

in exponential notation, the expression that is being multiplied

### **commutative property of addition**

two numbers may be added in either order without affecting the result; in symbols,

$$a + b = b + a$$

### **commutative property of multiplication**

two numbers may be multiplied in any order without affecting the result; in symbols,

$$a \cdot b = b \cdot a$$

### **constant**

a quantity that does not change value

### **distributive property**

the product of a factor times a sum is the sum of

the factor times each term in the sum; in symbols,

$$a \cdot (b + c) = a \cdot b + a \cdot c$$

**equation**

a mathematical statement indicating that two expressions are equal

**exponent**

in exponential notation, the raised number or variable that indicates how many times the base is being multiplied

**exponential notation**

a shorthand method of writing products of the same factor

**formula**

an equation expressing a relationship between constant and variable quantities

**identity property of addition**

there is a unique number, called the additive identity, 0, which, when added to a number, results in the original number; in symbols,  $a + 0 = a$

**identity property of multiplication**

there is a unique number, called the multiplicative identity, 1, which, when multiplied by a number, results in the original number; in symbols,

$$a \cdot 1 = a$$

**integers**

the set consisting of the natural numbers, their opposites, and 0:

$$\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$

**inverse property of addition**

for every real number  $a$ , there is a unique number, called the additive inverse (or opposite),



denoted  $-a$ , which, when added to the original number, results in the additive identity, 0; in symbols,  $a + (-a) = 0$

**inverse property of multiplication**

for every non-zero real number  $a$ , there is a unique number, called the multiplicative inverse (or reciprocal), denoted  $\frac{1}{a}$ , which, when multiplied by the original number, results in the multiplicative identity, 1; in symbols,  $a \cdot \frac{1}{a} = 1$

**irrational numbers**

the set of all numbers that are not rational; they cannot be written as either a terminating or repeating decimal; they cannot be expressed as a fraction of two integers

**natural numbers**

the set of counting numbers:  $\{1, 2, 3, \dots\}$

**order of operations**

a set of rules governing how mathematical expressions are to be evaluated, assigning priorities to operations

**rational numbers**

the set of all numbers of the form  $\frac{m}{n}$ , where  $m$  and  $n$  are integers and  $n \neq 0$ . Any rational number may be written as a fraction or a terminating or repeating decimal.

**real number line**

a horizontal line used to represent the real numbers. An arbitrary fixed point is chosen to

represent 0; positive numbers lie to the right of 0 and negative numbers to the left.

**real numbers**

the sets of rational numbers and irrational numbers taken together

**variable**

a quantity that may change value

**whole numbers**

the set consisting of 0 plus the natural numbers:

$\{0, 1, 2, 3, \dots\}$

## 2. Exponents and Scientific Notation

### *Learning Objectives*

In this section students will:

- Use the product rule of exponents.
- Use the quotient rule of exponents.
- Use the power rule of exponents.
- Use the zero exponent rule of exponents.
- Use the negative rule of exponents.
- Find the power of a product and a quotient.
- Simplify exponential expressions.
- Use scientific notation.

Mathematicians, scientists, and economists commonly encounter very large and very small numbers. But it may not be obvious how common such figures are in everyday life. For instance, a pixel is the smallest unit of light that can be perceived and recorded by a digital camera. A particular camera might record an image that is 2,048 pixels by 1,536 pixels, which is a very high resolution picture. It can also perceive a color depth (gradations in colors) of up to 48 bits per frame, and can shoot the equivalent of 24 frames per second. The maximum possible number of bits of information used to film a one-hour (3,600-second) digital film is then an extremely large number.

Using a calculator, we enter  $2,048 \times 1,536 \times 48 \times 24 \times 3,600$  and press ENTER. The calculator displays 1.304596316E13. What does this

mean? The “E13” portion of the result represents the exponent 13 of ten, so there are a maximum of approximately  $1.3 \times 10^{13}$  bits of data in that one-hour film. In this section, we review rules of exponents first and then apply them to calculations involving very large or small numbers.

## Using the Product Rule of Exponents

Consider the product  $x^3 \cdot x^4$ . Both terms have the same base,  $x$ , but they are raised to different exponents. Expand each expression, and then rewrite the resulting expression.

$$\begin{aligned} x^3 \cdot x^4 &= \overset{3 \text{ factors}}{x \cdot x \cdot x} \cdot \overset{4 \text{ factors}}{x \cdot x \cdot x \cdot x} \\ &= \overset{7 \text{ factors}}{x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x} \\ &= x^7 \end{aligned}$$

The result is that  $x^3 \cdot x^4 = x^{3+4} = x^7$ .

Notice that the exponent of the product is the sum of the exponents of the terms. In other words, when multiplying exponential expressions with the same base, we write the result with the common base and add the exponents. This is the *product rule of exponents*.

$$a^m \cdot a^n = a^{m+n}$$

Now consider an example with real numbers.

$$2^3 \cdot 2^4 = 2^{3+4} = 2^7$$

We can always check that this is true by simplifying each exponential expression. We find that  $2^3$  is 8,  $2^4$  is 16, and  $2^7$  is 128. The product  $8 \cdot 16$  equals 128, so the relationship is true. We can use the product rule of exponents to simplify expressions that are

a product of two numbers or expressions with the same base but different exponents.

### *The Product Rule of Exponents*

For any real number  $a$  and natural numbers  $m$  and  $n$ , the product rule of exponents states that

$$a^m \cdot a^n = a^{m+n}$$

### *Using the Product Rule*

Write each of the following products with a single base. Do not simplify further.

- a.  $t^5 \cdot t^3$
- b.  $(-3)^5 \cdot (-3)$
- c.  $x^2 \cdot x^5 \cdot x^3$

[reveal-answer q="fs-id1167339188489"]Show Solution[/reveal-answer]

[hidden-answer a="fs-id1167339188489"]

Use the product rule to simplify each expression.

- a.  $t^5 \cdot t^3 = t^{5+3} = t^8$
- b.  $(-3)^5 \cdot (-3) = (-3)^5 \cdot (-3)^1 = (-3)^{5+1} = (-3)^6$
- c.  $x^2 \cdot x^5 \cdot x^3$

At first, it may appear that we cannot simplify a product of three factors. However, using the associative property of multiplication, begin by simplifying the first two.

$$x^2 \cdot x^5 \cdot x^3 = (x^2 \cdot x^5) \cdot x^3 = (x^{2+5}) \cdot x^3 = x^7 \cdot x^3 = x^{7+3} = x^{10}$$

Notice we get the same result by adding the three exponents in one step.

$$x^2 \cdot x^5 \cdot x^3 = x^{2+5+3} = x^{10}$$

[/hidden-answer]

## Try It

Write each of the following products with a single base. Do not simplify further.

- a.  $k^6 \cdot k^9$

b.  $\left(\frac{2}{y}\right)^4 \cdot \left(\frac{2}{y}\right)$

c.  $t^3 \cdot t^6 \cdot t^5$

[reveal-answer q="fs-id1167339184743"]Show Solution[/reveal-answer]

[hidden-answer a="fs-id1167339184743"]

a.  $k^{15}$

b.  $\left(\frac{2}{y}\right)^5$

c.  $t^{14}$

[/hidden-answer]

## Using the Quotient Rule of Exponents

The *quotient rule of exponents* allows us to simplify an expression that divides two numbers with the same base but different exponents. In a similar way to the product rule, we can simplify

an expression such as  $\frac{y^m}{y^n}$ , where  $m > n$ . Consider the example

$\frac{y^9}{y^5}$ . Perform the division by canceling common factors.

$$\begin{aligned}\frac{y^9}{y^5} &= \frac{y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y}{y \cdot y \cdot y \cdot y \cdot y} \\ &= \frac{\overbrace{y \cdot y \cdot y \cdot y \cdot y}^{\text{)}y \cdot \text{)}y \cdot \text{)}y \cdot \text{)}y \cdot \text{)}y} \cdot y \cdot y \cdot y \cdot y}{\overbrace{y \cdot y \cdot y \cdot y \cdot y}^{\text{)}y \cdot \text{)}y \cdot \text{)}y \cdot \text{)}y \cdot \text{)}y}} \\ &= \frac{y \cdot y \cdot y \cdot y}{1} \\ &= y^4\end{aligned}$$

Notice that the exponent of the quotient is the difference between the exponents of the divisor and dividend.

$$\frac{a^m}{a^n} = a^{m-n}$$

In other words, when dividing exponential expressions with the same base, we write the result with the common base and subtract the exponents.

$$\frac{y^9}{y^5} = y^{9-5} = y^4$$

For the time being, we must be aware of the condition  $m > n$ . Otherwise, the difference  $m - n$  could be zero or negative. Those possibilities will be explored shortly. Also, instead of qualifying variables as nonzero each time, we will simplify matters and assume from here on that all variables represent nonzero real numbers.



## *The Quotient Rule of Exponents*

For any real number  $a$  and natural numbers  $m$  and  $n$ , such that  $m > n$ , the quotient rule of exponents states that

$$\frac{a^m}{a^n} = a^{m-n}$$

## *Using the Quotient Rule*

Write each of the following products with a single base. Do not simplify further.

a. 
$$\frac{(-2)^{14}}{(-2)^9}$$

b. 
$$\frac{t^{23}}{t^{15}}$$

c. 
$$\frac{(z\sqrt{2})^5}{z\sqrt{2}}$$

[reveal-answer q="fs-id1167339185892"]Show

Solution[/reveal-answer]

[hidden-answer a="fs-id1167339185892"]

Use the quotient rule to simplify each expression.

a. 
$$\frac{(-2)^{14}}{(-2)^9} = (-2)^{14-9} = (-2)^5$$

b. 
$$\frac{t^{23}}{t^{15}} = t^{23-15} = t^8$$

c. 
$$\frac{(z\sqrt{2})^5}{z\sqrt{2}} = (z\sqrt{2})^{5-1} = (z\sqrt{2})^4$$

[/hidden-answer]

## Try It

Write each of the following products with a single base. Do not simplify further.

a.  $\frac{s^{75}}{s^{68}}$

b.  $\frac{(-3)^6}{-3}$

c.  $\frac{(ef^2)^5}{(ef^2)^3}$

[reveal-answer q="fs-id1167339220838"]Show Solution[/reveal-answer]

[hidden-answer a="fs-id1167339220838"]

a.  $s^7$

b.  $(-3)^5$

c.  $(ef^2)^2$

[/hidden-answer]

## Using the Power Rule of Exponents

Suppose an exponential expression is raised to some power. Can we simplify the result? Yes. To do this, we use the *power rule of*

exponents. Consider the expression  $(x^2)^3$ . The expression inside the parentheses is multiplied twice because it has an exponent of 2. Then the result is multiplied three times because the entire expression has an exponent of 3.

$$\begin{aligned}
 (x^2)^3 &= \overset{\text{3 factors}}{(x^2) \cdot (x^2) \cdot (x^2)} \\
 &= \left( \overset{\text{2 factors}}{\underbrace{x \cdot x}} \right) \cdot \left( \overset{\text{2 factors}}{\underbrace{x \cdot x}} \right) \cdot \left( \overset{\text{2 factors}}{\underbrace{x \cdot x}} \right) \\
 &= x \cdot x \cdot x \cdot x \cdot x \cdot x \\
 &= x^6
 \end{aligned}$$

The exponent of the answer is the product of the exponents:  $(x^2)^3 = x^{2 \cdot 3} = x^6$ . In other words, when raising an exponential expression to a power, we write the result with the common base and the product of the exponents.

$$(a^m)^n = a^{m \cdot n}$$

<!--here-->

Be careful to distinguish between uses of the product rule and the power rule. When using the product rule, different terms with the same bases are raised to exponents. In this case, you add the exponents. When using the power rule, a term in exponential notation is raised to a power. In this case, you multiply the exponents.

Product Rule	but	Power Rule
$5^3 \cdot 5^4 = 5^{3+4} = 5^7$	but	$(5^3)^4 = 5^{3 \cdot 4} = 5^{12}$
$x^5 \cdot x^2 = x^{5+2} = x^7$	but	$(x^5)^2 = x^{5 \cdot 2} = x^{10}$
$(3a)^7 \cdot (3a)^{10} = (3a)^{7+10} = (3a)^{17}$	but	$((3a)^7)^{10} = (3a)^{7 \cdot 10} = (3a)^{70}$

## *The Power Rule of Exponents*

For any real number  $a$  and positive integers  $m$  and  $n$ , the power rule of exponents states that

$$(a^m)^n = a^{m \cdot n}$$

## *Using the Power Rule*

Write each of the following products with a single base. Do not simplify further.

- a.  $(x^2)^7$
- b.  $((2t)^5)^3$
- c.  $((-3)^5)^{11}$

[reveal-answer q="fs-id1167339223779"]Show Solution[/reveal-answer]

[hidden-answer a="fs-id1167339223779"]

Use the power rule to simplify each expression.

$$\text{a. } (x^2)^7 = x^{2 \cdot 7} = x^{14}$$

$$\text{b. } \left((2t)^5\right)^3 = (2t)^{5 \cdot 3} = (2t)^{15}$$

$$\text{c. } \left((-3)^5\right)^{11} = (-3)^{5 \cdot 11} = (-3)^{55}$$

[/hidden-answer]

## Try It

Write each of the following products with a single base. Do not simplify further.

$$\text{a. } \left((3y)^8\right)^3$$

$$\text{b. } (t^5)^7$$

$$\text{c. } \left((-g)^4\right)^4$$

[reveal-answer q="fs-id1167339219836"]Show Solution[/reveal-answer]  
 [hidden-answer a="fs-id1167339219836"]

- $(3y)^{24}$
- $t^{35}$
- $(-g)^{16}$

[/hidden-answer]

## Using the Zero Exponent Rule of Exponents

Return to the quotient rule. We made the condition that  $m > n$  so that the difference  $m - n$  would never be zero or negative. What would happen if  $m = n$ ? In this case, we would use the *zero exponent rule of exponents* to simplify the expression to 1. To see how this is done, let us begin with an example.

$$\frac{t^8}{t^8} = \frac{\overline{)t^8}}{\overline{)t^8}} = 1$$

If we were to simplify the original expression using the quotient rule, we would have

$$\frac{t^8}{t^8} = t^{8-8} = t^0$$

If we equate the two answers, the result is  $t^0 = 1$ . This is true for any nonzero real number, or any variable representing a real number.

$$a^0 = 1$$

The sole exception is the expression  $0^0$ . This appears later in more advanced courses, but for now, we will consider the value to be undefined.

### *The Zero Exponent Rule of Exponents*

For any nonzero real number  $a$ , the zero exponent rule of exponents states that

$$a^0 = 1$$

### *Using the Zero Exponent Rule*

Simplify each expression using the zero exponent rule of exponents.

a.  $\frac{c^3}{c^3}$



$$\text{b. } \frac{-3x^5}{x^5}$$

$$\text{c. } \frac{(j^2k)^4}{(j^2k) \cdot (j^2k)^3}$$

$$\text{d. } \frac{5(rs^2)^2}{(rs^2)^2}$$

[reveal-answer q="fs-id1167339227881"]Show  
Solution[/reveal-answer]

[hidden-answer a="fs-id1167339227881"]

Use the zero exponent and other rules to  
simplify each expression.

$$\begin{aligned} \text{a. } \frac{c^3}{c^3} &= c^{3-3} \\ &= c^0 \\ &= 1 \end{aligned}$$

$$\begin{aligned} \text{b. } \frac{-3x^5}{x^5} &= -3 \cdot \frac{x^5}{x^5} \\ &= -3 \cdot x^{5-5} \\ &= -3 \cdot x^0 \\ &= -3 \cdot 1 \\ &= -3 \end{aligned}$$

c. 
$$\frac{(j^2 k)^4}{(j^2 k) \cdot (j^2 k)^3} = \frac{(j^2 k)^4}{(j^2 k)^{1+3}}$$

Use the product rule in the denominator.

$$= \frac{(j^2 k)^4}{(j^2 k)^4}$$

Simplify.

$$= (j^2 k)^{4-4}$$

Use the quotient rule.

$$= (j^2 k)^0$$

Simplify.

$$= 1$$

d. 
$$\frac{5(rs^2)^2}{(rs^2)^2} = 5(rs^2)^{2-2}$$

Use the quotient rule.

$$= 5(rs^2)^0$$

Simplify.

$$= 5 \cdot 1$$

Use the zero exponent rule.

$$= 5$$

Simplify.

[/hidden-answer]

## Try It

Simplify each expression using the zero exponent rule of exponents.

a. 
$$\frac{t^7}{t^7}$$

b.  $\frac{(de^2)^{11}}{2(de^2)^{11}}$

c.  $\frac{w^4 \cdot w^2}{w^6}$

d.  $\frac{t^3 \cdot t^4}{t^2 \cdot t^5}$

[reveal-answer q="fs-id1167339240181"]Show Solution[/reveal-answer]

[hidden-answer a="fs-id1167339240181"]

a.  $1$

b.  $\frac{1}{2}$

c.  $1$

d.  $1$

[/hidden-answer]

## Using the Negative Rule of Exponents

Another useful result occurs if we relax the condition that  $m > n$  in the quotient rule even further. For example, can we simplify

$\frac{h^3}{h^5}$ ? When  $m < n$ —that is, where the difference  $m - n$  is

negative—we can use the *negative rule of exponents* to simplify the expression to its reciprocal.

Divide one exponential expression by another with a larger exponent. Use our example,  $\frac{h^3}{h^5}$ .

$$\begin{aligned}\frac{h^3}{h^5} &= \frac{h \cdot h \cdot h}{h \cdot h \cdot h \cdot h \cdot h} \\ &= \frac{\overline{)h \cdot )h \cdot )h}}{\overline{)h \cdot )h \cdot )h \cdot h \cdot h}} \\ &= \frac{1}{h \cdot h} \\ &= \frac{1}{h^2}\end{aligned}$$

If we were to simplify the original expression using the quotient rule, we would have

$$\begin{aligned}\frac{h^3}{h^5} &= h^{3-5} \\ &= h^{-2}\end{aligned}$$

Putting the answers together, we have  $h^{-2} = \frac{1}{h^2}$ . This is true

for any nonzero real number, or any variable representing a nonzero real number.

A factor with a negative exponent becomes the same factor with a positive exponent if it is moved across the fraction bar—from numerator to denominator or vice versa.

$$a^{-n} = \frac{1}{a^n} \quad \text{and} \quad a^n = \frac{1}{a^{-n}}$$

We have shown that the exponential expression  $a^n$  is defined when  $n$  is a natural number, 0, or the negative of a natural number. That

means that  $a^n$  is defined for any integer  $n$ . Also, the product and quotient rules and all of the rules we will look at soon hold for any integer  $n$ .

### *The Negative Rule of Exponents*

For any nonzero real number  $a$  and natural number  $n$ , the negative rule of exponents states that

$$a^{-n} = \frac{1}{a^n}$$

### *Using the Negative Exponent Rule*

Write each of the following quotients with a single base. Do not simplify further. Write answers with positive exponents.

a.  $\frac{\theta^3}{\theta^{10}}$

b.  $\frac{z^2 \cdot z}{z^4}$

c.  $\frac{(-5t^3)^4}{(-5t^3)^8}$

[reveal-answer q="fs-id1167339241827"]Show  
Solution[/reveal-answer]  
[hidden-answer a="fs-id1167339241827"]

a.  $\frac{\theta^3}{\theta^{10}} = \theta^{3-10} = \theta^{-7} = \frac{1}{\theta^7}$

b.  $\frac{z^2 \cdot z}{z^4} = \frac{z^{2+1}}{z^4} = \frac{z^3}{z^4} = z^{3-4} = z^{-1} = \frac{1}{z}$

c.  $\frac{(-5t^3)^4}{(-5t^3)^8} = (-5t^3)^{4-8} = (-5t^3)^{-4} = \frac{1}{(-5t^3)^4}$

[/hidden-answer]

## Try It

Write each of the following quotients with a single base. Do not simplify further. Write answers with positive exponents.

a.  $\frac{(-3t)^2}{(-3t)^8}$

b.  $\frac{f^{47}}{f^{49} \cdot f}$

c.  $\frac{2k^4}{5k^7}$

[reveal-answer q="fs-id1167339224503"]Show Solution[/reveal-answer]  
 [hidden-answer a="fs-id1167339224503"]

a.  $\frac{1}{(-3t)^6}$

b.  $\frac{1}{f^3}$

c.  $\frac{2}{5k^3}$

[/hidden-answer]

## Using the Product and Quotient Rules

Write each of the following products with a single base. Do not simplify further. Write answers with positive exponents.

a.  $b^2 \cdot b^{-8}$

b.  $(-x)^5 \cdot (-x)^{-5}$

c.  $\frac{-7z}{(-7z)^5}$

[reveal-answer q="fs-id1167339224799"]Show  
Solution[/reveal-answer]  
[hidden-answer a="fs-id1167339224799"]

a.  $b^2 \cdot b^{-8} = b^{2-8} = b^{-6} = \frac{1}{b^6}$

b.  $(-x)^5 \cdot (-x)^{-5} = (-x)^{5-5} = (-x)^0 = 1$

c.  $\frac{-7z}{(-7z)^5} = \frac{(-7z)^1}{(-7z)^5} = (-7z)^{1-5} = (-7z)^{-4} = \frac{1}{(-7z)^4}$

[/hidden-answer]



## Try It

Write each of the following products with a single base. Do not simplify further. Write answers with positive exponents.

a.  $t^{-11} \cdot t^6$

b.  $\frac{25^{12}}{25^{13}}$

[reveal-answer q="fs-id1167339225379"]Show Solution[/reveal-answer]

[hidden-answer a="fs-id1167339225379"]

a.  $t^{-5} = \frac{1}{t^5}$

b.  $\frac{1}{25}$

[/hidden-answer]

## Finding the Power of a Product

To simplify the power of a product of two exponential expressions, we can use the *power of a product rule of exponents*, which breaks up the power of a product of factors into the product of the powers of the factors. For instance, consider  $(pq)^3$ . We begin by using the associative and commutative properties of multiplication to regroup the factors.

$$\begin{aligned}(pq)^3 &= \overset{3 \text{ factors}}{(pq) \cdot (pq) \cdot (pq)} \\&= p \cdot q \cdot p \cdot q \cdot p \cdot q \\&= \overset{3 \text{ factors}}{p \cdot p \cdot p} \cdot \overset{3 \text{ factors}}{q \cdot q \cdot q} \\&= p^3 \cdot q^3\end{aligned}$$

In other words,  $(pq)^3 = p^3 \cdot q^3$ .

### *The Power of a Product Rule of Exponents*

For any real numbers  $a$  and  $b$  and any integer  $n$ , the power of a product rule of exponents states that

$$(ab)^n = a^n b^n$$

## Using the Power of a Product Rule

Simplify each of the following products as much as possible using the power of a product rule. Write answers with positive exponents.

a.  $(ab^2)^3$

b.  $(2t)^{15}$

c.  $(-2w^3)^3$

d.  $\frac{1}{(-7z)^4}$

e.  $(e^{-2}f^2)^7$

[reveal-answer q="fs-id1167339221295"]Show  
Solution[/reveal-answer]

[hidden-answer a="fs-id1167339221295"]

Use the product and quotient rules and the new definitions to simplify each expression.

a.  $(ab^2)^3 = (a)^3 \cdot (b^2)^3 = a^{1 \cdot 3} \cdot b^{2 \cdot 3} = a^3 b^6$

b.  $(2t)^{15} = (2)^{15} \cdot (t)^{15} = 2^{15} t^{15} = 32,768 t^{15}$

c.  $(-2w^3)^3 = (-2)^3 \cdot (w^3)^3 = -8 \cdot w^{3 \cdot 3} = -8w^9$

d.  $\frac{1}{(-7z)^4} = \frac{1}{(-7)^4 \cdot (z)^4} = \frac{1}{2,401z^4}$

$$\text{e.} \quad (e^{-2}f^2)^7 = (e^{-2})^7 \cdot (f^2)^7 = e^{-2 \cdot 7} \cdot f^{2 \cdot 7} = e^{-14}f^{14} = \frac{f^{14}}{e^{14}}$$

[/hidden-answer]

## Try It

Simplify each of the following products as much as possible using the power of a product rule. Write answers with positive exponents.

- a.  $(g^2h^3)^5$
- b.  $(5t)^3$
- c.  $(-3y^5)^3$
- d.  $\frac{1}{(a^6b^7)^3}$
- e.  $(r^3s^{-2})^4$

[reveal-answer q="fs-id1167339222354"]Show Solution[/reveal-answer]

[hidden-answer a="fs-id1167339222354"]

a.  $g^{10}h^{15}$

b.  $125t^3$

c.  $-27y^{15}$

d.  $\frac{1}{a^{18}b^{21}}$

e.  $\frac{r^{12}}{s^8}$

[/hidden-answer]

## Finding the Power of a Quotient

To simplify the power of a quotient of two expressions, we can use the *power of a quotient rule*, which states that the power of a quotient of factors is the quotient of the powers of the factors. For example, let's look at the following example.

$$(e^{-2}f^2)^7 = \frac{f^{14}}{e^{14}}$$

Let's rewrite the original problem differently and look at the result.

$$\begin{aligned}(e^{-2} f^2)^7 &= \left(\frac{f^2}{e^2}\right)^7 \\ &= \frac{f^{14}}{e^{14}}\end{aligned}$$

It appears from the last two steps that we can use the power of a product rule as a power of a quotient rule.

$$\begin{aligned}(e^{-2} f^2)^7 &= \left(\frac{f^2}{e^2}\right)^7 \\ &= \frac{(f^2)^7}{(e^2)^7} \\ &= \frac{f^{2 \cdot 7}}{e^{2 \cdot 7}} \\ &= \frac{f^{14}}{e^{14}}\end{aligned}$$

### *The Power of a Quotient Rule of Exponents*

For any real numbers  $a$  and  $b$  and any integer  $n$ , the power of a quotient rule of exponents states that

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

## Using the Power of a Quotient Rule

Simplify each of the following quotients as much as possible using the power of a quotient rule. Write answers with positive exponents.

a.  $\left(\frac{4}{z^{11}}\right)^3$

b.  $\left(\frac{p}{q^3}\right)^6$

c.  $\left(\frac{-1}{t^2}\right)^{27}$

d.  $(j^3k^{-2})^4$

e.  $(m^{-2}n^{-2})^3$

[reveal-answer q="fs-id1167339195210"]Show  
Solution[/reveal-answer]  
[hidden-answer a="fs-id1167339195210"]

a.  $\left(\frac{4}{z^{11}}\right)^3 = \frac{(4)^3}{(z^{11})^3} = \frac{64}{z^{11 \cdot 3}} = \frac{64}{z^{33}}$

$$\text{b. } \left(\frac{p}{q^3}\right)^6 = \frac{(p)^6}{(q^3)^6} = \frac{p^{1 \cdot 6}}{q^{3 \cdot 6}} = \frac{p^6}{q^{18}}$$

$$\text{c. } \left(\frac{-1}{t^2}\right)^{27} = \frac{(-1)^{27}}{(t^2)^{27}} = \frac{-1}{t^{2 \cdot 27}} = \frac{-1}{t^{54}} = -\frac{1}{t^{54}}$$

$$\text{d. } (j^3 k^{-2})^4 = \left(\frac{j^3}{k^2}\right)^4 = \frac{(j^3)^4}{(k^2)^4} = \frac{j^{3 \cdot 4}}{k^{2 \cdot 4}} = \frac{j^{12}}{k^8}$$

$$\text{e. } (m^{-2} n^{-2})^3 = \left(\frac{1}{m^2 n^2}\right)^3 = \frac{(1)^3}{(m^2 n^2)^3} = \frac{1}{(m^2)^3 (n^2)^3} = \frac{1}{m^{2 \cdot 3} \cdot n^{2 \cdot 3}} = \frac{1}{m^6 n^6}$$

[/hidden-answer]

## Try It

Simplify each of the following quotients as much as possible using the power of a quotient rule. Write answers with positive exponents.

$$\text{a. } \left(\frac{b^5}{c}\right)^3$$



b.  $\left(\frac{5}{u^8}\right)^4$

c.  $\left(\frac{-1}{w^3}\right)^{35}$

d.  $(p^{-4}q^3)^8$

e.  $(c^{-5}d^{-3})^4$

[reveal-answer q="fs-id1167339196679"]Show Solution[/reveal-answer]

[hidden-answer a="fs-id1167339196679"]

a.  $\frac{b^{15}}{c^3}$

b.  $\frac{625}{u^{32}}$

c.  $\frac{-1}{w^{105}}$

d.  $\frac{q^{24}}{p^{32}}$

e.  $\frac{1}{c^{20}d^{12}}$

[/hidden-answer]

## Simplifying Exponential Expressions

Recall that to simplify an expression means to rewrite it by combining terms or exponents; in other words, to write the expression more simply with fewer terms. The rules for exponents may be combined to simplify expressions.

### *Simplifying Exponential Expressions*

Simplify each expression and write the answer with positive exponents only.

- a.  $(6m^2n^{-1})^3$
- b.  $17^5 \cdot 17^{-4} \cdot 17^{-3}$
- c.  $\left(\frac{u^{-1}v}{v^{-1}}\right)^2$
- d.  $(-2a^3b^{-1})(5a^{-2}b^2)$

e.  $(x^2\sqrt{2})^4(x^2\sqrt{2})^{-4}$

f.  $\frac{(3w^2)^5}{(6w^{-2})^2}$

[reveal-answer q="fs-id1167339197420"]Show  
Solution[/reveal-answer]  
[hidden-answer a="fs-id1167339197420"]

a.	$\begin{aligned}(6m^2n^{-1})^3 &= (6)^3(m^2)^3(n^{-1})^3 \\ &= 6^3m^{2 \cdot 3}n^{-1 \cdot 3} \\ &= 216m^6n^{-3} \\ &= \frac{216m^6}{n^3}\end{aligned}$	<p>The power of a product rule</p> <p>The power rule</p> <p>Simplify.</p> <p>The negative exponent rule</p>
b.	$\begin{aligned}17^5 \cdot 17^{-4} \cdot 17^{-3} &= 17^{5-4-3} \\ &= 17^{-2} \\ &= \frac{1}{17^2} \text{ or } \frac{1}{289}\end{aligned}$	<p>The product rule</p> <p>Simplify.</p> <p>The negative exponent rule</p>
c.	$\begin{aligned}\left(\frac{u^{-1}v}{v^{-1}}\right)^2 &= \frac{(u^{-1}v)^2}{(v^{-1})^2} \\ &= \frac{u^{-2}v^2}{v^{-2}} \\ &= u^{-2}v^{2-(-2)} \\ &= u^{-2}v^4 \\ &= \frac{v^4}{u^2}\end{aligned}$	<p>The power of a quotient rule</p> <p>The power of a product rule</p> <p>The quotient rule</p> <p>Simplify.</p> <p>The negative exponent rule</p>
d.	$\begin{aligned}(-2a^3b^{-1})(5a^{-2}b^2) &= -2 \cdot 5 \cdot a^3 \cdot a^{-2} \cdot b^{-1} \cdot b^2 \\ &= -10 \cdot a^{3-2} \cdot b^{-1+2} \\ &= -10ab\end{aligned}$	<p>Commutative and associative laws of multiplication</p> <p>The product rule</p> <p>Simplify.</p>
e.	$\begin{aligned}(x^2\sqrt{2})^4(x^2\sqrt{2})^{-4} &= (x^2\sqrt{2})^{4-4} \\ &= (x^2\sqrt{2})^0 \\ &= 1\end{aligned}$	<p>The product rule</p> <p>Simplify.</p> <p>The zero exponent rule</p>
f.	$\begin{aligned}\frac{(3w^2)^5}{(6w^{-2})^2} &= \frac{(3)^5 \cdot (w^2)^5}{(6)^2 \cdot (w^{-2})^2} \\ &= \frac{3^5 w^{2 \cdot 5}}{6^2 w^{-2 \cdot 2}} \\ &= \frac{243w^{10}}{36w^{-4}} \\ &= \frac{27w^{10-(-4)}}{4} \\ &= \frac{27w^{14}}{4}\end{aligned}$	<p>The power of a product rule</p> <p>The power rule</p> <p>Simplify.</p> <p>The quotient rule and reduce fraction</p> <p>Simplify.</p>

[/hidden-answer]

## Try It

Simplify each expression and write the answer with positive exponents only.

a.  $(2uv^{-2})^{-3}$

b.  $x^8 \cdot x^{-12} \cdot x$

c.  $\left(\frac{e^2 f^{-3}}{f^{-1}}\right)^2$

d.  $(9r^{-5} s^3)(3r^6 s^{-4})$

e.  $\left(\frac{4}{9}tw^{-2}\right)^{-3} \left(\frac{4}{9}tw^{-2}\right)^3$

f.  $\frac{(2h^2 k)^4}{(7h^{-1} k^2)^2}$

[reveal-answer q="fs-id1167339229590"]Show Solution[/reveal-answer]  
[hidden-answer a="fs-id1167339229590"]

- a.  $\frac{v^6}{8u^3}$
- b.  $\frac{1}{x^3}$
- c.  $\frac{e^4}{f^4}$
- d.  $\frac{27r}{s}$
- e.  $\frac{1}{16h^{10}}$
- f.  $\frac{16h^{10}}{49}$

[/hidden-answer]

## Using Scientific Notation

Recall at the beginning of the section that we found the number  $1.3 \times 10^{13}$  when describing bits of information in digital images. Other extreme numbers include the width of a human hair, which is about 0.00005 m, and the radius of an electron, which is about 0.00000000000047 m. How can we effectively work read, compare, and calculate with numbers such as these?

A shorthand method of writing very small and very large numbers is called scientific notation, in which we express numbers in terms of exponents of 10. To write a number in scientific notation, move the decimal point to the right of the first digit in the number. Write the digits as a decimal number between 1 and 10. Count the number of places  $n$  that you moved the decimal point. Multiply the decimal number by 10 raised to a power of  $n$ . If you moved the decimal left as in a very large number,  $n$  is positive. If you moved the decimal right as in a small large number,  $n$  is negative.

For example, consider the number 2,780,418. Move the decimal left until it is to the right of the first nonzero digit, which is 2.

$$2,780,418 \longrightarrow 2.780418$$

6 places left

We obtain 2.780418 by moving the decimal point 6 places to the left. Therefore, the exponent of 10 is 6, and it is positive because we moved the decimal point to the left. This is what we should expect for a large number.

$$2.780418 \times 10^6$$

Working with small numbers is similar. Take, for example, the radius of an electron, 0.00000000000047 m. Perform the same series of steps as above, except move the decimal point to the right.

$$0.00000000000047 \longrightarrow 4.7$$

13 places right

Be careful not to include the leading 0 in your count. We move the decimal point 13 places to the right, so the exponent of 10 is 13.

The exponent is negative because we moved the decimal point to the right. This is what we should expect for a small number.

$$4.7 \times 10^{-13}$$

### *Scientific Notation*

A number is written in scientific notation if it is written in the form  $a \times 10^n$ , where  $1 \leq |a| < 10$  and  $n$  is an integer.

### *Converting Standard Notation to Scientific Notation*

Write each number in scientific notation.

- Distance to Andromeda Galaxy from Earth:  
24,000,000,000,000,000,000 m
- Diameter of Andromeda Galaxy:  
1,300,000,000,000,000,000 m
- Number of stars in Andromeda Galaxy:  
1,000,000,000,000
- Diameter of electron: 0.000000000000094 m
- Probability of being struck by lightning in any single year: 0.00000143

[reveal-answer q="fs-id1167339230108"]Show  
Solution[/reveal-answer]  
[hidden-answer a="fs-id1167339230108"]

a.  $24,000,000,000,000,000,000,000\text{ m}$   
 $24,000,000,000,000,000,000,000\text{ m}$   
 $\leftarrow 22\text{ places}$

$2.4 \times 10^{22}\text{ m}$   
 $1,300,000,000,000,000,000,000\text{ m}$   
 $1,300,000,000,000,000,000,000\text{ m}$   
b.  $\leftarrow 21\text{ places}$

$1.3 \times 10^{21}\text{ m}$   
 $1,000,000,000,000$   
 $1,000,000,000,000$   
c.  $\leftarrow 12\text{ places}$

$1 \times 10^{12}$   
 $0.00000000000094\text{ m}$   
 $0.00000000000094\text{ m}$   
d.  $\rightarrow 13\text{ places}$

$9.4 \times 10^{-13}\text{ m}$



0.00000143

0.00000143

e.

→6 places

$1.43 \times 10^{-6}$

[/hidden-answer]

### *Analysis*

Observe that, if the given number is greater than 1, as in examples a–c, the exponent of 10 is positive; and if the number is less than 1, as in examples d–e, the exponent is negative.

### Try It

Write each number in scientific notation.

- a. U.S. national debt per taxpayer (April 2014):

\$152,000

- b. World population (April 2014): 7,158,000,000
- c. World gross national income (April 2014):  
\$85,500,000,000,000
- d. Time for light to travel 1 m: 0.00000000334 s
- e. Probability of winning lottery (match 6 of 49  
possible numbers): 0.0000000715

[reveal-answer q="fs-id1167339230805"]Show Solution[/reveal-answer]  
[hidden-answer a="fs-id1167339230805"]

- a.  $\$1.52 \times 10^5$
- b.  $7.158 \times 10^9$
- c.  $\$8.55 \times 10^{13}$
- d.  $3.34 \times 10^{-9}$
- e.  $7.15 \times 10^{-8}$

[/hidden-answer]

## Converting from Scientific to Standard Notation

To convert a number in scientific notation to standard notation, simply reverse the process. Move the decimal  $n$  places to the right if  $n$  is positive or  $n$  places to the left if  $n$  is negative and add zeros

as needed. Remember, if  $n$  is positive, the value of the number is greater than 1, and if  $n$  is negative, the value of the number is less than one.

### *Converting Scientific Notation to Standard Notation*

Convert each number in scientific notation to standard notation.

- a.  $3.547 \times 10^{14}$
- b.  $-2 \times 10^6$
- c.  $7.91 \times 10^{-7}$
- d.  $-8.05 \times 10^{-12}$

[reveal-answer q="fs-id1167339216548"]Show  
Solution[/reveal-answer]

[hidden-answer a="fs-id1167339216548"]

$$3.547 \times 10^{14}$$

a.  $3.547000000000000$

→14 places

$$354,700,000,000,000$$

$$-2 \times 10^6$$

$$-2.000000$$

b.

→6 places

$$-2,000,000$$

$$7.91 \times 10^{-7}$$

$$0000007.91$$

c.

→7 places

$$0.000000791$$

$$-8.05 \times 10^{-12}$$

$$-000000000008.05$$

d.

→12 places

$$-0.00000000000805$$

[/hidden-answer]

## Try It

Convert each number in scientific notation to standard notation.

- a.  $7.03 \times 10^5$
- b.  $-8.16 \times 10^{11}$
- c.  $-3.9 \times 10^{-13}$
- d.  $8 \times 10^{-6}$

[reveal-answer q="fs-id1167339217110"]Show Solution[/reveal-answer]

[hidden-answer a="fs-id1167339217110"]

- a. 703,000
- b.  $-816,000,000,000$
- c.  $-0.000\,000\,000\,000\,39$
- d. 0.000008

[/hidden-answer]

## Using Scientific Notation in Applications

Scientific notation, used with the rules of exponents, makes calculating with large or small numbers much easier than doing so using standard notation. For example, suppose we are asked to calculate the number of atoms in 1 L of water. Each water molecule contains 3 atoms (2 hydrogen and 1 oxygen). The average drop of water contains around  $1.32 \times 10^{21}$  molecules of water and 1 L of water holds about  $1.22 \times 10^4$  average drops. Therefore, there are approximately  $3 \cdot (1.32 \times 10^{21}) \cdot (1.22 \times 10^4) \approx 4.83 \times 10^{25}$  atoms in 1 L of water. We simply multiply the decimal terms and add the exponents. Imagine having to perform the calculation without using scientific notation!

When performing calculations with scientific notation, be sure to write the answer in proper scientific notation. For example, consider the product  $(7 \times 10^4) \cdot (5 \times 10^6) = 35 \times 10^{10}$ . The answer is not in proper scientific notation because 35 is greater than 10. Consider 35 as  $3.5 \times 10$ . That adds a ten to the exponent of the answer.

$$(35) \times 10^{10} = (3.5 \times 10) \times 10^{10} = 3.5 \times (10 \times 10^{10}) = 3.5 \times 10^{11}$$

### Using Scientific Notation

Perform the operations and write the answer in scientific notation.

a.  $(8.14 \times 10^{-7}) (6.5 \times 10^{10})$

- b.  $(4 \times 10^5) \div (-1.52 \times 10^9)$   
 c.  $(2.7 \times 10^5) (6.04 \times 10^{13})$   
 d.  $(1.2 \times 10^8) \div (9.6 \times 10^5)$   
 e.  $(3.33 \times 10^4) (-1.05 \times 10^7) (5.62 \times 10^5)$

[reveal-answer q="fs-id1167339218020"]Show  
 Solution[/reveal-answer]

[hidden-answer a="fs-id1167339218020"]

- |    |   |  |
|----|---|--|
| a. | $(8.14 \times 10^{-7}) (6.5 \times 10^{10}) = (8.14 \times 6.5) (10^{-7} \times 10^{10})$<br>$= (52.91) (10^3)$<br>$= 5.291 \times 10^4$  | Commutative and associative<br>properties of multiplication<br>Product rule of exponents<br>Scientific notation  |
| b. | $(4 \times 10^5) \div (-1.52 \times 10^9) = \left(\frac{4}{-1.52}\right) \left(\frac{10^5}{10^9}\right)$<br>$\approx (-2.63) (10^{-4})$<br>$= -2.63 \times 10^{-4}$                         | Commutative and associative<br>properties of multiplication<br>Quotient rule of exponents<br>Scientific notation |
| c. | $(2.7 \times 10^5) (6.04 \times 10^{13}) = (2.7 \times 6.04) (10^5 \times 10^{13})$<br>$= (16.308) (10^{18})$<br>$= 1.6308 \times 10^{19}$  | Commutative and associative<br>properties of multiplication<br>Product rule of exponents<br>Scientific notation  |
| d. | $(1.2 \times 10^8) \div (9.6 \times 10^5) = \left(\frac{1.2}{9.6}\right) \left(\frac{10^8}{10^5}\right)$<br>$= (0.125) (10^3)$<br>$= 1.25 \times 10^2$                                      | Commutative and associative<br>properties of multiplication<br>Quotient rule of exponents<br>Scientific notation |
| e. | $(3.33 \times 10^4) (-1.05 \times 10^7) (5.62 \times 10^5) = [3.33 \times (-1.05) \times 5.62] (10^4 \times 10^7 \times 10^5)$<br>$\approx (-19.65) (10^{16})$<br>$= -1.965 \times 10^{17}$ | Commutative and associative<br>properties of multiplication<br>Product rule of exponents<br>Scientific notation  |

[/hidden-answer]

## Try It

Perform the operations and write the answer in scientific notation.

- a.  $(-7.5 \times 10^8)(1.13 \times 10^{-2})$
- b.  $(1.24 \times 10^{11}) \div (1.55 \times 10^{18})$
- c.  $(3.72 \times 10^9)(8 \times 10^3)$
- d.  $(9.933 \times 10^{23}) \div (-2.31 \times 10^{17})$
- e.  $(-6.04 \times 10^9)(7.3 \times 10^2)(-2.81 \times 10^2)$

[reveal-answer q="fs-id1167339235772"]Show Solution[/reveal-answer]

[hidden-answer a="fs-id1167339235772"]

- a.  $-8.475 \times 10^6$
- b.  $8 \times 10^{-8}$
- c.  $2.976 \times 10^{13}$
- d.  $-4.3 \times 10^6$
- e.  $\approx 1.24 \times 10^{15}$

[/hidden-answer]



## Applying Scientific Notation to Solve Problems

In April 2014, the population of the United States was about 308,000,000 people. The national debt was about \$17,547,000,000,000. Write each number in scientific notation, rounding figures to two decimal places, and find the amount of the debt per U.S. citizen. Write the answer in both scientific and standard notations.

[reveal-answer q="fs-id1167339235949"]Show  
Solution[/reveal-answer]

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The population was

$$308,000,000 = 3.08 \times 10^8.$$

The national debt was

$$\$17,547,000,000,000 \approx \$1.75 \times 10^{13}.$$

To find the amount of debt per citizen, divide the national debt by the number of citizens.

$$\begin{aligned}(1.75 \times 10^{13}) \div (3.08 \times 10^8) &= \left(\frac{1.75}{3.08}\right) \cdot \left(\frac{10^{13}}{10^8}\right) \\ &\approx 0.57 \times 10^5 \\ &= 5.7 \times 10^4\end{aligned}$$

The debt per citizen at the time was about

$$\$5.7 \times 10^4, \text{ or } \$57,000.$$

[/hidden-answer]

## Try It

An average human body contains around 30,000,000,000,000 red blood cells. Each cell measures approximately 0.000008 m long. Write each number in scientific notation and find the total length if the cells were laid end-to-end. Write the answer in both scientific and standard notations.

[reveal-answer q="fs-id1167339236318"]Show Solution[/reveal-answer]

[hidden-answer a="fs-id1167339236318"]

Number of cells:  $3 \times 10^{13}$ ; length of a cell:  $8 \times 10^{-6}$  m; total length:  $2.4 \times 10^8$  m or 240, 000, 000 m.

[/hidden-answer]

Access these online resources for additional instruction and practice with exponents and scientific notation.

- Exponential Notation
- Properties of Exponents
- Zero Exponent
- Simplify Exponent Expressions

- Quotient Rule for Exponents
- Scientific Notation
- Converting to Decimal Notation

## Key Equations

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### Rules of Exponents<

For nonzero real numbers  $a$  and  $b$  and integers  $m$  and  $n$

Product rule  $a^m \cdot a^n = a^{m+n}$

Quotient rule  $\frac{a^m}{a^n} = a^{m-n}$

Power rule  $(a^m)^n = a^{m \cdot n}$

Zero exponent rule  $a^0 = 1$

Negative rule  $a^{-n} = \frac{1}{a^n}$

Power of a product rule  $(a \cdot b)^n = a^n \cdot b^n$

Power of a quotient rule  $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

---

### Key Concepts

- Products of exponential expressions with the same base can be simplified by adding exponents. See (Figure).

- Quotients of exponential expressions with the same base can be simplified by subtracting exponents. See (Figure).
- Powers of exponential expressions with the same base can be simplified by multiplying exponents. See (Figure).
- An expression with exponent zero is defined as 1. See (Figure).
- An expression with a negative exponent is defined as a reciprocal. See (Figure) and (Figure).
- The power of a product of factors is the same as the product of the powers of the same factors. See (Figure).
- The power of a quotient of factors is the same as the quotient of the powers of the same factors. See (Figure).
- The rules for exponential expressions can be combined to simplify more complicated expressions. See (Figure).
- Scientific notation uses powers of 10 to simplify very large or very small numbers. See (Figure) and (Figure).
- Scientific notation may be used to simplify calculations with very large or very small numbers. See (Figure) and (Figure).

## Section Exercises

### Verbal

Is  $2^3$  the same as  $3^2$ ? Explain.

[reveal-answer q="fs-id1167339237109"]Show  
Solution[/reveal-answer]

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No, the two expressions are not the same. An exponent tells how many times you multiply the base. So  $2^3$  is the same as  $2 \times 2 \times 2$ , which is 8.  $3^2$  is the same as  $3 \times 3$ , which is 9.

[/hidden-answer]

When can you add two exponents?

What is the purpose of scientific notation?

[reveal-answer q="fs-id1167339237212"]Show  
Solution[/reveal-answer]

[hidden-answer a="fs-id1167339237212"]

It is a method of writing very small and very large numbers.

[/hidden-answer]

Explain what a negative exponent does.

*Numeric*

For the following exercises, simplify the given expression.  
Write answers with positive exponents.

$$9^2$$

[reveal-answer q="fs-id1167339237259"]Show  
Solution[/reveal-answer]

[hidden-answer a="fs-id1167339237259"]

81

[/hidden-answer]

$$15^{-2}$$

$$3^2 \times 3^3$$

[reveal-answer q="fs-id1167339237324"]Show  
Solution[/reveal-answer]  
[hidden-answer a="fs-id1167339237324"]  
243  
[/hidden-answer]

$$4^4 \div 4$$

$$(2^2)^{-2}$$

[reveal-answer q="fs-id1167339237400"]Show  
Solution[/reveal-answer]  
[hidden-answer a="fs-id1167339237400"]  
 $\frac{1}{16}$   
[/hidden-answer]

$$(5 - 8)^0$$

$$11^3 \div 11^4$$

[reveal-answer q="fs-id1167339237499"]Show  
Solution[/reveal-answer]

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$$\frac{1}{11}$$

[/hidden-answer]

$$6^5 \times 6^{-7}$$

$$(8^0)^2$$

[reveal-answer q="fs-id1167339237599"]Show  
Solution[/reveal-answer]

[hidden-answer a="fs-id1167339237599"]

$$1$$

[/hidden-answer]

$$5^{-2} \div 5^2$$

For the following exercises, write each expression with a single base. Do not simplify further. Write answers with positive exponents.

$$4^2 \times 4^3 \div 4^{-4}$$



[reveal-answer q="fs-id1167339237694"]Show  
Solution[/reveal-answer]

[hidden-answer a="fs-id1167339237694"]

$$4^9$$

[/hidden-answer]

$$\frac{6^{12}}{6^9}$$

$$(12^3 \times 12)^{10}$$

[reveal-answer q="fs-id1167339237808"]Show  
Solution[/reveal-answer]

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$$12^{40}$$

[/hidden-answer]

$$10^6 \div (10^{10})^{-2}$$

$$7^{-6} \times 7^{-3}$$

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Solution[/reveal-answer]

[hidden-answer a="fs-id1167339237944"]

$$\frac{1}{7^9}$$

[/hidden-answer]

$$(3^3 \div 3^4)^5$$

For the following exercises, express the decimal in scientific notation.

0.0000314

[reveal-answer q="fs-id1167339238034"]Show  
Solution[/reveal-answer]

[hidden-answer a="fs-id1167339238034"]

$$3.14 \times 10^{-5}$$

[/hidden-answer]

148,000,000

For the following exercises, convert each number in scientific notation to standard notation.

$$1.6 \times 10^{10}$$

[reveal-answer q="fs-id1167339238107"]Show  
Solution[/reveal-answer]  
[hidden-answer a="fs-id1167339238107"]  
16,000,000,000  
[/hidden-answer]

$$9.8 \times 10^{-9}$$

### *Algebraic*

For the following exercises, simplify the given expression.  
Write answers with positive exponents.

$$\frac{a^3 a^2}{a}$$

[reveal-answer q="fs-id1167339238194"]Show  
Solution[/reveal-answer]  
[hidden-answer a="fs-id1167339238194"]  
 $a^4$   
[/hidden-answer]

$$\frac{mn^2}{m^{-2}}$$

$$(b^3c^4)^2$$

[reveal-answer q="fs-id1167339238310"]Show  
Solution[/reveal-answer]

[hidden-answer a="fs-id1167339238310"]

$$b^6c^8$$

[/hidden-answer]

$$\left(\frac{x^{-3}}{y^2}\right)^{-5}$$

$$ab^2 \div d^{-3}$$

[reveal-answer q="fs-id1167339238449"]Show  
Solution[/reveal-answer]

[hidden-answer a="fs-id1167339238449"]

$$ab^2d^3$$

[/hidden-answer]

$$(w^0x^5)^{-1}$$

$$\frac{m^4}{n^0}$$

[reveal-answer q="fs-id1167339210809"]Show

Solution[/reveal-answer]

[hidden-answer a="fs-id1167339210809"]

$$m^4$$

[/hidden-answer]

$$y^{-4} (y^2)^2$$

$$\frac{p^{-4} q^2}{p^2 q^{-3}}$$

[reveal-answer q="fs-id1167339210953"]Show

Solution[/reveal-answer]

[hidden-answer a="fs-id1167339210953"]

$$\frac{q^5}{p^6}$$

[/hidden-answer]

$$(l \times w)^2$$

$$(y^7)^3 \div x^{14}$$

[reveal-answer q="fs-id1167339211086"]Show

Solution[/reveal-answer]

[hidden-answer a="fs-id1167339211086"]

$$\frac{y^{21}}{x^{14}}$$

[/hidden-answer]

$$\left(\frac{a}{2^3}\right)^2$$

$$5^2 m \div 5^0 m$$

[reveal-answer q="fs-id1167339211218"]Show

Solution[/reveal-answer]

[hidden-answer a="fs-id1167339211218"]

$$25$$

[/hidden-answer]

$$\frac{(16\sqrt{x})^2}{y^{-1}}$$

$$\frac{2^3}{(3a)^{-2}}$$

[reveal-answer q="fs-id1167339211355"]Show  
Solution[/reveal-answer]

[hidden-answer a="fs-id1167339211355"]

$$72a^2$$

[/hidden-answer]

$$(ma^6)^2 \frac{1}{m^3 a^2}$$

$$(b^{-3}c)^3$$

[reveal-answer q="fs-id1167339211497"]Show  
Solution[/reveal-answer]

[hidden-answer a="fs-id1167339211497"]

$$\frac{c^3}{b^9}$$

[/hidden-answer]

$$(x^2 y^{13} \div y^0)^2$$

$$(9z^3)^{-2}y$$

[reveal-answer q="fs-id1167339211652"]Show

Solution[/reveal-answer]

[hidden-answer a="fs-id1167339211652"]

$$\frac{y}{81z^6}$$

[/hidden-answer]

### *Real-World Applications*

To reach escape velocity, a rocket must travel at the rate of  $2.2 \times 10^6$  ft/min. Rewrite the rate in standard notation.

A dime is the thinnest coin in U.S. currency. A dime's thickness measures  $1.35 \times 10^{-3}$  m. Rewrite the number in standard notation.

[reveal-answer q="fs-id1167339211766"]Show

Solution[/reveal-answer]

[hidden-answer a="fs-id1167339211766"]

0.00135 m



[/hidden-answer]

The average distance between Earth and the Sun is 92,960,000 mi. Rewrite the distance using scientific notation.

A terabyte is made of approximately 1,099,500,000,000 bytes. Rewrite in scientific notation.

[reveal-answer q="fs-id1167339211783"]Show Solution[/reveal-answer]

[hidden-answer a="fs-id1167339211783"]

$$1.0995 \times 10^{12}$$

[/hidden-answer]

The Gross Domestic Product (GDP) for the United States in the first quarter of 2014 was  $\$1.71496 \times 10^{13}$ . Rewrite the GDP in standard notation.

One picometer is approximately  $3.397 \times 10^{-11}$  in. Rewrite this length using standard notation.

[reveal-answer q="fs-id1167339211896"]Show  
Solution[/reveal-answer]  
[hidden-answer a="fs-id1167339211896"]  
0.00000000003397 in.  
[/hidden-answer]

The value of the services sector of the U.S. economy in the first quarter of 2012 was \$10,633.6 billion. Rewrite this amount in scientific notation.

### *Technology*

For the following exercises, use a graphing calculator to simplify. Round the answers to the nearest hundredth.

$$\left( \frac{12^3 m^{33}}{4^{-3}} \right)^2$$

[reveal-answer q="fs-id1167339211998"]Show  
Solution[/reveal-answer]  
[hidden-answer a="fs-id1167339211998"]  
12,230,590,464  $m^{66}$

[/hidden-answer]

$$17^3 \div 15^2 x^3$$

### *Extensions*

For the following exercises, simplify the given expression.  
Write answers with positive exponents.

$$\left(\frac{3^2}{a^3}\right)^{-2} \left(\frac{a^4}{2^2}\right)^2$$

[reveal-answer q="fs-id1167339212196"]Show

Solution[/reveal-answer]

[hidden-answer a="fs-id1167339212196"]

$$\frac{a^{14}}{1296}$$

[/hidden-answer]

$$(6^2 - 24)^2 \div \left(\frac{x}{y}\right)^{-5}$$

$$\frac{m^2 n^3}{a^2 c^{-3}} \cdot \frac{a^{-7} n^{-2}}{m^2 c^4}$$

[reveal-answer q="fs-id1167339212439"]Show

Solution[/reveal-answer]

[hidden-answer a="fs-id1167339212439"]

$$\frac{n}{a^9 c}$$

[/hidden-answer]

$$\left( \frac{x^6 y^3}{x^3 y^{-3}} \cdot \frac{y^{-7}}{x^{-3}} \right)^{10}$$

$$\left( \frac{(ab^2 c)^{-3}}{b^{-3}} \right)^2$$

[reveal-answer q="fs-id1167339212695"]Show

Solution[/reveal-answer]

[hidden-answer a="fs-id1167339212695"]

$$\frac{1}{a^6 b^6 c^6}$$

[/hidden-answer]

Avogadro's constant is used to calculate the number of particles in a mole. A mole is a basic unit in chemistry to measure the amount of a substance. The constant is  $6.0221413 \times 10^{23}$ . Write Avogadro's constant in standard notation.

Planck's constant is an important unit of measure in quantum physics. It describes the relationship between energy and frequency. The constant is written as  $6.62606957 \times 10^{-34}$ . Write Planck's constant in standard notation.

[illegible][illegible][illegible][illegible][illegible][illegible]

## Glossary

### **scientific notation**

a shorthand notation for writing very large or very small numbers in the form  $a \times 10^n$  where  $1 \leq |a| < 10$  and  $n$  is an integer