NSCC Math For Bookkeeping

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Contents

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CHAPTER 1: SUCCEEDING IN BUSINESS MATHEMATICS

HOW TO USE THIS TEXTBOOK

Math!

You may really dread the "M" word.

Take a look at your world from a different perspective—notice that math is everywhere! Before you went to bed last night, you performed some math and solved a complex algebraic equation (and you probably didn't even realize it!). That equation helped you to figure out what time to get up this morning so that you could arrive at your destination on time. You factored in variables such as the required arrival time, commuting time (including the unknown variables of traffic and weather delays), morning routine time, and even snooze button time. When you solved this complex algebraic equation, you then set your alarm clock.

From the moment you woke up this morning, numbers have appeared everywhere. Perhaps you figured out how many calories are in your breakfast cereal. Maybe your car's gas tank was low, so you calculated whether you had enough gas for your commute today. You checked your bank account online to see if you had enough money to pay the bills. At Tim Hortons you figured out what size of coffee you could buy to go with your donut using only the $1.80 in change in your pocket. I hope you get my point.

Students ask, "But why do I want to learn about business math?" The answer is simply this: money. Our world revolves around money. We all work to earn money. We need money to purchase our necessities, such as homes, transportation, food, and utilities. We also need money to enjoy the pleasures of life, including vacations, entertainment, and hobbies. We need money to retire. And businesses need money to survive and prosper.

Quite simply, business math teaches you monetary concepts and how to make smart decisions with your money.

1.1: What Is Business Math?

Business math is the study of mathematics required by the field of business. By the fact that you are reading this textbook, you must be interested in a business field such as accounting, marketing, human resources, or economics. Regardless of your path, you cannot avoid dealing with money and numbers.

Both personally and in your career you certainly use elementary arithmetic such as addition, subtraction, multiplication, and division. However, there is a whole field of mathematics that deals specifically with money. You will work with taxes, gross earnings, product prices, and currency exchange; you will be offered loans, lines of credit, mortgages, leases, savings bonds, and other financial tools. Do you know what these are and how these financial tools can maximize your earnings and minimize your costs? Do you have what it takes to execute smart monetary decisions both personally and for your business? Do you know how interest works and how it gets calculated? If you can answer "yes" to these questions, then you are already off to a great start. If not, by the end of this textbook you will have a better understanding of all of these topics and more.

How Do I Learn about Business Math?

Let's be realistic. In some areas of life and business, you can achieve a reasonable degree of understanding just by reading. However, reading about business mathematics without doing it would be disastrous. To succeed, you must follow a structured approach:

- Always read the content prior to your professor covering the topic in class.
- Attend class, ask questions, and explore the topic to advance your understanding.
- Do the homework and assignments—you absolutely must practice, practice, and practice!
- Seek help immediately when you need it.

Learning mathematics is like constructing a building. Each floor of the building requires the floor below it to be completed first. In mathematics, each section of a textbook requires the concepts and techniques from the sections that preceded it. If you have trouble with a concept, you must fix it NOW before it causes a large ripple effect on your ability to succeed in subsequent topics.

So the bottom line is that you absolutely cannot replace this approach—you must follow it.

1.2: How to Use the Features of this Book to Your Advantage

HOW TO SOLVE A MATH PROBLEM

Do you find it difficult to start a math problem? What do you do first? What is the next step? The reason this is challenging to some students is that they have never been shown problem-solving techniques. This textbook extensively uses an author-created design called the PUPP model in all guided examples. Successfully working through a problem involves four steps.

PUPP

- Plan,
- Understand
    ◦ What You Already Know
    ◦ How Will You get There
- Perform, and
- Present

The figure below shows the PUPP model at work. This model provides a structured problem-solving approach that will aid you not only in mathematics but in any problem-solving situation you may encounter.

EXAMPLE: STRAIGHTFORWARD INTEREST RATE CALCULATION—FV AND PV KNOWN

When Sandra borrowed $7,100 from Sanchez, she agreed to reimburse him $8,615.19 three years from now including interest compounded quarterly. What interest rate is being charged?

Plan

Find the nominal quarterly compounded rate of interest (IY).

What You Already Know (Understand)

Step 1:

- The present value, future value, term, and compounding are known, as illustrated in the timeline.
- CY = quarterly = 4
- Term   =   3   years
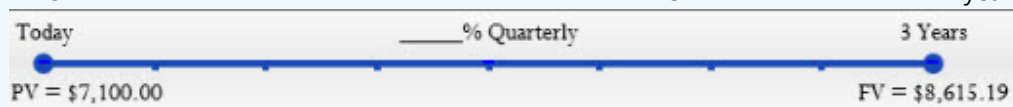
How You Will Get There (Understand)

Step 2: Calculate N using Formula 9.2.
Step 3: Substitute into Formula 9.3 and rearrange for i. Step 4: Substitute into Formula 9.1 and rearrange for IY.

Perform

Step 2:

- N = 4 × 3 = 12

Step 3:

- $8,615.10 = $7,100(1 + i)12
- 1.213394 = (1 + i)12
- 1
- 1.21339412 = 1 + ◇◇0.016249 = i

Step 4:

- 0.016249 = IY IY = 0.064996 = 0.065 or 6.5%
- 4

Calculator Instructions

N	I/Y	PV	PMT	FV	P/Y	C/Y
12	Answer: 6.499709	−7100	0	8615.10	4	4

Present

Sanchez is charging an interest rate of 6.5% compounded quarterly on the loan to Sandra.

Structure of the Loan
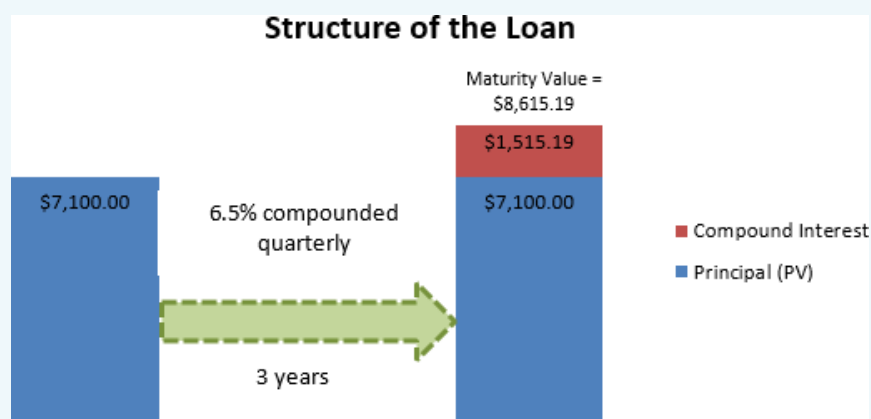
USING THE PUPP MODEL

To understand each of the steps in the PUPP model, let's plan a vacation.

Plan.

If you are heading out on vacation, the first thing you want to know is where you are going! Otherwise, how would you get there? In mathematics, you need to figure out what variable you are solving for before you can proceed. This step focuses your problem solving on the end goal. Many students mistakenly rush through this step and start punching numbers on a calculator or plugging numbers into formulas without understanding what it is they are doing. Spend plenty of time on this step, because the remaining three steps do not help at all if you solve for the wrong variable!

Understand – What You Already Know & How Will You get There.

In planning your vacation, once you figure out where you want to go you need to gather information about your destination. There are things you already know, but there are also things that you do not know, so you need a plan to figure these out. This step of the PUPP model helps your problem solving in two ways:

- ◦ **What You Already Know**First, you need to assess what information has already been provided to you. You need to assign values to variables. You may need to draw diagrams to understand

how all the numbers fit together.

- ◦ **How Will You get There**Second, you create a plan by which you will solve your problem. Your roadmap identifies how you go from Point A (some variable is unknown) to Point B (knowing what the variable is).

Perform.

Once your vacation is planned, all that is left is to go on vacation! In other words, execute your plan. This step involves all of the mechanics and computations of your roadmap that you developed in the previous step.

Present.

After you return from vacation, you tell everyone about what you just did. In mathematics, this means you must explain solutions in the context of the question asked. If you just calculated x = $700, what exactly does that represent and how should it be interpreted? Calculating a solution without understanding it is not good enough. A further benefit of this last step is that you can detect errors more easily when you have to explain the solution. For example, read this presentation statement: "If an investor puts $1,000 into a savings account that earns 10% interest, I have calculated thatafter one year the investor has $700 in her savings account." Did that make sense? If an account is earning interest, the balance should have become bigger, not smaller! Clearly, the solution of x = $700 must be an error!

Once you repeatedly practise this model and ingrain it in your thinking process, you have a structured approach to solving not just math problems but any life problem. When you are first learning the PUPP model you will need to write out every step of the model. As you keep practising, you'll find that the model becomes a way of thinking. You can never skip one of the steps, but you won't have to write them all out anymore because you'll execute some steps with rapid-fire thought!

UNDERSTANDING FORMULAS

You can't avoid formulas—this is math, after all. What helps, though, is to understand what the formulas do and how each component comes together to produce the solution. If you can understand, you do not need to memorize! This book lays formulas out visually, clearly labelling all symbols. In addition, each component is fully explained (see below). You are not required to cross-reference paragraph-style explanations to formulas, as the visual layout makes it all clear. The goal is for you to understand what exactly the formula does and how, so that you understand enough to recognize when solutions make sense or might be wrong.

N is Net Payment Amount: The net price is the amount after the discount has been applied. In other words, it is the dollar amount of the actual payment. If there is no cash discount on the payment, then the net price would be the same as the gross price.

Formula 6.1 – Single Discount: $N = L \times (1 - d)$

L is Invoice Balance: The "list" price is the amount of the invoice before any discounts are applied. In other words, it is the total dollar amount of the invoice and represents the balance owing.

d is Cash Discount Rate: This discount rate is the cash discount (in decimal format) that is deducted from the total invoice (the gross price) if payment is received within the discount period. This cash discount needs to be clearly stated in the terms of payment.

Additionally, you will not find any of those archaic algebraic symbols in this book. Representative symbols make the algebra easier to remember and understand. Those ancient mathematicians just loved to speak Greek (literally!), but this text speaks modern-day English and understands that we have technology here in the twenty-first century! We'll use symbols like N for net price, L for list price, P for profit, and E for expenses, letters that actually remind you of the variable they stand for!

DOES ANYONE ACTUALLY DO ANY OF THIS IN THE REAL WORLD?

As you read through the guided examples and do your homework, pause to consider how the question applies to you. Does it demonstrate a business application you can use to be a smarter business professional? How can you extend the question to similar situations? This textbook shows you examples from both business and personal situations that you either have already encountered or probably will encounter in the future. Remember: Math is all around you!

- **Business.** Why do business programs require a business math course in the first term or two? How is it relevant to your future? This textbook provides numerous examples of applications in marketing, finance, economics, accounting, and more. Once you complete this textbook, you should see clearly how business mathematics is relevant and important to your chosen future career.

- **Personal.** You will see companies and products from your everyday life. Real-world situations show you how to put a few extra dollars in your pocket by making smart mathematical choices. What if you could save $100 per month simply by making smarter choices? Over 40 years at a very conservative rate of interest, that is approximately $100,000 in your pocket. You will also find life lessons. Are you ready to start planning your RRSP? Would you like to know the basics on how to do this? If you are thinking of buying a home, do you know how the numbers work on your mortgage? To learn how to buy a car at the lowest cost, read on!

Paths To Success

This feature is designed to make you feel like part of the "in" crowd by letting you take advantage of shortcuts or tricks of the trade, such as some easy way to remember a particular technique or formula. It helps you see how some of the math could be made a little simpler or performed in a different way.

Things to Watch Out For

Awareness of common pitfalls and sources of error in mathematics reduces your chances of making a mistake. A reminder also helps when important concepts are about to be used, reused, or combined in a novel way. It gives you a "heads up" when needed.

Give it Some Thought

Mastering business math requires two key skills:

- Executing the required techniques and calculations
- Understanding and successfully integrating various mathematical concepts. (For example, when a variable changes, can you estimate how this affects the final result?)

The second skill is much harder to acquire than the first. The Give It Some Thought section, poses scenarios and questions for which no calculations are required. You need to visualize and conceptualize various mathematical theorems. Ultimately, you perform a self-test to see if you are "getting it."

Practice Questions

At the end of almost every section of this book you will find approximately 18 practice questions covering the concepts specifically introduced in that section, with final solutions listed to all questions at the end of this textbook. These questions are divided into three categories to help develop your mathematical skills and assess your abilities:

- Mechanics.
  This group of questions focuses on fundamental mathematical skills and formula usage. They are your first taste of using the section formulas and working with the mathematical concepts at an introductory level. Successful completion of the Mechanics section means that you:

  - Have at least a basic understanding of the variables at play,

  - Show rudimentary application of concepts, and

  - Can calculate solutions using the formulas.

- Applications.
  Applying your knowledge. These questions are more typical of business math course expectations (check with your instructor). The key difference from the Mechanics section is that you must now execute your problem-solving skills. These questions require you to determine the unknown variable and figure out how the various pieces of the puzzle come together in the solution. Successful completion of these questions means that you

  - Understand concepts at a satisfactory level,

  - Can problem solve typical business math applications, and

  - Can successfully integrate concepts at an acceptable performance level.

- Challenge, Critical Thinking, and Other Applications.
  These questions put your knowledge and understanding to the test. The difficulty bar is set high, as

these questions commonly require multistep solutions with advanced problem-solving techniques. As well, success might require you to integrate formulas, concepts, or procedures in novel ways. You will be challenged! Successful completion of these questions means that you

- ◦ Understand concepts at an above-average level,

- ◦ Can problem solve in difficult situations, and

- ◦ Can successfully integrate concepts at a higher level of thinking.

At this point, I hope that you can see that all the features of this textbook are built for you! They are here to help you learn, understand, and perform math. This book gives you every opportunity to succeed!

Now that you know what business mathematics is and why it is important to you, let's get going! Remember to take advantage of all the features of this textbook along the way.

CHAPTER 2: BACK TO THE BASICS

Where can you go in life and not be exposed to numbers and mathematics? Whether you are figuring out the price of a product (including shipping) on eBay, or balancing your bank accounts, you use your elementary mathematical skills from both your primary and secondary education.

Think about the math you perform every day:

- At the grocery store, you compare products to calculate the best value. One brand of potato chips retails for $3.99 for 300 g, while the equally satisfying brand beside it is priced at $3.49 for 250 g. Which offers the better value?

- As the host for a large gathering, you are preparing a homemade lasagna and need to triple the original recipe, which calls for 1 ⅔ cups of tomato sauce. In the expanded recipe, how many cups of tomato sauce do you need?

- If you are a sports fan, you know plenty of statistics about your favourite players and teams. Many come in percentage form, such as three-point throws for NBA stars or save percentages for NHL goaltenders. What exactly do those percentages mean?

- Many employers pay out bonuses. Perhaps in your company managers get twice as large a bonus as employees. Your company has five managers and 25 employees. If it announces a $35,000 total bonus, what is your share as an employee?

Mathematics and numbers surround you in the business world, where you must read many numerical reports, interpret how the numbers fit together, and create your own reports showing such metrics as sales and profit projections.

Away from work, you must manage your income and pay your bills. This is a mathematical problem you likely solve on a daily basis, ensuring that the money flowing out of your bank account does not exceed the money flowing in. To purchase groceries, vacations, or entertainment, you need numbers.

This chapter gives you a refresher on your basic mathematical skills, which are essential for success in later chapters.

Some instructors will review this chapter with you, while others will leave it up to you to complete this chapter independently. Either way, this chapter is important and should be used to test your basic abilities. For example, it would be unfortunate if you got a leasing calculation wrong because you made an error by breaking an order of operation rule even though you understood basic leasing concepts.

Approach this chapter with confidence, and if you encounter any difficulties ensure that you master the concepts before moving on to future chapters.

2.1 Order of Operations

PROCEED IN AN ORDERLY MANNER

You have just won $50,000 in a contest. Congratulations! But before you can claim it, you are required to answer a mathematical skill-testing question, and no calculators are permitted. After you hand over your winning ticket to the redemption agent, she hands you your time-limited skill-testing question: 2 × 5 + 30 ÷ 5. As the clock counts down, you think of the various possibilities. Is the answer 8, 14, 16, or something else altogether? Would it not be terrible to lose $50,000 because you cannot solve the question! If you figured out the solution is 16, you are on the right path. If you thought it was something else, this is a great time to review order of operations.

THE SYMBOLS

While some mathematical operations such as addition use a singular symbol (+), there are other operations, like multiplication, for which multiple representations are acceptable. With the advent of computers, even more new symbols have crept into mathematical symbology. The table below lists the various mathematical operations and the corresponding mathematical symbols you can use for them.

Mathematical Operation	Symbol or Appearance	Comments
Brackets	() or [] or {}	In order, these are known as round, square, and curly brackets.
Exponents	2^3 or 2^3	In 23, 3 is the exponent; an exponent is always written as a superscript. On a computer, the exponent is recognized with the ^ symbol; 23 is represented by 2^3.
Multiplication	× or * or • or 2(2) or (2)(2)	In order, these are known as times, star, or bullet. The last two involve implied multiplication, which is when two terms are writtennext to each other joined only by brackets.
Division	/ or ÷ or $^4/_2$	In order, these are known as the slash, divisor, and divisor line. Note: in the last example that the division is represented by the horizontal line.
Addition	+	There are no other symbols.
Subtraction	–	There are no other symbols.

You may wonder if the different types of brackets mean different things. Although mathematical fields like calculus use specialized interpretations for the different brackets, business math uses all the brackets to help the reader visually pair up the brackets. Consider the following two examples:

> Examples

> Example 1: 3 × (4 / (6 – (2 + 2)) + 2)
> Example 2: 3 × [4 / {6 – (2 + 2)} + 2]

Notice that in the second example you can pair up the brackets much more easily, but changing the shape of the brackets did not change the mathematical expression. This is important to understand when using a calculator, which usually has only round brackets. Since the shape of the bracket has no mathematical impact, solving example 1 or example 2 would involve the repeated usage of the round brackets.

BEDMAS

In the section opener, your skill-testing question was 2 × 5 + 30 ÷ 5. Do you just solve this expression from left to right, or should you start somewhere else? To prevent any confusion over how to resolve these mathematical operations, there is an agreed-upon sequence of mathematical steps commonly referred to as BEDMAS.

> BEDMAS
>
> **BEDMAS** is an acronym for **B**rackets, **E**xponents, **D**ivision, **M**ultiplication, **A**ddition, and **S**ubtraction.

How It Works

Step 1: Brackets must be resolved first. As brackets can be nested inside of each other, you must resolve the innermost set of brackets first before proceeding outwards to the next set of brackets. When resolving a set of brackets, you must perform the mathematical operations within the brackets by following the remaining steps in this model (EDMAS). If there is more than one set of brackets but the sets are not nested, work from left to right and top to bottom.

Step 2: If the expression has any exponents, you must resolve these next. Remember that an exponent indicates how many times you need to multiply the base against itself. For example,

$$2^3 = 2 \times 2 \times 2.$$

Step 3: The order of appearance for multiplication and division does not matter. However, you must resolve these operations in order from left to right and top to bottom as they appear in the expression.

Step 4: The last operations to be completed are addition and subtraction. The order of appearance doesn't matter; however, you must complete the operations working left to right through the expression.

IMPORTANT NOTES

Before proceeding with the Texas Instruments BAII Plus calculator, you must change some of the factory defaults, as explained in the table below. To change the defaults, open the Format window on your calculator. If for any reason your calculator is reset (either by removing the battery or pressing the reset button), you must perform this sequence again.

Buttons Pushed	Calculator Display	What It Means
2nd Format	DEC=2.00	You have opened the Format window to its first setting. DEC tells your calculator how to round the calculations. In business math, it is important to be accurate. Therefore, we will set the calculator to what is called a floating display, which means your calculator will carry all of the decimals and display as many as possible on the screen.
9 Enter	DEC=9.	The floating decimal setting is now in place. Let us proceed.
↓	DEG	This setting has nothing to do with business math and is just left alone. If it does not read DEG, press 2nd Set to toggle it.
↓	US 12-31-1990	Dates can be entered into the calculator. North Americans and Europeans use slightly different formats for dates. Your display is indicating North American format and is acceptable for our purposes. If it does not read US, press 2nd Set to toggle it.
↓	US 1,000	In North America it is common to separate numbers into blocks of 3 using a comma. Europeans do it slightly differently. This setting is acceptable for our purposes. If your display does not read US, press 2nd Set to toggle it.
↓	Chn	There are two ways that calculators can solve equations. This is known as the Chain method, which means that your calculator will simply resolve equations as you punch it in without regard for the rules of BEDMAS. This is not acceptable and needs to be changed.
2nd Set	AOS	AOS stands for Algebraic Operating System. This means that the calculator is now programmed to use BEDMAS in solving equations.
2nd Quit	0.	Back to regular calculator usage.

Also note that on the BAII Plus calculator you have two ways to key in an exponent:

1. If the exponent is squaring the base (e.g., 3^2), press 3 x^2. It calculates the solution of 9.
2. If the exponent is anything other than a 2, you must use the y^x button. For 23, you press 2 y^x 3 =. It calculates the solution of 8.

THINGS TO WATCH OUT FOR

Negative Signs

Remember that mathematics uses both positive numbers (such as +3) and negative numbers (such as −3).

Positive numbers do not need to have the + sign placed in front of them since it is implied. Thus +3 is written as just 3. Negative numbers, though, must have the negative sign placed in front of them. Be careful not to confuse the terminology of a negative number with a subtraction or minus sign. For example, 4 + (−3) is read as "four plus negative three" and not "four plus minus three." To key a negative number on a calculator, enter the number first followed by the ± button, which switches the sign of the number.

The Horizontal Divisor Line

One of the areas in which people make the most mistakes involves the "hidden brackets." This problem almost always occurs when the horizontal line is used to represent division. Consider the following mathematical expression:

$$(4 + 6) \div (2 + 3)$$

If you rewrite this expression using the horizontal line to represent the divisor, it looks like this:

$$\frac{4+6}{2+3}$$

Notice that the brackets disappear from the expression when you write it with the horizontal divisor line because they are implied by the manner in which the expression appears. Your best approach when working with a horizontal divisor line is to reinsert the brackets around the terms on both the top and bottom. Thus, the expression looks like this:

$$\frac{(4+6)}{(2+3)}$$

Employing this technique will ensure that you arrive at the correct solution, especially when using calculators.

PATHS TO SUCCESS

Hidden and Implied Symbols

If there are hidden or implied symbols in the expressions, your first step is to reinsert those hidden symbols in their correct locations. In the example below, note how the hidden multiplication and brackets are reinserted into the expression:

$$4\frac{3+2^2 \times 3}{(2+8) \div 2}$$

transforms into

$$4x\frac{3+2^2 \times 3}{(2+8) \div 2}$$

Once you have reinserted the symbols, you are ready to follow the BEDMAS model.

Calculators are not programmed to be capable of recognizing implied symbols. If you key in "3(4 + 2)" on your calculator, failing to input the multiplication sign between the "3" and the "(4 + 2)," you get a solution of 6. Your calculator ignores the "3" since it doesn't know what mathematical operation to perform on it. To have your calculator solve the expression correctly, you must punch the equation through as "3 × (4 + 2) =". This produces the correct answer of 18.

Simplifying Negatives

If your question involves positive and negative numbers, it is sometimes confusing to know what symbol to put when simplifying or solving. Remember these two rules:

Key Takeaways

Rule #1: A pair of the same symbols is always positive.
Thus "4 + (+3)" and "4 − (−3)" both become "4 + 3."
Rule #2: A pair of the opposite symbols is always negative.
Thus "4 + (−3)" and "4 − (+3)" both become "4 − 3."

A simple way to remember these rules is to count the total sticks involved, where a "+" sign has two sticks and a "−" sign has one stick. If you have an odd number of total sticks, the outcome is a negative sign. If you have an even number of total sticks, the outcome is a positive sign.

Note the following examples:

- 4 + (−3) =
- → 3 total sticks is odd and therefore simplifies to negative
- → 4 − 3 = 1
- (−2) × (−2) =
- → 2 total sticks is even and therefore simplifies to positive
- → (−2) ×(− 2) = +4

EXAMPLE 2.1 A: SOLVING EXPRESSIONS USING BEDMAS

Evaluate each of the following expressions.

a. $2 \times 5 + 30 \div 5$

b. $(6+3)^2 + 18 \div 2$

c. $4 \times [\dfrac{\{3 + 2^2 \times 3\}}{\{(2+8) \div 2\}}]$

Plan

You need to evaluate each of the expressions. This means you must solve each expression.

Understand – What You Already Know

You are provided with the mathematical expressions in a formula format. These expressions are ready for you to solve.

How You Will Get There

Employ the knowledge of BEDMAS to solve each operation in its correct order.

Perform

Question A

2 x 5 + 30 ÷ 5	Step 1: No Brackets
	Step 2: No exponents
	Step 3: Working left to right, resolve the multiplication first
10 + 30 ÷ 5	Step 4: Now resolve the division
10 + 6	Step 5: Perform the remaining addition
16	Final Solution

Question B

$(6+3)^2$ + 18 ÷ 2	Step 1: Start with the brackets and perform BEDMAS; you have only an addition to perform
9 + 18 ÷ 2	Step 2: Resolve the exponent
81 + 18 ÷ 2	Step 3: Perform the division
81 + 9	Step 4: Perform the addition
90	Final Solution

Question C

$$4 \times \left[\frac{\{3 + 2^2 \times 3\}}{\{(2 + 8) \div 2\}}\right]$$

Step 1: Start with the innermost set of brackets (2 + 8) and perform BEDMAS. In this bracket you only have one addition to resolve.

$$4 \times \left[\frac{\{3 + 2^2 \times 3\}}{\{10 \div 2\}}\right]$$

The innermost brackets are complete, so you now drop them. You still have two sets of inner {} brackets. Start with the top one and perform BEDMAS.

$$4 \times \left[\frac{\{3 + 4 \times 3\}}{\{10 \div 2\}}\right]$$

Exponent resolved.

$$4 \times \left[\frac{\{3 + 12\}}{\{10 \div 2\}}\right]$$

Multiplication performed.

$$4 \times \left[\frac{15}{10 \div 2\}}\right]$$

Addition performed. You no longer need the brackets so you drop them. Step 1: Now do the bottom brackets performing EDMAS. You have only one division to resolve.

$$4 \times \left[\frac{15}{5}\right]$$

Division is completed. You no longer need the brackets so you drop them.
Step 1: One last set of brackets to go! Only 1 division left.

$$4 \times 3$$

Division performed. You no longer need the brackets so you drop them. Step 2: No exponents. Step 3: perform the multiplication.

$$12$$

Step 4: Not Needed – Final Solution

Final Solutions

a. 16
b. 90
c. 12

SECTION 2.1 EXERCISES

Hint: If a question involves money, round the answers to the nearest cent.

Mechanics

1.	81 ÷ 27 + 3 × 4
2.	100 ÷ (5 × 4 – 5 × 2)
3.	33 – 9 + (1 + 7 × 3)
4.	(6 + 3)2 – 17 × 3 + 70
5.	100 – (42 + 3) – (3 + 9 × 3 – 4)

Applications

6.	$[(7^2 - \{-41\}) - 5 \times 2] \div (80 \div 10)$
7.	$\$1,000(1 + 0.09 \times \dfrac{88}{365})$
8.	$3[\$2,000(1 + 0.003)^8] + \$1,500$
9.	$\dfrac{20,000}{1 + 0.07 \times 712}$
10.	$4 \times [(52 + 15)^2 \div (13^2 - 9)]^2$
11.	$500[\dfrac{(1 + 0.00875)^{43} - 1}{0.00875}]$
12	$1,000(1 + \dfrac{0.12}{6})^{15}$

Challenge, Critical Thinking, & Other Applications

13. $$\frac{\$2,500}{1+0.10} + \frac{\$7,500}{(1+0.10)^2} + \frac{-\$1,500}{(1+0.10)^3} + \frac{-\$2,000}{(1+0.10)^4}$$

14. $$\$175,000(1+0.07)^{15} + \$14,000\frac{(1+0.07)^{20}-1}{0.07}$$

15. $$\$5,000[\frac{\{1+[(1+0.08)^{0.5}-1]\}^{75}-1}{(1+0.08)^{0.5}-1}0]$$

16. $$\$800[\frac{(1+0.07)^{20}-(1+0.03)^{20}}{0.07-0.03}]$$

17. $$\$60,000(1+0.0058)^{80} - \$450[\frac{(1+0.0058)^{25}-1}{0.07}]$$

18. $$(\frac{0.08}{2})\$1,000[\frac{1-\frac{1}{\{1+(\frac{0.08}{2})\}^{16}}}{\frac{0.08}{2}}] + \$1,000[1+(\frac{0.08}{2})]^{16}$$

19. $$\$1,475\frac{(1+\frac{0.06}{4})^{16}-1}{\frac{0.06}{4}}]$$

20. $$\$6,250(1+0.525)^{10} + \$325[\frac{(1+0.0525)^{10}-1}{0.0525}]$$

2.2 Fractions, Decimals, & Rounding

JUST ONE SLICE OF PIE, PLEASE

Your local newspaper quotes a political candidate as saying, "The top half of the students are well-educated, the bottom half receive extra help, but the middle half we are leaving out."[1] You stare at the sentence for a moment and then laugh. To halve something means to split it into two. However, there are three halves here! You conclude that the speaker was not thinking carefully.

In coming to this conclusion, you are applying your knowledge of fractions. In this section, you will review fraction types, convert fractions into decimals, perform operations on fractions, and also address rounding issues in business mathematics.

TYPES OF FRACTIONS

To understand the characteristics, rules, and procedures for working with fractions, you must become familiar with fraction terminology. First of all, what is a fraction? A **fraction** is a part of a whole. It is written in one of three formats:

$$1/2 \text{ or } \quad {}^{1}/_{2} \text{ or } \quad \frac{1}{2}$$

Each of these formats means exactly the same thing. The number on the top, side, or to the left of the line is known as the **numerator**. The number on the bottom, side, or to the right of the line is known as the **denominator**. The slash or line in the middle is the **divisor line**. In the above example, the numerator is 1 and the denominator is 2. There are five different types of fractions, as explained in the table below.

---

1. 1 Neal, Marcia. Candidate for the 3rd Congressional District Colorado State Board of Education, as quoted in Perez, Gayle. 2008. "Retired School Teacher Seeks State Board Seat." Pueblo Chieftain.

Fraction	Terminology	Characteristics	Result of Division*
$\dfrac{2}{5}$	Proper	The numerator is smaller than the denominator.	Answer is between 0 and 1
$\dfrac{5}{2}$	Improper	The numerator is larger than the denominator.	Answer is greater than 1
$3\dfrac{2}{5}$	Compound	A fraction that combines an integer with either a proper or improper fraction. When the division is performed, the proper or improper fraction is *added* to the integer.	Answer is greater than the integer
$3\dfrac{\frac{2}{5}}{7}$	Complex	A fraction that has fractions within fractions, combining elements of compound, proper, or improper fractions together. It is important to follow BEDMAS in resolving these fractions.	Answer varies depending on the fractions involved
$\dfrac{1}{2} and \dfrac{2}{4}$	Equivalent	Two or more fractions of any type that have the same numerical value upon completion of the division. Note that both of these examples work out to 0.5.	Answers are equal

*Assuming all numbers are positive.

HOW IT WORKS

First, focus on the correct identification of proper, improper, compound, equivalent, and complex fractions. In the next section, you will work through how to accurately convert these fractions into their decimal equivalents.

Equivalent fractions require you to either solve for an unknown term or express the fraction in larger or smaller terms.

Solving For An Unknown Term

These situations involve two fractions where only one of the numerators or denominators is missing. Follow this four-step procedure to solve for the unknown:

Step 1: Set up the two fractions.

Step 2: Note that your equation contains two numerators and two denominators. Pick the pair for which you know both values.

Step 3: Determine the multiplication or division relationship between the two numbers.

Step 4: Apply the same relationship to the pair of numerators or denominators containing the unknown.

Assume you are having a party and one of your friends says he would like to eat one-third of the pizza. You notice the pizza has been cut into nine slices. How many slices would you give to your friend?

Step 1: Your friend wants one out of three pieces. This is one-third. You want to know how many pieces out of nine

to give him. Assign a meaningful variable to represent your unknown, so have *s* represent the number of slices to give; you need to give him *s* out of 9 pieces, or *s*/9.

$$\frac{1}{3} = \frac{s}{9}$$

Step 2: Work with the denominators 3 and 9 since you know both of them.

Step 3: Take the larger number and divide it by the smaller number. We have 9 ÷ 3 = 3. Therefore, the denominator on the right is three times larger than the denominator on the left.

Step 4: Take the 1 and multiply it by 3 to get the *s*.
Therefore, *s* = 1 × 3 = 3.

$$\frac{1 \times 3}{3 \times 3} = \frac{3}{9}$$

You should give your friend **three** slices of pizza.

Expressing The Fraction In Larger Or Smaller Terms.

When you need to make a fraction easier to understand or you need to express it in a certain format, it helps to try to express it in larger or smaller terms. To express a fraction in larger terms, multiply both the numerator and denominator by the same number. To express a fraction in smaller terms, divide both the numerator and denominator by the same number.

Larger terms:

$$\frac{2}{12}$$

expressed with terms twice as large would be

$$\frac{2 \times 2}{12 \times 2} = \frac{4}{24}$$

Smaller terms: 2 expressed with terms half as large would be 2 ÷ 2 = 1

When expressing fractions in higher or lower terms, you do not want to introduce decimals into the fraction unless there would be a specific reason for doing so. For example, if you divided 4 into both the numerator and denominator of

$$\frac{2}{12}$$

,you would have

$$\frac{0.5}{3},$$

which is not a typical format. To find numbers that divide evenly into the numerator or denominator (called factoring), follow these steps:

- Pick the smallest number in the fraction.
- Use your multiplication tables and start with 1× before proceeding to 2×, 3×, and so on. When you find a number that works, check to see if it also divides evenly into the other number.

For example, if the fraction is

$$\frac{12}{18}$$

you would factor the numerator of 12. Note that 1 × 12 = 12; however, 12 does not divide evenly into the denominator. Next you try 2 × 6 and discover that 6 does divide evenly into the denominator.

Therefore, you reduce the fraction to smaller terms by dividing by 6, or

$$\frac{12 \div 6}{18 \div 6} = \frac{2}{3}$$

.

THINGS TO WATCH OUT FOR

With complex fractions, it is critical to obey the rules of BEDMAS. As suggested in Section 2.1, always reinsert the hidden symbols before solving. Note in the following example that an addition sign and two sets of brackets were hidden: You should rewrite

$$3\frac{\frac{2}{5}}{7} \, as \, 3 + [\frac{\frac{2}{5}}{7}]$$

before you attempt BEDMAS.

PATHS TO SUCCESS

What do you do when there is a negative sign in front of a fraction, such as

$$-\frac{1}{2}?$$

Do you put the negative with the numerator or the denominator? The common solution is to multiply the numerator by negative 1, resulting in

$$\frac{(-1) \times 1}{2} = \frac{-1}{2}$$

.

In the special case of a compound fraction, multiply the entire fraction by −1. Thus,

$$-1\frac{1}{2} = (-1) \times (1 + \frac{1}{2}) = -1 - \frac{1}{2}$$

.

EXAMPLE 2.2 A IDENTIFYING TYPES OF FRACTIONS

Identify the type of fraction represented by each of the following:

A. $\dfrac{2}{3}$

B. $6\dfrac{7}{8}$

C. $12\dfrac{\frac{4}{3}}{6\frac{4}{5}}$

D. $\dfrac{15}{11}$

E. $\dfrac{5}{6}$

F. $\dfrac{3}{4} \& \dfrac{9}{12}$

Plan

For each of these six fractions, identify the type of fraction.

What You Already Know

There are five types of fractions, including proper, improper, compound, complex, or equivalent

How You Will Get There

Examine each fraction for its characteristics and match these characteristics with the definition of the fraction.

Perform

a. $\dfrac{2}{3}$

The numerator is smaller than the denominator. This matches the characteristics of a proper fraction.

b. $6\dfrac{7}{8}$

This fraction combines an integer with a proper fraction (since the numerator is smaller than the denominator). This matches the characteristics of a compound fraction.

c. $12\dfrac{\frac{4}{3}}{6\frac{4}{5}}$

There are lots of fractions involving fractions nested inside other fractions. The fraction as a whole is a compound fraction, containing an integer with a proper fraction (since the numerator is smaller than the denominator). Within the proper fraction, the numerator is an improper fraction $\dfrac{4}{3}$ and the denominator is a compound fraction containing an integer and a proper fraction $6\dfrac{4}{5}$. This all matches the definition of a complex fraction: nested fractions combining elements of compound, proper, and improper fractions together.

d. $\dfrac{15}{11}$

The numerator is larger than the denominator. This matches the characteristics of an improper fraction.

e. $\dfrac{5}{6}$

The numerator is smaller than the denominator. This matches the characteristics of a proper fraction.

f. $\dfrac{3}{4} \& \dfrac{9}{12}$

There are two proper fractions here that are equal to each other. If you were to complete the division, both fractions calculate to 0.75. These are equivalent fractions.

Present

Of the six fractions examined, there are two proper fractions (a and e), one improper fraction (d), one compound fraction (b), one complex fraction (c), and one equivalent fraction (f).

EXAMPLE 2.2 B

a   Solve for the unknown term x
. $$\frac{7}{12} = \frac{49}{x}$$

b   Express this fraction in lower case terms
. $$\frac{5}{50}$$

Plan

a. Find the value of the unknown term, x.
b. Take the proper fraction and express it in a lower term.

What You Already Know

The needed fractions in a ready-to-solve format are provided.

How You Will Get There

a. Apply the four-step technique to solving equivalent fractions. The first step has already been done for you, in that the equation is already set up.

b. Find a common divisor that divides evenly into both the numerator and denominator. As only 1 and 5 go into the number 5, it makes sense that you should choose 5 to divide into both the numerator and denominator.

Note that 5 factors evenly into the denominator, 50, meaning that no remainder or decimals are left over.

Perform

a.Step 2: You have both of the numerators, so work with that pair.

Step 3: Take the larger number and divide by the smaller number, or 49 ÷ 7 = 7. Therefore, multiply the fraction on the left by 7 to get the fraction on the right.

Step 4: Applying the same relationship, 12 × 7 = 84.

$$\frac{5 \div 5}{50 \div 5} = \frac{1}{10}$$

CONVERTING TO DECIMALS

Although fractions are common, many people have trouble interpreting them. For example, in comparing $^{27}/_{37}$ to $^{57}/_{73}$, which is the larger number? The solution is not immediately apparent. As well, imagine a retail world where your local Walmart was having a $^{3}/_{20}$ th off sale! It's not that easy to realize that this equates to 15% off. In other words, fractions are converted into decimals by performing the division to make them easier to understand and compare.

How It Works

The rules for converting fractions into decimals are based on the fraction types.

Proper and Improper Fractions

Resolve the division. For example, $^{3}/_{4}$ is the same as 3 ÷ 4 = 0.75.
As well, 5 = 5 ÷ 4 = 1.25.

Compound Fractions

The decimal number and the fraction are joined by a hidden addition symbol. Therefore, to convert to a decimal you need to reinsert the addition symbol and apply BEDMAS:

$$3\,^{4}/_{5} = 3 + 4 \div 5 = 3 + 0.8 = 3.8$$

Complex Fractions.

The critical skill here is to reinsert all of the hidden symbols and then apply the rules of BEDMAS:

$$2\frac{\frac{11}{14}}{1\frac{1}{4}} = 2 + [\frac{(11 \div 4)}{(1 + 1 \div 4)}] = 2 + [\frac{(11 \div 4)}{(1 + 0.25)}] = 2 + [\frac{2.75}{1.25}] = 2 + 2.2 = 4.2$$

EXAMPLE 2.2 C CONVERTING FRACTIONS TO DECIMALS

a.
$$\frac{2}{5}$$

b.
$$6\frac{7}{8}$$

c.
$$12\frac{\frac{9}{12}}{1\frac{2}{10}}$$

Plan

Take these fractions and convert them into decimal numbers.

What You Know Already

The three fractions are provided and ready to convert.

How Will You Get There

a. This is a proper fraction requiring you to complete the division.
b. This is a compound fraction requiring you to reinsert the hidden addition symbol and then apply BEDMAS.
c. This is a complex fraction requiring you to reinsert all hidden symbols and apply BEDMAS.

Perform

a.
2/5 = 2 ÷ 5 = 0.4

b.
6 7/8 = 6 + 7 ÷ 8 = 6 + 0.875 = 6.875

c.
$$12\frac{\frac{9}{2}}{1\frac{2}{10}} = 12 + [\frac{9 \div 2}{1 + 2 \div 10}] = 12+]\frac{9 \div 2}{1 + 0.2}] = 12 + [\frac{4.5}{1.2}] = 12 + 3.75 = 15.75$$

> **Present**
>
> In decimal format, the fractions have converted to 0.4, 6.875, and 15.75, respectively.

ROUNDING PRINCIPLE

Your company needs to take out a loan to cover some short-term debt. The bank has a posted rate of 6.875%. Your bank officer tells you that, for simplicity, she will just round off your interest rate to 6.9%. Is that all right with you? It shouldn't be!

What this example illustrates is the importance of rounding. This is a slightly tricky concept that confuses most students to some degree. In business math, sometimes you should round your calculations off and sometimes you need to retain all of the digits to maintain accuracy.

How It Works

To round a number off, you always look at the number to the right of the digit being rounded. If that number is 5 or higher, you add one to your digit; this is called *rounding up*. If that number is 4 or less, you leave your digit alone; this is called *rounding down*.

For example, if you are rounding 8.345 to two decimals, you need to examine the number in the third decimal place (the one to the right). It is a 5, so you add one to the second digit and the number becomes 8.35.

For a second example, let's round 3.6543 to the third decimal place. Therefore, you look at the fourth decimal position, which is a 3. As the rule says, you would leave the digit alone and the number becomes 3.654.

NONTERMINATING DECIMALS

What happens when you perform a calculation and the decimal doesn't terminate?

You need to assess if there is a pattern in the decimals

The Nonterminating Decimal without a Pattern: For example, $6/17$ = 0.352941176 … with no apparent ending decimal and no pattern to the decimals.

The Nonterminating Decimal with a Pattern: For example, $2/11$ = 0.18181818 … endlessly. You can see that the numbers 1 and 8 repeat. A shorthand way of expressing this is to place a horizontal line above the digits that repeat. Thus, you can rewrite 0.18181818 … as 0.18.

You need to know if the number represents an interim or final solution to a problem

Interim Solution: You must carry forward all of the decimals in your calculations, as the number should not be rounded until you arrive at a final answer. If you are completing the question by hand, write out as many decimals as possible; to save space and time, you can use the shorthand horizontal bar for repeating decimals. If you are completing the question by calculator, store the entire number in a memory cell.

Final Solution: To round this number off, an industry protocol or other clear instruction must apply. If these do not exist, then you would make an arbitrary rounding choice, subject to the condition that you must maintain enough precision to allow for reasonable interpretation of the information.

IMPORTANT NOTES

To assist in your calculations, particularly those that involve multiple steps to resolve, your calculator has 10 memory cells. Your display is limited to 10 digits, but when you store a number into a memory cell the calculator retains all of the decimals associated with the number, not just those displaying on the screen. Your calculator can, in fact, carry up to 13 digit positions. It is strongly recommended that you take advantage of this feature where needed throughout this textbook.

Let's say that you just finished keying in $^6/_{17}$ on your calculator, and the resultant number is an interim solution that you need for another step. With 0.352941176 on your display, press STO followed by any numerical digit on the keypad of your calculator. STO stands for *store*. To store the number into memory cell 1, for example, press STO 1. The number with 13 digits is now in permanent memory. If you clear your calculator (press CE/C) and press RCL # (where # is the memory cell number), you will bring the stored number back. RCL stands for *recall*. Press RCL 1. The stored number 0.352941176 reappears on the screen.

EXAMPLE 2.2 D: ROUNDING NUMBERS

A.   6/13

B.   4/9

C.   4/11

D.   3/22

E. $5\dfrac{\frac{1}{7}}{\frac{10}{27}}$

Plan

Convert each of the fractions into decimal format, then round any final answer to four decimals or use repeating decimal notation.

What You Know Already

The fractions and clear instructions on how to round them have been provided.

How Will You Get There

To convert the fractions to decimals, you need to complete the division by obeying the rules of BEDMAS. Watch for hidden symbols and adhere to the rules of rounding.

Perform

a.   6/13 = 0.461538. The fifth decimal is a 3, so round down. The answer is 0.4615.

b.   4/9 = 0.444444. Note the repeating decimal of 4. Using the horizontal bar, write 0. 4

c.   4/11 = 0.363636. Note the repeating decimals of 3 and 6. Using the horizontal bar, write 0. 36

d.   3/22 = 0.136363. Note the repeating decimals of 3 and 6 after the 1. Using the horizontal bar, write 0.136

e.
$$5\frac{\frac{1}{7}}{\frac{10}{27}} = 5 + \frac{(1 \div 7)}{(10 \div 27)} = 5 + \frac{0.142857}{0.370} = 5 + 0.385714 = 5.385714.$$

Since the fifth digit is a 1, round down. The answer is 5.3857.

Present

According to the rounding instructions, the solutions are 0.4615, 0. 4, 0. 36, 0.136 and 5.3857, respectively.

ROUNDING RULES

One of the most common sources of difficulties in math is that different people sometimes use different standards for rounding. This seriously interferes with the consistency of final solutions and makes it hard to assess accuracy. So that everyone arrives at the same solution to the exercises/examples in this textbook, these rounding rules apply throughout the book:

1.   Never round an interim solution unless there is a logical reason or business process that forces the

number to be rounded. Here are some examples of logical reasons or business processes indicating you should round:

      a. You withdraw money or transfer it between different bank accounts. In doing so, you can only record two decimals and therefore any money moving between the financial tools must be rounded to two decimals.

      b. You need to write the numbers in a financial statement or charge a price for a product. As our currency is in dollars and cents, only two decimals can appear.

2. When you write nonterminating decimals, show only the first six (or up to six) decimals. Use the horizontal line format for repeating decimals. If the number is not a final solution, then assume that all decimals or as many as possible are being carried forward.

3. Round all final numbers to six decimals in decimal format and four decimals in percentage format unless instructions indicate otherwise.

4. Round final solutions according to common business practices, practical limitations, or specific instructions. For example, round any final answer involving dollar currency to two decimals. These types of common business practices and any exceptions are discussed as they arise at various points in this textbook.

5. Generally avoid writing zeros, which are not required at the end of decimals, unless they are required to meet a rounding standard or to visually line up a sequence of numbers. For example, write 6.340 as 6.34 since there is no difference in interpretation through dropping the zero.

Paths To Success

Does your final solution vary from the actual solution by a small amount? Did the question involve multiple steps or calculations to get the final answer? Were lots of decimals or fractions involved? If you answer yes to these questions, the most common source of error lies in rounding. Here are some quick error checks for answers that are "close":

1. Did you remember to obey the rounding rules laid out above? Most importantly, are you carrying decimals for interim solutions and rounding only at final solutions?

2. Did you resolve each fraction or step accurately? Check for incorrect calculations or easy-to-make errors, like transposed numbers.

3. Did you break any rules of BEDMAS?

EXERCISES

Mechanics

In each of the following equations, identify the value of the unknown term.

1 Convert each of the following fractions into decimal format.

a.	b.	c.	d.	e.	f.	g.	h.
1/8	3 3/4	10/9	34/49	$1\dfrac{\frac{2}{3}}{9\frac{5}{3}}$	56/27	$\dfrac{10\frac{1}{5}}{9}$	6/11

2. In each of the following equations, identify the value of the unknown term.

a. $\dfrac{3}{4} = \dfrac{x}{36}$

b. $\dfrac{y}{8} = \dfrac{16}{64}$

c. $\dfrac{2}{z} = \dfrac{18}{5}$

d. $\dfrac{5}{6} = \dfrac{75}{p}$

3. Take each of the following fractions and provide one example of the fraction expressed in both higher and lower terms.

a. $\dfrac{5}{10}$

b. $\dfrac{6}{8}$

4. Convert each of the following fractions into decimal format.

a. $\dfrac{7}{8}$

b. $15\dfrac{3}{4}$

c. $\dfrac{13}{5}$

d. $133\dfrac{\frac{17}{2}}{3\frac{2}{5}}$

5. Convert each of the following fractions into decimal format and round to three decimals.

a. $\dfrac{7}{8}$

b. $15\dfrac{3}{4}$

c. $\dfrac{10}{9}$

d. $\dfrac{15}{32}$

6. Convert each of the following fractions into decimal format and express in repeating decimal notation.

a. $\dfrac{1}{12}$

b. $5\dfrac{8}{33}$

c. $\dfrac{4}{3}$

d. $\dfrac{-34}{110}$

Applications

7. Calculate the solution to each of the following expressions. Express your answer in decimal format.

a. $\dfrac{1}{5} + 3\dfrac{1}{4} + \dfrac{5}{2}$

b. $\dfrac{1\frac{3}{8}}{2} - \dfrac{11}{40} + 19\dfrac{1}{2} \times \dfrac{3}{4}$

8. Calculate the solution to each of the following expressions. Express your answer in decimal format with two decimals.

a. $(1 + \dfrac{0.11}{12})^4$

b. $1 - 0.05 \times \dfrac{263}{365}$

c. $200[1 - \dfrac{1}{(1 + \frac{0.10}{4})^2}]$

9. Calculate the solution to each of the following expressions. Express your answer in repeating decimal notation as needed.

a. $\dfrac{1}{11} + 3\dfrac{1}{9}$

b. $\dfrac{5}{3} - \dfrac{7}{6}$

Questions 10–14 involve fractions. For each, evaluate the expression and round your answer to the nearest cent.

10. $134,000 (1+ 0.14 \times 23/365)$

11. $\$10,000(1 + \dfrac{0.0525}{2})13$

12. $\$2,995(1 + 0.13 \times \dfrac{90}{365}) - \dfrac{\$400}{1 + 0.13 \times \frac{15}{365}}$

13. $\dfrac{\$535,000}{(1 + \frac{0.07}{12})^3}$

14. $\dfrac{\$155,600}{(1 + \frac{0.06}{12})^8}$

Challenge, Critical Thinking, & Other Applications

Questions 15–20 involve more complex fractions and reflect business math equations encountered later in this textbook. For each, evaluate the expression and round your answer to the nearest cent.

15. $\dfrac{\$648}{\frac{0.0575}{12}}\left[1 - \dfrac{1}{(1 + \frac{0.0575}{12})^7}\right]$

16. $\dfrac{\$10,000}{(1 + \frac{0.115}{4})^2} + \$68\dfrac{\left[1 - \frac{1}{(1 + \frac{0.115}{4})^2}\right]}{\frac{0.115}{4}}$

17. $\dfrac{\$2,000,000}{\left[\frac{(1 + \frac{0.065}{2})^{12} - 1}{\frac{0.065}{2}}\right]}$

18. $\$8,500\left[1 - \dfrac{1 - (\frac{1}{1.08})^4}{1.08}\right] + \$19,750(\dfrac{1}{1.08})^4 - \$4,350$

19. $\$15,000\left[\dfrac{(1 + \frac{0.058}{4})^{16} - 1}{\frac{0.058}{4}}\right]$

20. $\dfrac{0.08}{2}(\$1000)\left[\dfrac{1 - \frac{1}{(1 + \frac{0.07}{2})}}{\frac{0.07}{2}}\right] + \$1,000\dfrac{1}{(1 + \frac{0.07}{2})^{10}}$

2.3 Percentages

HOW DOES IT ALL RELATE?

Your class just wrote its first math quiz. You got 13 out of 19 questions correct, or $^{13}/_{19}$. In speaking with your friends Sandhu and Illija, who are in other classes, you find out that they also wrote math quizzes; however, theirs were different.
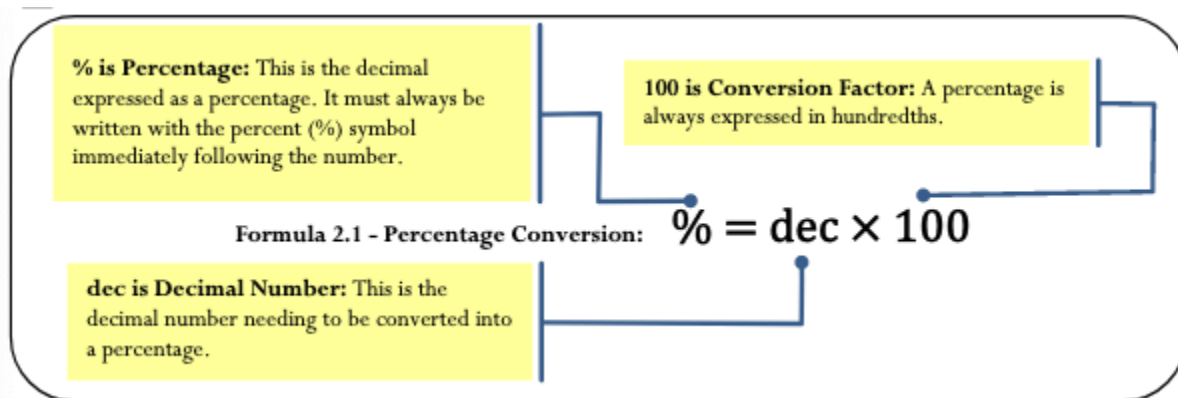
Sandhu scored 16 out of 23, or $^{16}/_{23}$, while Illija got 11 out of 16, or $^{11}/_{16}$. Who achieved the highest grade? Who had the lowest? The answers are not readily apparent, because fractions are difficult to compare.

Now express your grades in percentages rather than fractions. You scored 68%, Sandhu scored 70%, and Illija scored 69%. Notice you can easily answer the questions now. The advantage of percentages is that they facilitate comparison and comprehension.

CONVERTING DECIMALS TO PERCENTAGES

A **percentage** is a part of a whole expressed in hundredths. In other words, it is a value out of 100. For example, 93% means 93 out of 100, or $^{93}/_{100}$.

Formula 2.1

% is Percentage: This is the decimal expressed as a percentage. It must always be written with the percent (%) symbol immediately following the number.

100 is Conversion Factor: A percentage is always expressed in hundredths.

Formula 2.1 – Percentage Conversion: $\% = dec \times 100$

dec is Decimal Number: This is the decimal number needing to be converted into a percentage.

HOW IT WORKS

Assume you want to convert the decimal number 0.0875 into a percentage. This number represents the dec variable in the formula. Substitute into Formula 2.1:

% = 0.0875 × 100 = 8.75%

Important Notes

You can also solve this formula for the decimal number. To convert any percentage back into its decimal form, you need to perform a mathematical opposite. Since a percentage is a result of multiplying by 100, the mathematical opposite is achieved by dividing by 100. Therefore, to convert 81% back into decimal form, you take 81% ÷ 100 = 0.81.

Things To Watch Out For

Your Texas Instruments BAII Plus calculator has a % key that can be used to convert any percentage number into its decimal format. For example, if you press 81 and then %, your calculator displays 0.81.

While this function works well when dealing with a single percentage, it causes problems when your math problem involves multiple percentages. For example, try keying 4% + 3% = into the calculator using the % key. This should be the same as 0.04 + 0.03 = 0.07. Notice, however, that your calculator has 0.0412 on the display.

Why is this? As a business calculator, you BAII Plus is programmed to take portions of a whole. When you key 3% into the calculator, it takes 3% of the first number keyed in, which was 4%. As a formula, the calculator sees 4% + (3% × 4%). This works out to 0.04 + 0.0012 = 0.0412.

To prevent this from happening, your best course of action is not to use the % key on your calculator. It is best to key all percentages as decimal numbers whenever possible, thus eliminating any chance that the % key takes a portion of your whole. Throughout this textbook, all percentages are converted to decimals before calculations take place.

PATHS TO SUCCESS

When working with percentages, you can use some tricks for remembering the formula and moving the decimal point.

Remembering the Formula

When an equation involves only multiplication of all terms on one side with an isolated solution on the other side, use a mnemonic called the triangle technique. In this technique, draw a triangle with a horizontal line through its middle. Above the line goes the solution, and below the line, separated by vertical lines, goes each of the terms involved in the multiplication. The figure to the right shows how Formula 2.1 would be drawn using the triangle technique.

To use this triangle, identify the unknown variable, which you then calculate from the
other variables in the triangle:

1. Anything on the same line gets multiplied together. If solving for %, then
   the other variables are on the same line and multiplied as dec × 100.

2. Any pair of items with one above the other is treated like a fraction and
   divided. If solving for dec, then the other variables are above/below each
   other and are divided as %/100 .

Moving the Decimal

Another easy way to work with percentages is to remember that multiplying or dividing by 100 moves the decimal
over two places.

1. If you are multiplying by 100, the decimal position moves two positions to the right.
$$0.73 \times 100 = 0.73 = 73\%$$
2 positions to the right

2. If you are dividing by 100, the decimal position moves two positions to the left (see Figure 2.5).
$$73\% \div 100 = 73. = 0.73$$
2 positions to the left

EXAMPLE 2.3A: WORKING WITH PERCENTAGES

1. Convert (a) and (b) into percentages.

2. Convert (c) back into decimal format.

a. $\dfrac{3}{8}$

b.	1.3187
c.	12.399 %

Plan

For questions (a) and (b), you need to convert these into percentage format.
For question (c), you need to convert it back to decimal format.

What You Already Know

1. This is a fraction to be converted into a decimal, or dec.

2. This is dec.

3. This is %.

How You Will Get There

1. Convert the fraction into a decimal to have dec.

    Then apply Formula 2.1:
    % = dec × 100 to get the percentage.

2. As this term is already in decimal format, apply Formula 2.1: % = dec × 100
    to get the percentage.

3. This term is already in percentage format. Using the triangle technique, calculate the decimal number
    through dec = %/100.

Perform

A. $$\frac{3}{8} = 3 \div 8 = 0.375 \qquad\qquad \frac{0}{0} = 0.375 \times 100 \qquad\qquad \frac{0}{0} = 37.5\%$$

B. $$\% = \frac{0}{0} = 1.3187 \times 100 \qquad\qquad \frac{0}{0} = 131.87\%$$

C. $$dec = \frac{12.399\%}{100} = 0.12399$$

Present

In percentage format, the first two numbers are 37.5% and 131.87%. In decimal format, the last number is 0.12399.

RATE, PORTION, BASE

In your personal life and career, you will often need to either calculate or compare various quantities involving fractions. For example, if your income is $3,000 per month and you can't spend more than 30% on housing, what is your maximum housing dollar amount? Or perhaps your manager tells you that this year's sales of $1,487,003 are 102% of last year's sales. What were your sales last year?

Formula 2.2

Rate is The Relationship: The **rate** is the decimal form expressing the relationship between the portion and the base. Convert it to a percentage if needed by applying Formula 2.1. This variable can take on any value, whether positive or negative.
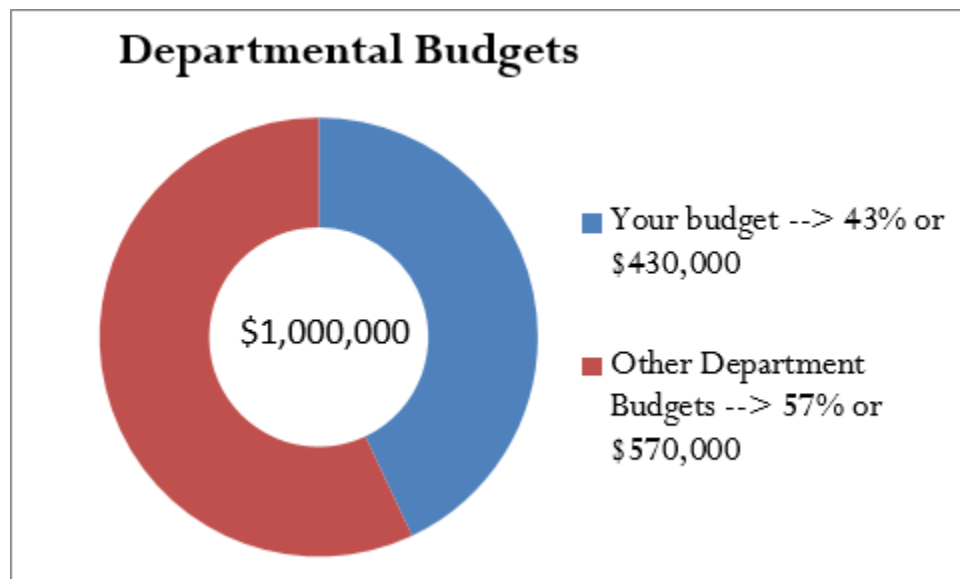
Portion is The Part Of The Quantity: The portion represents the part of the whole. Compare it against the base to assess the rate.

Formula 2.2 – Rate, Portion, Base: $$\text{Rate} = \frac{\text{Portion}}{\text{Base}}$$

Base is The Entire Quantity: The base is the entire amount or quantity that is of concern. It represents a whole, standard, or benchmark that you assess the portion against.

HOW IT WORKS

Assume that your company has set a budget of $1,000,000. This is the entire amount of the budget and represents your *base*. Your department gets $430,000 of the budget—this is your department's part of the whole and represents the *portion*. You want to know the relationship between your budget and the company's budget. In other words, you are looking for the *rate*.

Departmental Budgets

$1,000,000

■ Your budget --> 43% or $430,000

■ Other Department Budgets --> 57% or $570,000

- Apply Formula 2.2, where rate = $430,000 / $1,000,000.
- Your budget is 0.43, or 43%, of the company's budget.

Important Notes

There are three parts to this formula. Mistakes commonly occur through incorrect assignment of a quantity to its associated variable. The table below provides some tips and clue words to help you make the correct assignment.

Variable	Key Words	Example
Base	of	If your department can spend 43% of the company's total budget **of $1,000,000**, what is your maximum departmental spending?
Portion	is, are	If your department can spend 43% of the company's total budget of $1,000,000, what is your maximum departmental spending?
Rate	%, percent, rate	If your department can spend **43%** of the company's total budget of $1,000,000, what is your maximum departmental spending?

Things To Watch Out For

In resolving the rate, you must express all numbers in the same units—you cannot have apples and oranges in the same sequence of calculations. In the above example, both the company's budget and the departmental budget are in the units of dollars. Alternatively, you would not be able to calculate the rate if you had a base expressed in kilometres and a portion expressed in metres. Before you perform the rate calculation, express both in kilometres or both in metres.

PATHS TO SUCCESS

Formula 2.2 is another formula you can use the triangle technique for. You do not need to memorize multiple versions of the formula for each of the variables. The triangle appears to the right.

Be very careful when performing operations involving rates, particularly in summing or averaging rates.

Summing Rates

Summing rates requires each rate to be a part of the same whole or base. If Bob has 5% of the kilometres travelled and Sheila has 6% of the oranges, these are not part of the same whole and cannot be added. If you did, what does the 11% represent? The result has no interpretation. However, if there are 100 oranges of which Bob has 5% and Sheila has 6%, the rates can be added and you can say that in total they have 11% of the oranges.
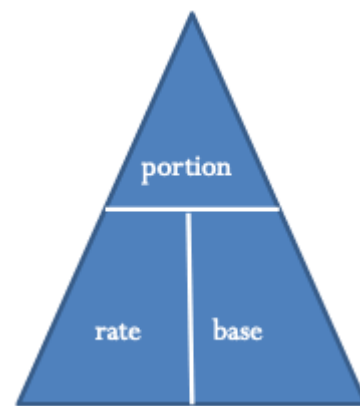
Averaging Rates

Simple averaging of rates requires each rate to be a measure of the same variable with the same base. If 36% of your customers are female and 54% have high income, the average of 45% is meaningless since each rate measures a different variable. Recall that earlier in this chapter you achieved 68% on your test and Sandhu scored 70%. However, your test involved 19 questions and Sandhu's involved 23 questions. These rates also cannot be simply averaged to 69% on the reasoning that (68% + 70%) / 2 = 69%, since the bases are not the same. When two variables measure the same characteristic but have different bases (such as the math quizzes), you must use a weighted-average technique.

When can you average rates? Hypothetically, assume Sandhu achieved his 70% by writing the same test with 19 questions. Since both rates measure the same variable and have the same base, the simple average of 69% is now calculable.

GIVE IT SOME THOUGHT

Consider the following situations and select the best answer without performing any calculations.

1. If the rate is 0.25%, in comparison to the base the portion is

    a. just a little bit smaller than the base.

    b. a lot smaller than the base.

    c. just a little bit bigger than the base.

    d. a lot bigger than the base.

2. If the portion is $44,931 and the base is $30,000, the rate is

    a. smaller than 100%.

    b. equal to 100%.

    c. larger than 100% but less than 200%.

    d. larger than 200%.

3. If the rate is 75% and the portion is $50,000, the base is

    a. smaller than $50,000.

    b. larger than $50,000.

    c. the same as the portion and equal to $50,000.

> **Solutions**
>
> 1. b (0.25% is 0.0025, resulting in a very small portion)
> 2. c (the portion is larger than the base, but not twice as large)
> 3. b (the portion represents 75% of the base, meaning the base must be larger

EXAMPLE 2.3B: RATE, PORTION, BASE

Solve for the unknown in the following three scenarios.

1. If your total income is $3,000 per month and you can't spend more than 30% on housing, what is the maximum amount of your total income that can be spent on housing?

2. Your manager tells you that 2014 sales are 102% of 2013 sales. The sales for 2014 are

$1,487,003. What were the sales in 2013?

3.   In Calgary, total commercial real estate sales in the first quarter of 2008 were $1.28 billion. The industrial, commercial, and institutional (ICI) land sector in Calgary had sales of $409.6 million. What percentage of commercial real estate sales is accounted for by the ICI land sector?

Plan

1.   You are looking for the maximum amount of your income that can be spent on housing.

2.   You need to figure out the sales for 2013.

3.   You must determine the percentage of commercial real estate sales accounted for by the ICI land sector in Calgary.

What You Already Know

1.   Look for key words in the question: "what **is** the maximum amount" and "**of** your total income." The total income is the base, and the maximum amount is the portion.
Base = $3,000Rate = 30% Portion = maximum amount

2.   Look for key words in the question: "sales for 2014 are $1,487,003" and "of 2013 sales." The 2014 sales is the portion, and the 2013 sales is the base.
Portion = $1,487,003   Rate = 102%   Base = 2013 sales

3.   Look for key words in the question: "**of** commercial real estate sales" and "**are** accounted for by the ICI land sector." The commercial real estate sales are the base, and the ICI land sector sales are the portion.
Base=$1.28 billion   Portion=$409.6 million   Rate=percentage

How You Will Get There

1.   Apply Formula 2.2, but rearrange using the triangle technique to have:
Portion = Rate × Base.

2.   Apply Formula 2.2, but rearrange using the triangle technique to have:
Base=Portion/Rate.

3.   Apply Formula 2.2:
Rate=Portion/Base.

Perform

$$1 \quad Portion = 30\% \times \$3,000 = 0.3 \times \$3,000 = \$900$$

$$2 \quad Base = \frac{\$1,487,003}{102\%} = \frac{\$1,487,003}{102} = \$1,457,846.08$$

$$3 \quad Rate = \frac{\$409.6 million}{\$1.28 billion} = \frac{\$409,600,000}{\$1,280,000,000} = 0.32 \, or \, 32\%$$

Present

1. The maximum you can spend on housing is $900 per month.

2. 2013 sales were $1,457,846.08.

3. The ICI land sector accounted for 32% of commercial real estate sales in Calgary for the first quarter of 2008.

SECTION 2.3 EXERCISES

Mechanics

1. Convert the following decimals into percentages.
   a. 0.4638   b. 3.1579   c. 0.000138   d. 0.015

2. Convert the following fractions into percentages.

   $a.\dfrac{3}{8}$ $\qquad\qquad$ $b.\dfrac{17}{32}$ $\qquad\qquad$ $c.\dfrac{42}{12}$ $\qquad\qquad$ $d.2\dfrac{4}{5}$

3. Convert the following fractions into percentages. Round to four decimals or express in repeating decimal format as needed.

   $a.\dfrac{46}{12}$ $\qquad\qquad$ $b.\dfrac{2}{9}$ $\qquad\qquad$ $c.\dfrac{3}{11}$ $\qquad\qquad$ $d.\dfrac{48}{93}$

4. Convert the following percentages into decimal form.

    a. 15.3%   b. 0.03%   c. 153.987%   d. 14.0005%

5. What percentage of $40,000 is $27,000?

6. What is ½% of $500,000?

7. $0.15 is 4,900% of what number?

APPLICATIONS

1. In February 2009, 14,676 mortgages were in arrears in Canada, which represented 0.38% of all mortgages. How many total mortgages were in the Canadian market at that time?

2. In 2009, medical experts predicted that one out of two Manitobans would contract some form of the H1N1 virus. If the population of Manitoba in 2009 was 1,217,200, how many Manitobans were predicted to become ill?

3. In August 2004, Google Inc. offered its stocks to the public at $85 per share. In October 2007, the share price had climbed to $700.04. Express the 2007 share price as a percentage of the 2004 price.

4. During Michael Jordan's NBA career (1984–2003), he averaged a free throw completion percentage of 83.5% in regular season play. If Jordan threw 8,772 free throws in his career, how many completed free throws did he make?

5. If total advertising expenditures on television advertising declined 4.1% to $141.7 billion in the current year, how much was spent on television advertising in the previous year? Round your answer to one decimal.

6. If the new minimum wage of $8.75 per hour is 102.9412% of the old minimum wage, what was the old minimum wage?

7. A machine can produce 2,500 products per hour. If 37 of those products were defective, what is the defect rate per hour for the machine?

Challenge, Critical Thinking, & Other Applications

1. In 2011, Manitoba progressive income tax rates were 10.8% on the first $31,000, 12.75% on the next $36,000, and 17.4% on any additional income. If your gross taxable earnings for the year were $85,000, what percentage of your earnings did you pay in taxes?

2. In 2011, the maximum amount that you could have contributed to your RRSP (Registered Retirement Savings Plan) was the lesser of $22,450 or 18% of your earned income from the previous year. How much income do you need to claim a $22,450 contribution in 2011?

3. Maria, a sales representative for a large consumer goods company, is paid 3% of the total profits earned by her company. Her company averages 10% profit on sales. If Maria's total income for the year was $60,000, what total sales did her company realize?

4. A house was purchased six years ago for $214,000. Today it lists at a price that is 159.8131% of the original purchase price. In dollars, how much has the price of the house increased over the six years?

5. An investor buys 1,000 shares of WestJet Airlines at $10.30 per share. A few months later, the investor sells the shares when their value hits 120% of the original share price. What is the price of a WestJet share when the investor sells these shares? How much money did the investor make?

6. A Honda Insight has fuel economy of 3.2 litres consumed per 100 kilometres driven. It has a fuel tank capacity of 40 litres. A Toyota Prius is rated at 4.2 L per 100 km driven. It has a fuel tank capacity of 45 L. What percentage is the total distance driveable (rounded to the nearest kilometre before calculating) of a Honda Insight compared to that of a Toyota Prius?

2.4 Mathematical Symbols and Operations

One of the most difficult areas of mathematics involves translating words into mathematical symbols and operations. To assist in this translation, the table below lists some common language and the mathematical symbol that is typically associated with the word or phrase.

Language			Math Symbol
Sum Addition	In addition to In excess	Increased by Plus	$+$
Divide Division	Divisible Quotient	Per	$\div$
Becomes Is/Was/Were	Will be	Results in Totals	$=$
Less than	Lower than		$<$
Less than or equal to			$\leq$

Language			Math Symbol
Subtract Decreased by Diminished by	Less Minus	Difference Reduced by	$-$
Multiplied by Times	Percentage of	Product of Of	$\times$
More than Greater than			$>$
Greater than or equal to			$\geq$
Not equal to			$\neq$

CHAPTER 3: MARKETING APPLICATIONS

WHAT IS IT GOING TO COST ME?

When you buy an iPod, it is very important that the right price is set. The price should

- be seen as fair by you the buyer,
- pay for the costs (plastics, battery, buttons, circuit boards, headset) and expenses (employees, factory, electricity, distribution) of making the iPod, and
- allow the seller's business to make some extra money as profit so that it can grow its business further.

If your business is selling a product, you will need to pay close attention to price adjustments because they affect profitability. Various discounts, like putting items on sale, may increase sales while lowering the amount of profit per transaction. How do you know where to set the balance to maximize profit overall?

As a student in a business program, consider this chapter essential to the success of any business. Whether your pricing strategy is high or low, the company must ensure that it can still pay its bills as a minimum requirement. And that requires careful juggling of many factors. If it fails to manage its pricing properly, the company will go bankrupt!

This chapter will make you a smarter business professional and a wiser consumer. You shop retail almost every day and regularly purchase goods and services. If you understand how product pricing works, you can make sense of "deals." You can easily explain why the same product sells for two different prices at two different stores.

In this chapter, you must learn the language of marketers to perform merchandising mathematics involving product costs, expenses, prices, markups, markdowns, and ultimately profitability. Once the study of the various pricing components is complete, we will see how the various pieces of the pricing puzzle fit together into a cohesive merchandising environment.

Adaptation note: Chapter 6 in Business Math: A Step-by-Step handbook

# 3.1 Figuring Out the Cost: Discounts

HOW MUCH?

You mutter in exasperation, "Why can't they just set one price and stick with it?" Your mind boggles at all the competing discounts you encounter at the mall in your search for that perfect Batman toy for your nephew. Walmart is running their Rollback promotion and is offering a Batmobile for 25% off, regularly priced at $49.99. Toys R' Us has an outlet in the parking lot where the regular price for the same toy is $59.99, but all Batman products are being cleared out at 40% off. You head over to The Bay for a warehouse clearance event that has the same toy priced at $64.99 but at 35% off. It is also Bay Days, which means you can scratch and win a further 10% to 20% off the sale price. You go to Dairy Queen for a Blizzard to soothe your headache while you figure things out.

The **cost** of a product is the amount of money required to obtain the merchandise. If you are a consumer, the ticketed price tag on the product is your cost. If you are a reseller (also known as a *middleman* or *intermediary*), what you pay to your supplier for the product is your cost. If you are a manufacturer, then your cost equals all of the labour, materials, and production expenditures that went into creating the product.

A **discount** is a reduction in the price of a product. As a consumer, you are bombarded with discounts all the time. Retailers use various terms for discounts, including *sales* or *clearance*. If your business purchases a product from a supplier, any discount it receives lowers how much the business pays to acquire the product. When a business buys products, the price paid is the cost to the business. Therefore, a lower price means a lower cost.

If your business is the one selling the product, any discount offered lowers the selling price and reduces revenue per sale. Since the revenue must cover all costs and expenses associated with the product, the lower price means that the business reduces profits per sale. In business, it is common practice to express a discount as a percentage off the regular price.

HOW DISTRIBUTION AND PRICING WORK
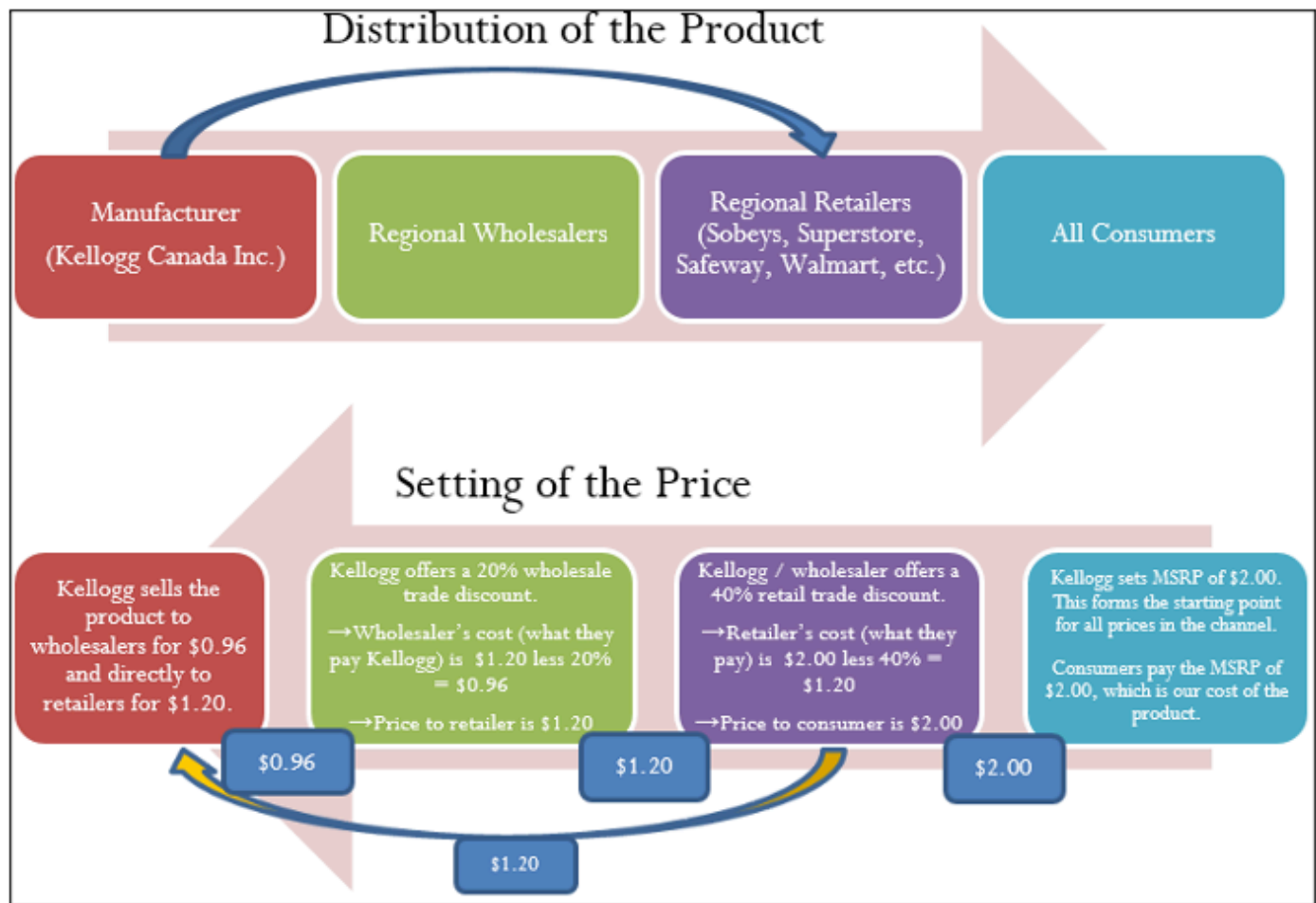
Start with distribution in the top half of the figure and work left to right. As an example, let's look at a manufacturer such as Kellogg Canada Inc. (which makes such products as Pop-Tarts, Eggo Waffles, and Rice Krispies). Kellogg's Canadian production plant is located in London, Ontario. To distribute its products to the rest of Canada, Kellogg Canada uses various regional wholesalers. Each wholesaler then resells the product to retailers in its local trade area; however, some retailers (such as the Real Canadian Superstore) are very large, and Kellogg Canada distributes directly to these organizations, bypassing the wholesaler as represented by the blue arrow. Finally, consumers shop at these retailers and acquire Kellogg products.

The relationship of distribution to pricing is illustrated in the bottom half of the figure, working right to left. For now, focus on understanding how pricing works; the mathematics used in the figure will be explained later in this chapter. Kellogg Canada sets a **manufacturer's suggested retail price**, known as the MSRP. This is a recommended retail price based on consumer market research. Since grocery retailers commonly carry thousands or tens of thousands of products, the MSRP helps the retailer to determine the retail price at which the product should be listed. In this case, assume a $2.00 MSRP, which is the price consumers will pay for the product.

The retailer must pay something less than $2.00 to make money when selling the product. Kellogg Canada understands its distributors and calculates that to be profitable most retailers must pay approximately 40% less than the MSRP. Therefore, it offers a 40% discount. If the retailer purchases directly from Kellogg, as illustrated by the yellow arrow, the price paid by the retailer to acquire the product is $2.00 less 40%, or $1.20. Smaller retailers

acquire the product from a wholesaler for the same price. Thus, the retailer's cost equals the wholesaler's price (or Kellogg Canada's price if the retailer purchases it directly from Kellogg).

The wholesaler's price is $1.20. Again, Kellogg Canada, knowing that the wholesaler must pay something less than $1.20 to be profitable, offers an additional 20% discount exclusively to the wholesaler. So the price paid by the wholesaler to acquire the product from Kellogg Canada is $1.20 less 20%, or $0.96. This $0.96 forms Kellogg Canada's price to the wholesaler, which equals the wholesaler's cost.

In summary, this discussion illustrates two key pricing concepts:

1. Companies higher up in the distribution channel pay lower prices than those farther down the channel. Companies receive discounts off the MSRP based on their level in the distribution system. This may result in multiple discounts, such as a wholesaler receiving both the retailer's discount and an additional discount for being a wholesaler.

2. One organization's price becomes the next organization's cost (assuming the typical distribution channel structure):

   ◦ Manufacturer's Price = Wholesaler's Cost

   ◦ Wholesaler's Price = Retailer's Cost

   ◦ Retailer's Price = Consumer's Cost

TYPES OF DISCOUNTS

You will perform discount calculations more effectively if you understand how and why single pricing discounts and multiple pricing discounts occur. Businesses or consumers are offered numerous types of discounts, of which five of the most common are trade, quantity, loyalty, sale, and seasonal.

Trade Discounts

A **trade discount** is a discount offered to businesses only based on the type of business and its position in the distribution system (e.g., as a retailer, wholesaler, or any other member of the distribution system that resells the product). Consumers are ineligible for trade discounts. In the discussion of the figure, two trade discounts are offered. The first is a 40% retail trade discount, and the second is a 20% wholesale trade discount. Typically, a business that is higher up in the distribution system receives a combination of these trade discounts. For example, the wholesaler receives both the 40% retail trade discount and the 20% wholesale trade discount from the MSRP. The wholesaler's cost is calculated as an MSRP of $2.00 less 40% less 20% = $0.96.

Quantity Discounts

A **quantity discount** (also called a *volume discount*) is a discount for purchasing larger quantities of a certain product. If you have ever walked down an aisle in a Real Canadian Superstore, you probably noticed many shelf tags that indicate quantity discounts, such as "Buy one product for $2" or "Take two products for $3." Many Shell gas stations offer a Thirst Buster program in which customers who purchase four Thirst Busters within a three-month period get the fifth one free. If the Thirst Busters are $2.00 each, this is equivalent to buying five drinks for $10.00 less a $2.00 quantity discount.

Loyalty Discounts.

A **loyalty discount** is a discount that a seller gives to a purchaser for repeat business. Usually no time frame is specified; that is, the offer is continually available. As a consumer, you see this regularly in marketing programs such as Air Miles or with credit cards that offer cash back programs. For example, Co-op gas stations inManitoba track consumer gasoline purchases through a loyalty program and mails an annual loyalty discount cheque to its customers, recently amounting to 12.5¢ per litre purchased. In business-to-business circles, sellers typically reward loyal customers by deducting a loyalty discount percentage, commonly ranging from 1% to 5%, from the selling price.

Sale Discounts

A **sale discount** is a temporary lowering of the price from a product's regular selling price. Businesses put items on sale for a variety of reasons, such as selling excess stock or attracting shoppers. You see such promotional events all the time: LED monitors are on sale at Best Buy; Blu-Ray discs are half off at Walmart; The Brick is having a door crasher event Saturday morning.

Seasonal Discounts

A **seasonal discount** is a discount offered to consumers and businesses for purchasing products out of season. At the business level, manufacturers tend to offer seasonal discounts encouraging retailers, wholesalers, or distributors to purchase products before they are in season. Bombardier Inc. manufactures Ski-Doos, which are sold in Canada from approximately November through March—a time of year when most of the country has snow and consumers would want to buy one. To keep production running smoothly from April through October, Bombardier could offer seasonal discounts to its wholesalers and retailers for the coming winter season. At a retail level, the examples are plentiful. On November 1 most retailers place their Halloween merchandise on seasonal discount to clear out excess inventory, and many retailers use Boxing Day (or Boxing Week) to clear their out-of-season merchandise.
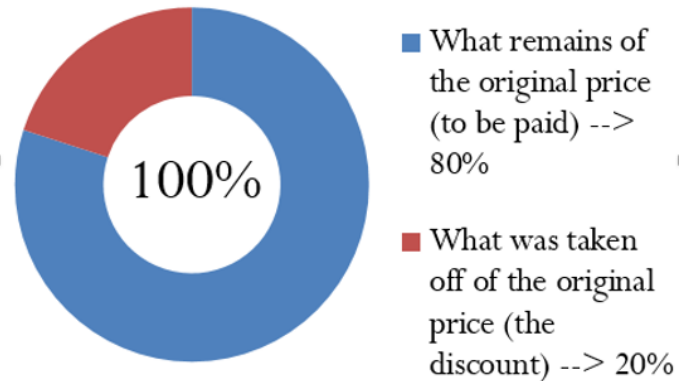
SINGLE DISCOUNTS

Let's start by calculating the cost when only one discount is offered. Later in this section you will learn how to calculate a cost involving multiple discounts.

The Formula

Figuring out the price after applying a single discount is called a net price calculation. When a business calculates the net price of a product, it is interested in what you still have to pay, not in what has been removed. Note in Formula 6.1 below that you take 1 and subtract the discount rate to determine the rate owing. If you are eligible for a 20% discount, then you must pay 80% of the list price, as illustrated in the figure to the right.

Formula 6.1

20% Discount Interpretation

- ■ What remains of the original price (to be paid) --> 80%

- ■ What was taken off of the original price (the discount) --> 20%

N is Net Price: The **net price** is the price of the product after the discount is removed from the list price. It is a dollar amount representing what price remains after you have applied the discount.
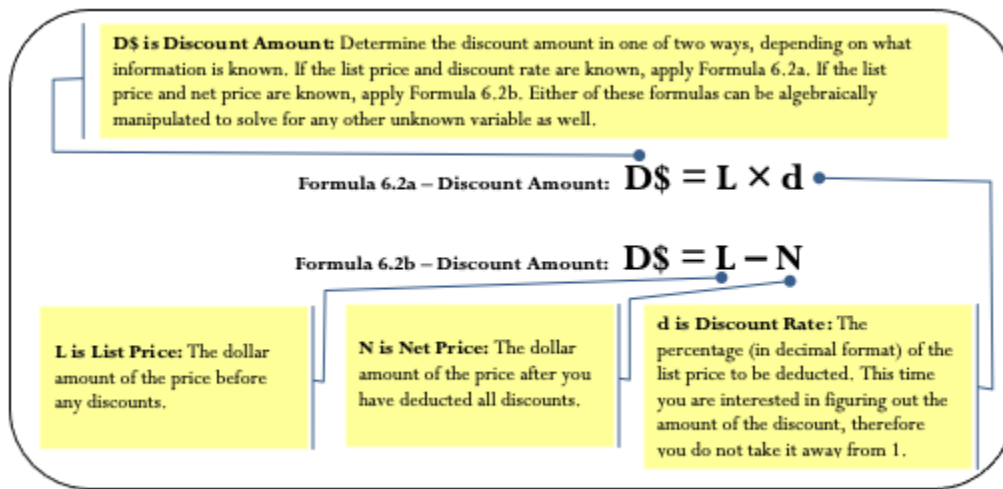
Formula 6.1 – Single Discount: $N = L \times (1 - d)$

L is List Price: The **list price** is the normal or regular dollar price of the product before any discounts. It is the **Manufacturer's Suggested Retail Price (MSRP** - a price for a product that has been published or advertised in some way), or any dollar amount before you remove the discount.

d is Discount Rate: The discount rate represents the percentage (in decimal format) of the list price that is deducted.

Formula 6.1 once again applies Formula 2.2 on rate, portion, and base, where the list price is the base, the (1 – d) is the rate, and the net price represents the portion of the price to be paid.

Notice that Formula 6.1 requires the discount to be in a percentage (decimal) format; sometimes a discount is expressed as a dollar amount, though, such as "Save $5 today." Formulas 6.2a and 6.2b relate the discount dollar amount to the list price, discount percent, and net price. Choose one formula or the other depending on which variables are known.

Formula 6.2

D$ is Discount Amount: Determine the discount amount in one of two ways, depending on what information is known. If the list price and discount rate are known, apply Formula 6.2a. If the list price and net price are known, apply Formula 6.2b. Either of these formulas can be algebraically manipulated to solve for any other unknown variable as well.

Formula 6.2a – Discount Amount: $D\$ = L \times d$

Formula 6.2b – Discount Amount: $D\$ = L - N$

L is List Price: The dollar amount of the price before any discounts.

N is Net Price: The dollar amount of the price after you have deducted all discounts.

d is Discount Rate: The percentage (in decimal format) of the list price to be deducted. This time you are interested in figuring out the amount of the discount, therefore you do not take it away from 1.

How It Works

Follow these steps to calculate the net price involving a single discount. These steps are adaptable if the net price is a known variable and one of the other variables is unknown.

Step 1: Identify any known variables, including list price, discount rate, or discount amount.

Step 2: If the list price is known, skip this step. Otherwise, solve for list price using an appropriate formula.

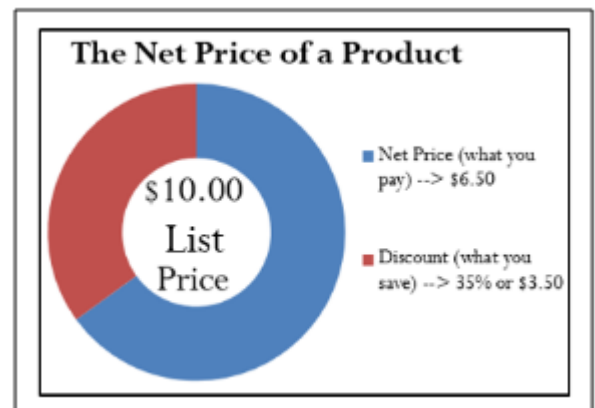**Step 3**: Calculate the net price.

  • If the list price and discount are are known, apply Formula 6.1.
  • If the list price and discount amount are known, apply Formula 6.2b and rearrange for N.

Assume a product sells for $10 and is on sale at 35% off the regular price. Calculate the net price for the product.

Step 1: The list price of the product is L = $10. It is on sale with a discount rate of d = 0.35.

Step 2: List price is known, so this step is not needed.

Step 3: Applying Formula 6.1 results in a new price of N = $10 × (1 − 0.35) = $6.50. Note that if you are interested in learning the discount amount, you apply Formula 6.2b to calculate D$ = $10 − $6.50 = $3.50.

The Net Price of a Product

$10.00 List Price

■ Net Price (what you pay) --> $6.50

■ Discount (what you save) --> 35% or $3.50

Important Notes

You can combine Formula 6.1 and either version of Formula 6.2 in a variety of ways to solve any single discount situation for any of the three variables. As you deal with increasingly complicated pricing formulas, your algebraic skills in solving linear equations and substitution become very important.

Many of the pricing problems take multiple steps that combine various formulas, so you need to apply the PUPP

model systematically. In any pricing problem, you must understand which variables are provided and match them up to the known formulas. To get to your end goal, you must look for formulas in which you know all but one variable. In these cases, solving for variables will move you forward toward solving the overall pricing problem.

If you find you cannot produce a formula with only one unknown variable, can you find two formulas with the same two unknowns? If so, recall from Section 2.5 that you can use your algebraic skills to find the roots of the two equations simultaneously. Alternatively, you can solve one formula for a variable then substitute it into the other formula, allowing you to isolate the remaining variable. Throughout the examples in this chapter you will see many applications of these algebraic skills.

Things To Watch Out For

Remember to apply the rounding rules discussed in Chapter 2:

1. Until you arrive at the final solution, avoid rounding any interim numbers unless you have some special reason to do so.
2. Round all dollar amounts to the nearest cent. If the dollar amount has no cents, you may write it either without the cents or with the ".00" at the end.
3. Round all percentages to four decimals when in percent format.

Paths To Success

When working with single discounts, you are not always solving for the net price. Sometimes you must calculate the discount percent or the list price. At other times you know information about the discount amount but need to solve for list price, net price, or the discount rate. The triangle technique discussed in Section 2.3 can remind you how to rearrange the formulas for each variable, as illustrated in the figure to the right.

Formula 6.1: Single Discount   Formula 6.2a: Discount Amount
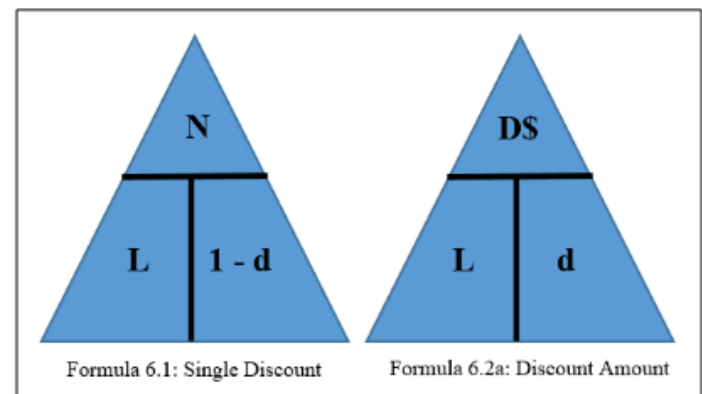
Give It Some Thought

1. Will you pay more than, less than, or exactly $10.00 for a product if you are told that you are paying:
    1. a net price of $10.00 when there is a discount of 25%?
    2. a list price of $10.00 when there is a discount of 25%?
2. If an item is subject to a 40% discount, will the net price be more than or less than half of the list price of the product?

> ### Solutions
>
> 1a. Exactly $10. The net price is the price after the discount.
> 1b. Less than $10. The discount needs to be removed from the list price.

2. More than half. A 40% discount means that you will pay 60% of the list price

EXAMPLE 6.1A: DETERMINING THE RETAILER'S NET PRICE FOR A PAIR OF JEANS

A manufacturer that sells jeans directly to its retailers uses market research to find out it needs to offer a 25% trade discount. In doing so, the retailers will then be able to price the product at the MSRP of $59.99. What price should retailers pay for the jeans?

Plan

Calculate how much a retailer should pay for the jeans after the regular price has been discounted to accommodate the trade discount. This is called the net price for the product, or N.

What You Already Know

Step 1: The list price and the discount rate are known:
L =$59.99d = 0.25

How You Will Get There

Step 2: List price is known, so skip this step.
Step 3: Apply Formula 6.1.

Perform

Step 3: N = $59.99 × (1 – 0.25) = $59.99 × 0.75 = $44.99

Present

The manufacturer should sell the jeans to the retailers for $44.99

EXAMPLE 6.1B: DETERMINING THE LIST PRICE OF A JACKET

Winners pays a net price of $27.50 for a winter jacket after receiving a retail trade discount of 45%. What was the MSRP of the jacket?

Plan

Calculate the MSRP for the jacket before Winners received the retail trade discount to arrive at the net price. This is called the list price for the product, or L.

What You Already Know

Step 1: The net price and the discount rate are known:
N = $27.50d = 0.45

How You Will Get There

Step 2: List price is the unknown variable; skip this step.
Step 3: Apply Formula 6.1, rearranging for L.

Perform

Step 3: $27.50 = L (1 − 0.45) L = $27.50 ÷ (1 − 0.45)
L = $27.50 ÷ 0.55 = $50.00

Present

The MSRP, or list price, of the winter jacket is $50.00.
You are shopping at Mountain Equipment Co-op for a new environmentally friendly water bottle. The price tag reads
$14.75, which is $10.24 off the regular price. Determine the discount rate applied.

EXAMPLE 6.1C: DETERMINING THE DISCOUNT PERCENT AND DISCOUNT AMOUNT

You are shopping at Mountain Equipment Co-op for a new environmentally friendly water bottle. The price tag reads $14.75, which is $10.24 off the regular price. Determine the discount rate applied.

Plan

You need to find out how the sale price translates into the discount rate, or d.

What You Already Know

Step 1: The discount amount and net price are known:
D$ = $10.24N = $14.75

How You Will Get There

Step 2: Use Formula 6.2b to calculate the list price, rearranging for L.
Step 3: Convert the discount amount into a percentage by applying Formula 6.2a, rearranging for d.

Perform

Step 2: List price: $10.24 = L – $14.75, which results in L = $24.99 Step 3: Discount rate: d = D$ ÷ L
d = $10.24 = 0.409764 or 40.9764%
$24.99

Present

The water bottle today has been reduced in price by the amount of $10.24. This represents a sale discount of 40.9674%.

MULTIPLE DISCOUNTS

You are driving down the street when you see a large sign at Old Navy that says, "Big sale, take an additional 25% off already reduced prices!" In other words, products on sale (the first discount) are being reduced by an additional 25% (the second discount). Because Formula 6.1 handles only a single discount, you must use an extended formula in this case.

The Formula

Businesses commonly receive more than one discount when they make a purchase. Consider a transaction in which a business receives a 30% trade discount as well as a 10% volume discount. First, you have to understand that this is not a 30% + 10% = 40% discount. The second discount is always applied to the net price after the first discount is applied. Therefore, the second discount has a smaller base upon which it is calculated. If there are more than two discounts, you deduct each subsequent discount from continually smaller bases. Formula 6.3 expresses how to calculate the net price when multiple discounts apply.

Formula 6.3

N is Net Price: The dollar amount of the price after all discounts have been deducted.

L is List Price: The dollar amount of the price before any discounts.

Formula 6.3 – Multiple Discounts:

$$N = L \times (1-d_1) \times (1-d_2) \times \ldots \times (1-d_n)$$

d_1 **is First Discount,** d_2 **is Second Discount,** d_n **is nth Discount:** When there is more than one discount, you must extend beyond Formula 6.1 by multiplying another discount expression. These discounts are represented by the same d symbol; however, each discount receives a subscript to make its symbol unique. Therefore, the first discount receives the symbol of d_1, the second discount receives the symbol d_2, and so on. Recall that the symbol n represents the number of pieces of data (a count), so you can expand or contract this formula to the exact number of discounts being offered.

It is often difficult to understand exactly how much of a discount is being received when multiple discounts are involved. Often it is convenient to summarize the multiple discount percentages into a single percentage. This makes it easier to calculate the net price and aids in understanding the discount benefit. Simplifying multiple percent discounts into a single percent discount is called finding the **single equivalent discount**. Whether you apply the multiple discounts or just the single equivalent discount, you arrive at the same net price. The conversion of multiple discount percentages into a single equivalent discount percent is illustrated in Formula 6.4.

Formula 6.4

d_{equiv} **(or just d)** is the **single equivalent discount rate** that is equal to the series of multiple discounts. Recall that taking $(1-d)$ calculates what you pay. Therefore, if you take 1, which represents the entire amount, and reduce it by what you pay, the rate left over must be what you did not pay. In other words, it is the discount rate.

Formula 6.4 – Single Equivalent Discount:

$$d_{equiv} = 1-(1-d_1) \times (1-d_2) \times \ldots \times (1-d_n)$$

d_1 **is First Discount,** d_2 **is Second Discount,** d_n **is nth Discount:** This is the same notation as in Formula 6.3. Since there are multiple discounts, each discount receives a numerical subscript to give it a unique identifier. You can expand or contract the formula to the exact number of discounts being offered.

How It Works

Refer back to the steps in calculating net price. The procedure for calculating a net price involving a single discount extends to a more generic procedure involving multiple discounts. As with the single discount procedures, you can adapt the model if the net price is known and one of the other variables is unknown. Follow these steps to calculate the net price involving any number of discounts:

Step 1: Identify any known variables, including list price, discount rate(s), or discount amount.

Step 2: If the list price is known, skip this step. Otherwise, solve for list price.

- If only one discount is involved, apply Formula 6.2a.

- If more than one discount is involved, the discount amount represents the total discount amount received from all of the discounts combined. This requires you first to convert the multiple discount rates into an equivalent single discount rate using Formula 6.4 and then to apply Formula 6.2a.

Step 3: Calculate the net price.

- If the list price and only a single known discount rate are involved, apply Formula 6.1.

- If the list price and multiple discount rates are known and involved, apply Formula 6.3.

- If the list price and the total discount amount are known, apply Formula 6.2b and rearrange for N.

Assume a product with an MSRP of $100 receives a trade discount of 30% and a volume discount of 10%.
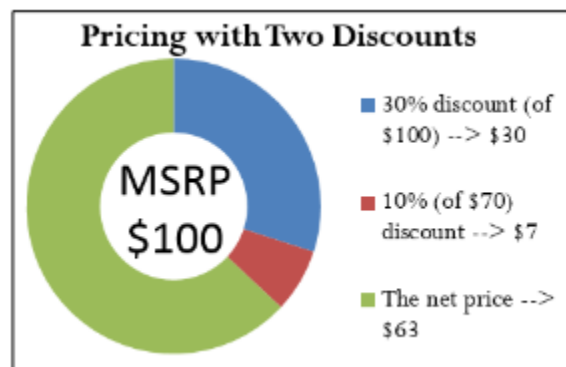
Calculate the net price.

- Step 1: The list price and discounts are L = $100, d1 = 0.30 and d2 = 0.10.
- Step 2: List price is known, so skip this step.
- Step 3: Apply Formula 6.3 to calculate the net price:

$$\Diamond = \$100 \times (1 - 0.30) \times (1 - 0.10) = \$63$$
The net price is $63, which is illustrated above.

If you are solely interested in converting multiple discounts into a single equivalent discount, you need only substitute into Formula 6.4. In the above example, the product received a trade discount of 30% and a volume discount of 10%. To calculate the single equivalent discount, apply Formula 6.4:

$$d_{equiv} = 1 - (1 - 0.30) \times (1 - 0.10)$$

$$d_{equiv} = 1 - (0.70)(0.90)$$

$$d_{equiv} = 1 - 0.63 = 0.37$$

Therefore, whenever discounts of 30% and 10% are offered together, the single equivalent discount is 37%. Whether it is the multiple discounts or just the single equivalent discount that you apply to the list price, the net price calculated is always the same.

Important Notes

Order of Discounts: The order of the discounts *does not* matter in determining the net price. Remember from the rules of BEDMAS that you can complete multiplication in any order. Therefore, in the above example you could have arrived at the
$63 net price through the following calculation:

$$\$100 \times (1 - 0.10) \times (1 - 0.30) = \$63$$

The order of the discounts *does* matter if trying to interpret the value of any single discount. If the trade discount is applied before the quantity discount and you are wanting to know the quantity discount amount, then the quantity discount needs to be second. Thus,

$$\$100 \times (1-0.30) = \$70$$

$$\$70 \times 0.10 = \$7$$

which is the amount of the quantity discount.

Price Does Not Affect Single Equivalent Discount

Notice in Formula 6.4 that the list price and the net price are not involved in the calculation of the single equivalent discount. When working with percentages, whether you have a net price of $6.30 and a list price of $10, or a net price of $63 and a list price of $100, the equivalent percentage always remains constant at 37%.

Things To Watch Out For

A common mistake when working with multiple discounts is to add the discounts together to calculate the single equivalent discount. This mistaken single discount is then substituted into Formula 6.1 to arrive at the wrong net price. Remember that if two discounts of 30% and 10% apply, you cannot sum these discounts. The second discount of 10% is applied on a smaller price tag, not the original price tag. To calculate the net price you must apply Formula 6.3.

Paths To Success

If you happen to know any two of the net price (N), list price (L), or the total discount amount (D$), then you could also use Formula 6.2 to solve for the single equivalent discount, dequiv. For example, if you know the net price is $63 and the total discount amount for all discounts is $37, you could use Formula 6.2b to figure out that the

list price is $100, then convert the discount amount into a percentage using Formula 6.2a. This method will also produce a single equivalent discount of 37%.

Another method of calculating the single equivalent discount is to recognize Formula 6.2a as an application of Formula 3.1 involving percent change. The variable d is a discount rate, which you interpret as a negative percent change. The discount amount, D$, is the difference between the list price (representing the Old price) and the net price (representing the New price after the discount). Therefore, Formula 6.2a can be rewritten as follows:

| D$ = L X d |
| becomes |
| New – Old = Old x % |

$$\frac{New - Old}{Old} = \triangle\%$$

Therefore, any question about a single equivalent discount where net price and list price are known can be solved as a percent change. Using our ongoing net price example, you have:

$$\frac{\$63 - \$100}{\$100} = -0.37 \, or -37\%$$

This is a discount of 37%.

Give It Some Thought

If you are offered discounts in the amount of 25%, 15%, 10%, and 5%, will your total discount percent be 55%, less than 55%, or more than 55%?

Solution

Less than 55%. Each percent discount is calculated from a smaller base.

EXAMPLE 6.1D: RETAILER PURCHASING SKI-DOOS WITH MULTIPLE DISCOUNTS

A retail dealership purchases some Expedition TUV Yeti II Ski-Doos to stock in its stores. Examining the merchandising terms of the manufacturer, Bombardier, the dealership notices that it would be eligible to receive a 35% trade discount, 15% volume discount, and 3% loyalty discount. Because it is June and Ski-Doos are out of season, Bombardier offers a seasonal discount of 12% for purchases

made before June 30. If the MSRP for the Ski-Doo is $12,399.00 and the dealership purchases this item on June 15, what price would it pay?

Plan

You are looking for the net price that the retail dealership will pay for the Ski-Doo, or N.

What You Already Know

Step 1: The retail dealership is eligible for all four discounts (it qualifies for the seasonal discount since it is purchasing before June 30).
Therefore,L = $12,399.00, d1 = 0.35, d2 = 0.15, d3 = 0.03,and d4 = 0.12

How You Will Get There

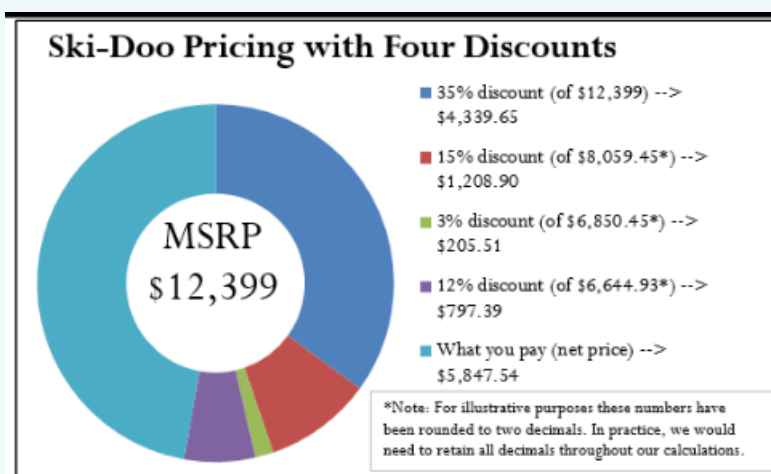Step 2: You know the list price, so skip this step.
Step 3: Apply Formula 6.3.

Perform

Step 3: N = $12,399 × (1 – 0.35) × (1 – 0.15) × (1 – 0.03) × (1 – 0.12)
N = $12,399 × 0.65 × 0.85 × 0.97 × 0.88 = $5,847.54

Present

After all four discounts, the retail dealership could purchase the Ski- Doo for $5,847.54.

Ski-Doo Pricing with Four Discounts

- 35% discount (of $12,399) --> $4,339.65
- 15% discount (of $8,059.45*) --> $1,208.90
- 3% discount (of $6,850.45*) --> $205.51
- 12% discount (of $6,644.93*) --> $797.39
- What you pay (net price) --> $5,847.54

MSRP $12,399

*Note: For illustrative purposes these numbers have been rounded to two decimals. In practice, we would need to retain all decimals throughout our calculations.

EXAMPLE 6.1E: REDUCING MULTIPLE DISCOUNTS TO A SINGLE EQUIVALENT DISCOUNT

The retail dealership in Example 6.1D purchases more products subject to the same discounts. It needs to simplify its calculations. Using the information from Example 6.1D, what single equivalent discount is equal to the four specified discounts?

Plan

You are looking for a single equivalent discount that is equal to the four discount percentages, or dequiv (or just d).

What You Already Know

You know the discount rates:
d1 = 0.35, d2 = 0.15, d3 = 0.03, and d4 = 0.12

How You Will Get There

Apply Formula 6.4.

Perform

$d_{equiv} = 1 - (1 - 0.35) \times (1 - 0.15) \times (1 - 0.03) \times (1 - 0.12)$
$d_{equiv} = 1 - (0.65)(0.85)(0.97)(0.88) = 1 - 0.471614 = 0.528386$ or 52.8386%

Present

The retail dealership can apply a 52.8386% discount to all the products it purchases.

EXAMPLE 6.1F: MAKING A SMART CONSUMER PURCHASE

You are shopping on Boxing Day for an 80" HDTV. You have just one credit card in your wallet, a cashback Visa card, which allows for a 1% cash rebate on all purchases. While scanning flyers for the best deal, you notice that Visions is selling the TV for $5,599.99 including taxes, while Best Buy is selling it for $5,571.99 including taxes. However, because of a computer glitch Best Buy is unable to accept Visa today. Where should you buy your television?

Plan

You want to know which store you should buy the television at. You must calculate the net price (N) for each of the stores.

What You Already Know

Step 1: You know the list price for each of the stores. You also know the discount available from Visa. Thus,

$L_{Best\ Buy}$ = $5,571.99 with no discounts since Visa cannot be used there

$L_{Visions}$ = $5,599.99, d = 0.01, since you can use your Visa card there

How You Will Get There

Step 2: List price is known, so skip this step.

Step 3: Apply Formula 6.1.

Perform

Step 3: Best Buy: No discounts apply, so the list price equals the net price and N = $5,571.99.

Visions: N = $5,599.99 × (1 − 0.01) = $5,599.99 × 0.99 = $5,543.99

Present

The net price for Visions is $5,543.99. You save $5,571.99 − $5,543.99 = $28.00 by purchasing your TV at Visions.

EXAMPLE 6.1G: UNDERSTANDING THE PRICE

An advertisement claims that at 60% off, you are saving $18. However, today there is an additional 20% off. What price should you pay for this item? What percent savings does this represent?

Plan

You are looking for how much you should pay after the discounts (N), and the single equivalent percentage that represents the two discounts (dequiv or just d).

What You Already Know

Step 1: You know the discount amount for the first discount only, as well as the two discount rates: D$1 = $18, d1 = 0.60, and d2 = 0.20.

How You Will Get There

Step 2: Calculate the list price by applying Formula 6.2a and rearranging for L.
Step 3: To calculate the net price, apply Formula 6.3. Step 4: To calculate the single equivalent discount, apply Formula 6.4.

Perform

Step 2: $18.00 = L × 0.60
L = $18.00 ÷ 0.6 = $30.00
Step 3: N = $30 × (1 − 0.60) × (1 − 0.20) = $30 × 0.40 × 0.80 = $9.60 Step 4: dequiv = 1 − (1 − 0.60) × (1 − 0.20) = 1 − (0.40)(0.80) = 1 − 0.32 = 0.68 or 68%

Present

You should pay $9.60 for the item, which represents a 68

EXERCISES

Round all money to two decimals and percentages to four decimals in each of the following questions.

Mechanics

For questions 1–4, solve for the unknown variables (identified with a ?) based on the information provided. "N/A" indicates that the particular variable is not applicable in the question.

Question	List Price of MRSP	First Discount	Second Discount	Third Discount	Net price	Equivalent Single Discount Rate	Total Discount Amount
1.	$980.00	42%	N/A	N/A	?	N/A	?
2.	?	25%	N/A	N/A	$600.00	N/A	?
3.	$1,975.00	25%	15%	10%	?	?	?
4.	?	18%	4%	7%	$366.05	?	?

Applications

5. A wholesaler of stereos normally qualifies for a 35% trade discount on all electronic products purchased from its manufacturer. If the MSRP of a stereo is $399.95, what net price will the wholesaler pay?

6. Mary is shopping at the mall where she sees a sign that reads, "Everything in the store is 30% off, including sale items!" She wanders in and finds a blouse on the clearance rack. A sign on the clearance rack states, "All clearance items are 50% off." If the blouse is normally priced at $69.49, what price should Mary pay for it?

7. A distributor sells some shoes directly to a retailer. The retailer pays $16.31 for a pair of shoes that has a list price of $23.98. What trade discount percent is the distributor offering to its retailers?

8. A retailer purchases supplies for its head office. If the retailer pays $16.99 for a box of paper and was eligible for a 15% volume discount, what was the original MSRP for the box of paper?

9. Mountain Equipment Co-op has purchased a college backpack for $29 after discounts of 30%, 8%, and 13%. What is the MSRP for the backpack? What single discount is equivalent to the three discounts?

10. Walmart purchased the latest CD recorded by Selena Gomez. It received a total discount of $10.08 off the MSRP for the CD, which represents a discount percent of 42%.

    a. What was the MSRP?

    b. What was the net price paid for the CD?

11. Best Buy just acquired an HP Pavilion computer for its electronics department. The net price on the computer is $260.40 and Best Buy receives discounts of 40% and 38%.

    a. What single discount is equivalent to the two discounts?

    b. What is the list price?

    c. What is the total discount amount?

12. TELUS retails a Samsung cellphone at the MSRP of $399.99. TELUS can purchase the phone from its supplier and receive a 20% trade discount along with a 5% volume discount.

    a. What is the single equivalent percent discount?

    b. What net price does TELUS pay for the phone?

    c. How much of a discount in dollars does this represent?

13. A wholesaler offers the following discounts: 10% seasonal discount for all purchases made between March 1 and May 1, 15% cumulative quantity discount whenever more than 5,000 units are purchased in any month, 5% loyalty discount for customers who have made regular purchases every month for at least one year, and a 33% trade discount to any retailer. Ed's Retail Superstore makes a purchase of 200 watches, MSRP $10, from the wholesaler on April 29. This month alone, Ed's has ordered more than

5,000 watches. However, Ed's has purchased from the wholesaler for only the past six months. Determine the total price that Ed's should pay for the watches.

14. If a distributor is eligible for a 60% trade discount, 5% volume discount, and 3% seasonal discount, what single equivalent discount rate would it be eligible to receive? If the trade discount is applied first and equals a trade discount of $48, calculate the net price for the item.

Challenge, Critical Thinking, & Other Applications

As mentioned in one of the "Paths to Success" sections, discount percentages share a commonality with negative percent changes (Section 3.1). Use the formulas from this chapter to solve questions 15–17 involving percent change.

15. A human resource manager needs to trim labour costs in the following year by 3%. If current year labour costs are $1,231,498, what are the labour costs next year?

16. At an accounting firm, the number of accountants employed is based on the ratio of 1:400 daily manual journal entries. Because of ongoing increases in automation, the number of manual journal entries declines at a constant rate of 4% per year. If current entries are 4,000 per year, how many years and days will it take until the firm needs to lay off one accountant? (Hint: An accountant is laid off when the number of journal entries drops below 3,600.)

17. An economist is attempting to understand how Canada reduced its national debt from 1999 to 2008. In 1999, Canada's national debt was $554.143 billion. In 2008, the national debt stood at $457.637 billion. What percentage had the national debt been reduced by during this time period?

18. Sk8 is examining an invoice. The list price of a skateboard is $109.00, and the invoice states it received a trade discount of 15% and quantity discount of 10% as well as a loyalty discount. However, the amount of the loyalty discount is unspecified.

    a. If Sk8 paid $80.88 for the skateboard, what is the loyalty discount percent?

    b. If the loyalty discount is applied after all other discounts, what amount of loyalty dollars does Sk8 save per skateboard?

19. Currently, a student can qualify for up to six different tuition discounts at a local college based on such factors as financial need or corporate sponsorships. Mary Watson just applied to the college and qualifies for all six discounts: 20%, 15%, 23%, 5%, 3%, and 1%.

    a. She is confused and wants the college to tell her what single discount percent she is receiving. What should the college tell her?

    b. If her total list tuition comes to $6,435.00, how much should she pay?

20. Sumandeep is very loyal to her local hairstylist. Because she is loyal, her hairstylist gives her three different discounts: 10%, 5%, and 5%. These discounts amount to $14.08 in savings.

    a. What was the list price her hairstylist charged her?

    b. What amount did she pay her hairstylist?

    c. If her hairstylist increases prices by 5%, what are the list price, net price, and total discount amount?

Adapted from unit 6.1 in Business Math: A Step-by-Step Handbook

3.2 Markup: Setting the Regular Price

NEED TO STAY IN BUSINESS

As you wait in line to purchase your Iced Caramel Macchiato at Starbucks, you look at the pricing menu and think that $4.99 seems like an awful lot of money for a frozen coffee beverage. Clearly, the coffee itself doesn't cost anywhere near that much. But then gazing around the café, you notice the carefully applied colour scheme, the comfortable seating, the high- end machinery behind the counter, and a seemingly well-trained barista who answers customer questions knowledgeably.

Where did the money to pay for all of this come from? You smile as you realize your $4.99 pays not just for the macchiato, but for everything else that comes with it.

The process of taking a product's cost and increasing it by some amount to arrive at a selling price is called **markup**. This process is critical to business success because every business must ensure that it does not lose money when it makes a sale. From the consumer perspective, the concept of markup helps you make sense of the prices that businesses charge for their products or services. This in turn helps you to judge how reasonable some prices are (and hopefully to find better deals).

THE COMPONENTS IN A SELLING PRICE

Before you learn to calculate markup, you first have to understand the various components of a selling price. Then, in the next section, markup and its various methods of calculation will become much clearer.

When your business acquires merchandise for resale, this is a monetary outlay representing a cost. When you then resell the product, the price you charge must recover more than just the product cost. You must also recover all the selling and operating expenses associated with the product. Ultimately, you also need to make some money, or profit, as a result of the whole process.

THE FORMULA

Most people think that marking up a product must be a fairly complex process. It is not. Formula 6.5 illustrates the relationship between the three components of cost, expenses, and profits in calculating the selling price.

Formula 6.5

S is Selling Price: Once you calculate what the business paid for the product (cost), the bills it needs to cover (expenses), and how much money it needs to earn (profit), you arrive at a selling price by summing the three components.

C is Cost: The cost is the amount of money that the business must pay to purchase or manufacture the product. If manufactured, the cost represents all costs incurred to make the product. If purchased, this number results from applying an appropriate discount formula from Section 6.1. There is a list price from which the business will deduct discounts to arrive at the net price. The net price paid for the product equals the cost of the product. If a business purchases or manufactures a product for $10 then it must sell the product for at least $10. Otherwise, it fails to recover what was paid to acquire or make the product in the first place—a path to sheer disaster!

Formula 6.5: The Selling Price of a Product: $S = C + E + P$

E is Expenses: Expenses (also called overhead) are the financial outlays involved in selling the product. Beyond just purchasing the product, the business has many more bills to pay, including wages, taxes, leases, equipment, electronics, insurance, utilities, fixtures, décor, and many more. These expenses must be recovered and may be calculated as

a. A fixed dollar amount per unit
b. A percentage of the product cost. For example, if a business forecasts total merchandise costs of $100,000 for the coming year and total business expenses of $50,000, then it may set a general guideline of adding 50% ($50,000 ÷ $100,000) to the cost of a product to cover expenses.
c. A percentage of the product selling price based on a forecast of future sales. For example, if a business forecasts total sales of $250,000 and total business expenses of $50,000, then it may set a general guideline of adding 20% ($50,000 ÷ $250,000) of the selling price to the cost of a product to cover expenses.

P is Profit: Profit is the amount of money that remains after a business pays all of its costs and expenses. A business needs to add an amount above its costs and expenses to allow it to grow. If it adds too much profit, though, the product's price will be too high, in which case the customer may refuse to purchase it. If it adds too little profit, the product's price may be too low, in which case the customer may perceive the product as shoddy and once again refuse to purchase it. Many businesses set general guidelines on how much profit to add to various products. As with expenses, this profit may be expressed as

a. A fixed dollar amount per unit
b. A percentage of the product cost
c. A percentage of the selling price

How it Works

Follow these steps to solve pricing scenarios involving the three components:

> **Step 1**: Four variables are involved in Formula 6.5. Identify the known variables. Note that you may have to calculate the product's cost by applying the single or multiple discount formulas (Formulas 6.1 and 6.3, respectively). Pay careful attention to expenses and profits to capture how you calculate these amounts.

> **Step 2**: Apply Formula 6.5 and solve for the unknown variable.

Assume a business pays a net price of $75 to acquire a product. Through analyzing its finances, the business estimates expenses at $25 per unit, and it figures it can add $50 in profit. Calculate the selling price.

> Step 1: The net price paid for the product is the product cost. The known variables are C = $75, E = $25, and P = $50.

Step 2: According to Formula 6.5, the unit selling price is S = C + E + P = $75 + $25 + $50 = $150.

Important Notes

In applying Formula 6.5 you must adhere to the basic rule of linear equations requiring all terms to be in the same unit. That is, you could use Formula 6.5 to solve for the selling price of an individual product, where the three components are the unit cost, unit expenses, and unit profit. When you add these, you calculate the unit selling price. Alternatively, you could use Formula 6.5 in an aggregate form where the three components are total cost, total expenses, and total profit. In this case, the selling price is a total selling price, which is more commonly known as total revenue. But you cannot mix individual components with aggregate components.

Things To Watch Out For

The most common mistake in working with pricing components occurs in the "Understand" portion of the PUPP model. It is critical to identify and label information correctly. You have to pay attention to details such as whether you are expressing the expenses in dollar format or as a percentage of either cost or selling price. Systematically work your way through the information provided piece by piece to ensure that you do not miss an important detail.

Give It Some Thought

- What three components make up a selling price? In what units are these components commonly expressed?
- In what three ways are expenses and profits expressed?
- What is the relationship between net price and cost?

Solutions

1. Cost, expenses (also called overhead), and profit. They are expressed either per unit or as a total.
2. A specific dollar amount, a percentage of cost, or a percentage of the selling price.
3. The net price paid for a product is the same as the cost of the product.

EXAMPLE 3.2A: SETTING A PRICE ON FASHION IN DOLLARS

Mary's Boutique purchases a dress for resale at a cost of $23.67. The owner determines that each dress must contribute$5.42 to the expenses of the store. The owner also wants this dress to earn $6.90 toward profit. What is the regular selling price for the dress?

Plan

You are looking for the regular selling price for the dress, or S.

What You Already Know

Step 1: The unit cost of the dress and the unit expense and the unit profit are all known: C = $23.67, E = $5.42, P = $6.90

How You Will Get There

Step 2: Apply Formula 6.5.

Perform

Step 2: S = $23.67 + $5.42 + $6.90 = $35.99

Present

Mary's Boutique will set the regular price of the dress at $35.99.

EXAMPLE 3.2B: SETTING THE PRICE USING PERCENTAGE OF COST

John's Discount Store just completed a financial analysis. The company determined that expenses average 20% of the product cost and profit averages 15% of the product cost. John's Discount Store purchases Chia Pets from its supplier for anMSRP of $19.99 less a trade discount of 45%. What will be the regular selling price for the Chia Pets?

Plan

You are looking for the regular selling price for the Chia pets, or S.

What You Already Know

Step 1: The list price, discount rate, expenses, and profit are known:

- L = $19.99

- d = 0.45

- E = 20% of cost, or 0.20C

- P = 15% of cost, or 0.15C

How You Will Get There

Step 1 (continued): Although the cost of the Chia Pets is not directly known, you do know the MSRP (list price) and the trade discount. The cost is equal to the net price. Apply Formula 6.1.
Step 2: To calculate the selling price, apply Formula 6.5.

Perform

Step 1: N = $19.99 × (1 – 0.45) = $19.99 × 0.55 = $10.99 = C
Step 2: S = $10.99 + 0.20C + 0.15C
S = $10.99 + 0.20($10.99) + 0.15($10.99)
S = $10.99 + $2.20 + $1.65 = $14.84

Present

John's Discount Store will sell the Chia Pet for $14.84.

The Price Components for a Chia Pet

$14.84

■ Cost -->
$10.99

■ Expenses -->
$2.20
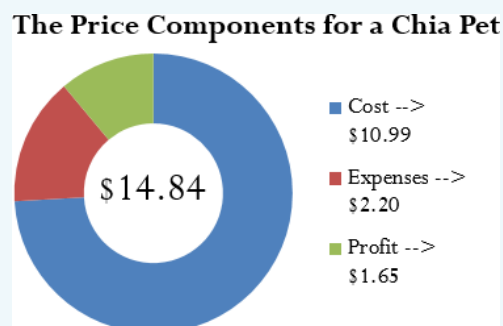
■ Profit -->
$1.65

EXAMPLE 3.2C: SETTING THE PRICE USING PERCENTAGE OF SELLING PRICE

Based on last year's results, Benthal Appliance learned that its expenses average 30% of the regular selling price. It wants a 25% profit based on the selling price. If Benthal Appliance purchases a fridge for $1,200, what is the regular unit selling price?

Plan

You are looking for the regular unit selling price for the fridge, or S.

What You Already Know

Step 1: The cost, expenses, and profit for the fridge are known:

- E = 30% of S, or 0.3
- SP = 25% of S, or 0.25
- SC = $1,200.00

How You Will Get There

Step 2: Apply Formula 6.5

Perform

Step 2: S = $1,200.00 + 0.3S + 0.25S
S = $1,200.00 + 0.55S
S − 0.55S = $1,200.00
0.45S = $1,200.00
S = $2,666.6

Present

Benthal Appliance should set the regular selling price of the fridge at $2,666.67.

Price Components for a Fridge

$2,666.67

- Cost --> $1,200.00
- Expenses --> 0.3($2,666.67) = $800.00
- Profit --> 0.25($2,666.67) = $666.67
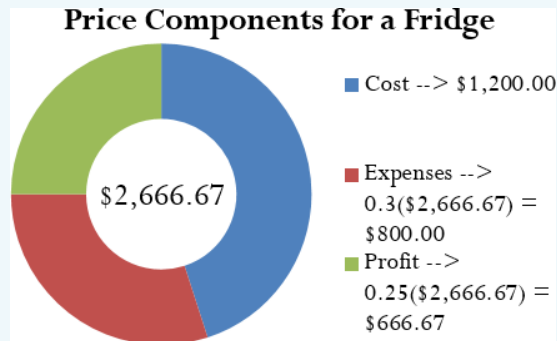
EXAMPLE 3.2D: USING SELLING PRICE TO FIGURE OUT THE COST

If a company knows that its profits are 15% of the selling price and expenses are 30% of cost, what is the cost of an MP3 player that has a regular selling price of $39.99?

Plan

You are looking for the cost of the MP3 player, or C.

What You Already Know

Step 1: The expenses, profits, and the regular unit selling price are as follows:

- S = $39.99
- P = 15% of S, or 0.15S
- E = 30% of cost, or 0.3C

How You Will Get There

Step 2: Apply Formula 6.5, rearranging for C.

Perform

Step 2: $39.99 = C + 0.3C + 0.15($39.99)

$39.99 = 1.3C + $6.00
$33.99 = 1.3C
$26.15 = C

Present

The cost of the MP3 Player is $26.15.

Price Components of an MP3 Player

$39.99

- Cost --> $26.15
- Expenses -->
  $0.3(\$26.15) = \$7.84$
- Profit -->
  $0.15(\$39.99) = \$6.00$
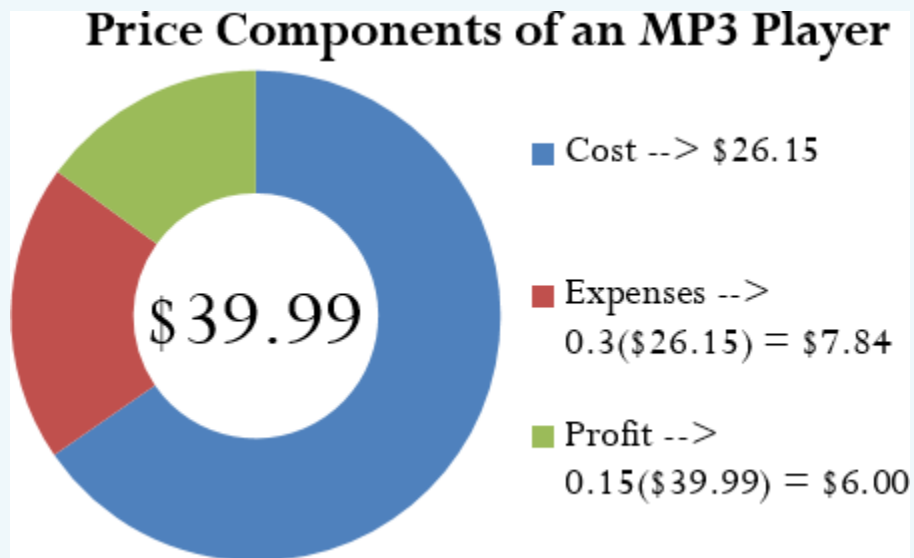
EXAMPLE 3.2E: DETERMINING PROFITABILITY

Peak of the Market considers setting the regular unit selling price of its strawberries at $3.99 per kilogram. If it purchases these strawberries from the farmer for $2.99 per kilogram and expenses average 40% of product cost, does Peak of the Market make any money?

Plan

In asking whether Peak of the Market makes money, you are looking for the profit, or P.

What You Already Know

Step 1: The cost, expenses, and proposed regular unit selling price for the strawberries are as follows:

- S = $3.99
- C = $2.99
- E = 40% of cost, or 0.4C

How You Will Get There

Step 2: Apply Formula 6.5, rearranging for P.

Perform

Step 2: $3.99 = $2.99 + 0.4($2.99) + P
$3.99 = $4.19 + P
−$0.20 = P

Present

The negative sign on the profit means that Peak of the Market would take a loss of $0.20 per kilogram if it sells the strawberries at $3.99. Unless Peak of the Market has a marketing reason or sound business strategy for doing this, the company should reconsider its pricing.
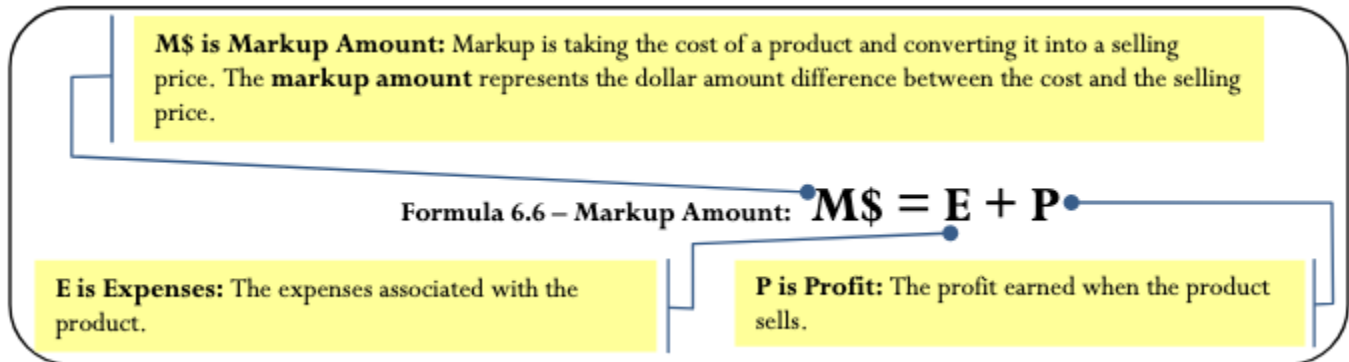
CALCULATING THE MARKUP DOLLARS

Most companies sell more than one product, each of which has different price components with varying costs, expenses, and profits. Can you imagine trying to compare 50 different products, each with three different components? You would have to juggle 150 numbers! To make merchandising decisions more manageable and comparable, many companies combine expenses and profit together into a single quantity, either as a dollar amount or a percentage. This section focuses on the markup as a dollar amount.
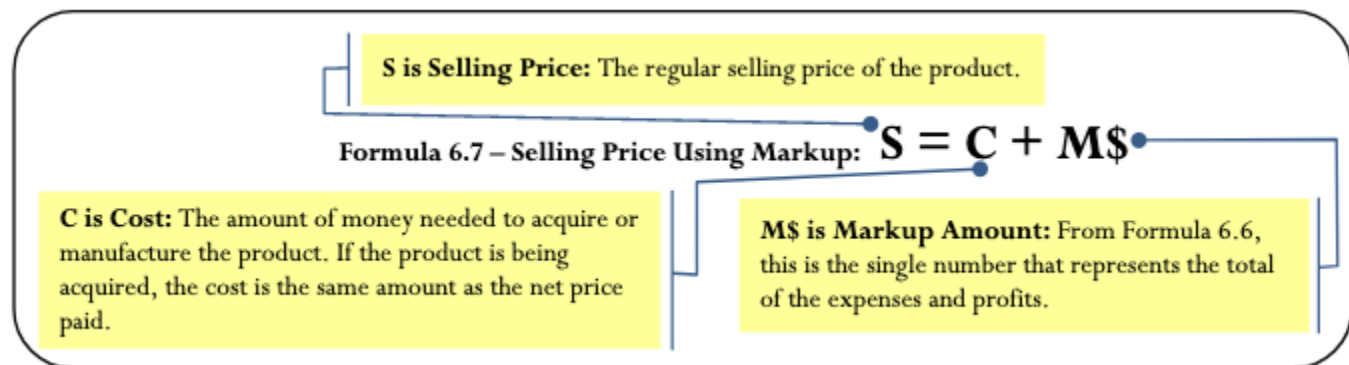
The Formula

One of the most basic ways a business simplifies its merchandising is by combining the dollar amounts of its expenses and profits together as expressed in Formula 6.6.

Formula 6.6

M$ is Markup Amount: Markup is taking the cost of a product and converting it into a selling price. The **markup amount** represents the dollar amount difference between the cost and the selling price.

Formula 6.6 – Markup Amount: $M\$ = E + P$

E is Expenses: The expenses associated with the product.

P is Profit: The profit earned when the product sells.

Note that since the markup amount (M$) represents the expenses (E) and profit (P) combined, you can substitute the variable for markup amount into Formula 6.5 to create Formula 6.7, which calculates the regular selling price.

S is Selling Price: The regular selling price of the product.

Formula 6.7 – Selling Price Using Markup: $S = C + M\$$

C is Cost: The amount of money needed to acquire or manufacture the product. If the product is being acquired, the cost is the same amount as the net price paid.

M$ is Markup Amount: From Formula 6.6, this is the single number that represents the total of the expenses and profits.

How It Works

Follow these steps when you work with calculations involving the markup amount:

> **Step 1**: You require three variables in either Formula 6.6 or Formula 6.7. At least two of the variables must be known. If the amounts are not directly provided, you may need to calculate these amounts by applying other discount or markup formulas.

> **Step 2**: Solve either Formula 6.6 or Formula 6.7 for the unknown variable. Recall from Example 6.2D that the MP3 player's expenses are $7.84, the profit is $6.00, and the cost is $26.15.
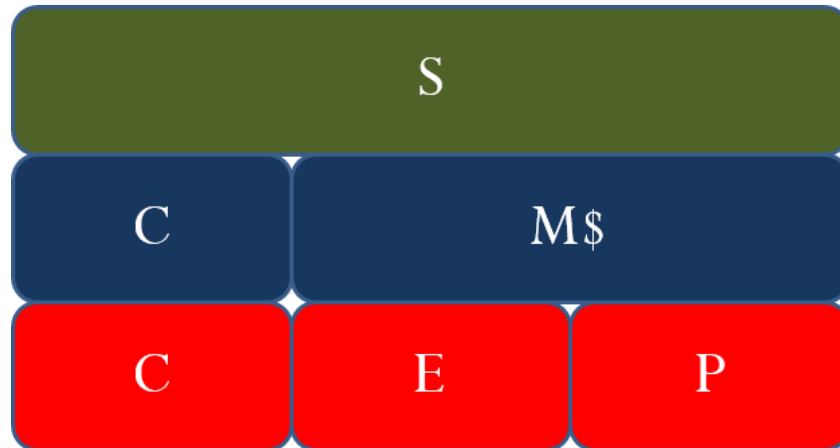
Calculate the markup amount and the selling price.

> Step 1: The known variables are E = $7.84, P = $6.00, and C = $26.15.

> Step 2: According to Formula 6.6, the markup amount is the sum of the expenses and profit, or M$ = $7.84 + $6.00 =$13.84.

Step 2 (continued): Applying Formula 6.7, add the markup amount to the cost to arrive at the regular selling price, resulting in S = $26.15 + $13.84 = $39.99

Paths To Success

You might have already noticed that many of the formulas in this chapter are interrelated. The same variables appear numerous times but in different ways. To help visualize the relationship between the various formulas and variables, many students have found it helpful to create a markup chart [see below].

This chart demonstrates the relationships between Formulas 6.5, 6.6, and 6.7. It is evident that the selling price (the green line) consists of cost, expenses, and profit (the red line representing Formula 6.5); or it can consist of cost and the markup amount (the blue line representing Formula 6.7). The markup amount on the blue line consists of the expenses and profit on the red line (Formula 6.6).

EXAMPLE 3.2F: MARKUP AS A DOLLAR AMOUNT

A cellular retail store purchases an iPhone with an MSRP of $779 less a trade discount of 35% and volume discount of 8%. The store sells the phone at the MSRP.What is the markup amount?If the store knows that its expenses are 20% of the cost, what is the store's profit?

Plan

First of all, you need to calculate the markup amount (M$). You also want to find the store's profit (P).

What You Already Know

Step 1: The smartphone MSRP and the two discounts are known, along with the expenses and selling price:

- L = $270 d1 = 0.35 d2 = 0.08
- E = 20% of cost, or 0.2CS = $779

How You Will Get There

Step 1 (continued): Calculate the cost of the iPhone by applying Formula 6.3.
Step 2: Calculate the markup amount using Formula 6.7.
Step 2 (continued): Calculate the profit by applying Formula 6.6, rearranging for P.

Perform

Step 1 (continued):

- N = $779.00 × (1 − 0.35) × (1 − 0.08)
- N = $779.00 × 0.65 × 0.92 = $465.84 = C

Step 2:

- $779.00 = $465.84 + M$
- $313.16 = M$

Step 2 (continued):

- $313.16 = 0.2($465.84) + P
- $313.16 = $93.17 + P
- $219.99 = P

Present

The markup amount for the iPhone is $313.16. When the store sells the phone for $779.00, its profit is$219.99.

The Price Components of an Apple iPhone

$779

Cost --> $465.84

Expenses --> $93.17

Profit --> $219.99

Calculating the Markup Percent

It is important to understand markup in terms of the actual dollar amount; however, it is more common in business practice to calculate the markup as a percentage. There are three benefits to converting the markup dollar amount into a percentage:

1. Easy comparison of different products having vastly different price levels and costs, to help you see how each product contributes toward the financial success of the company. For example, if a chocolate bar has a 50¢ markup included in a selling price of $1, while a car has a $1,000 markup included in a selling price of $20,000, it is difficult to compare the profitability of these items. If these numbers were expressed as a percentage of the selling price such that the chocolate bar has a 50% markup and the car has a 5% markup, it is clear that more of every dollar sold for chocolate bars goes toward list profitability.

2. Simplified translation of costs into a regular selling price—a task that must be done for each product, making it helpful to have an easy formula, especially when a company carries hundreds, thousands, or even tens of thousands of products. For example, if all products are to be marked up by 50% of cost, an item with a $100 cost can be quickly converted into a selling price of $150.

3. An increased understanding of the relationship between costs, selling prices, and the list profitability for any given product. For example, if an item selling for $25 includes a markup on selling price of 40% (which is $10), then you can determine that the cost is 60% of the selling price ($15) and that $10 of every $25 item sold goes toward list profits.

You can translate the markup dollars into a percentage using two methods, which express the amount either as a percentage of cost or as a percentage of selling price:

Method 1: Markup as a Percentage of Cost.

This method expresses the markup rate using cost as the base. Many companies use this technique internally because most accounting is based on cost information. The result, known as the **markup on cost percentage**, allows a reseller to convert easily from a product's cost to its regular unit selling price.

Method 2: Markup as a Percentage of Selling Price.

This method expresses the markup rate using the regular selling price as the base. Many other companies use this method, known as the **markup on selling price percentage**, since it allows for quick understanding of the portion of the selling price that remains after the cost of the product has been recovered. This percentage represents the list profits before the deduction of expenses and therefore is also referred to as the *list profit margin*.

THE FORMULA

Both formulas are versions of Formula 2.2 on rate, portion, and base. The markup on cost percentage is expressed in Formula 6.8, while the markup on selling price percentage is expressed in Formula 6.9.

How it Works

Refer back to the steps in calculating markup. The steps you must follow for calculations involving markup percent are almost identical to those for working with markup dollars:

> **Step 1**: Three variables are required in either Formula 6.8 or Formula 6.9. For either formula, at least two of the variables must be known. If the amounts are not directly provided, you may need to calculate these amounts by applying other discount or markup formulas.

> **Step 2**: Solve either Formula 6.8 or Formula 6.9 for the unknown variable.

Continuing to work with Example 6.2D, recall that the cost of the MP3 player is $26.15, the markup amount is $13.84, and the selling price is $39.99. Calculate both markup percentages.

> Step 1: The known variables are C = $26.15, M$ = $13.84, and S = $39.99.

> Step 2: To calculate the markup on cost percentage, apply Formula 6.8:

$$MoC\% = \frac{\$13.84}{\$26.15} \times 100 = 52.9254\%$$

In other words, you must add 52.9254% of the cost on top of the unit cost to arrive at the regular unit selling price of $39.99. Step 2 (continued): To calculate the markup on selling price percentage, apply Formula 6.9:

$$MoC\% = \frac{\$13.84}{\$39.99} \times 100 = 34.6087\%$$

In other words, 34.6087% of the selling price represents list profits after the business recovers the $26.15 cost of the MP3 player.

Important Notes

Businesses are very focused on profitability. Your Texas Instruments BAII Plus calculator is programmed with the markup on selling price percentage. The function is located on the second shelf above the number three. To use this function, open the window by pressing 2nd 3. You can scroll between lines using your ↑ and ↓ arrows.

There are three variables:

- CST is the cost. Use the symbol C.
- SEL is the selling price. Use the symbol S.
- MAR is the markup on selling price percentage. Use the symbol MoS%.

As long as you know any two of the variables, you can solve for the third. Enter any two of the three variables (you need to press ENTER after each), making sure the window shows the output you are seeking, and press CPT.

Things To Watch Out For

Merchandising involves many variables. Nine formulas have been established so far, and a few more are yet to be introduced. Though you may feel bogged down by all of these formulas, just remember that you have encountered most of these merchandising concepts since you were very young and that you interact with retailers and pricing every day. This chapter merely formalizes calculations you already perform on a daily basis, whether at work or at home. The calculation of discounts is no different than going to Walmart and finding your favourite CD on sale. You know that when a business sells a product, it has to recoup the cost of the product, pay its bills, and make some money. And you have worked with percentages since elementary school.

Do not get stuck in the formulas. Think about the concept presented in the question. Change the scenario of the question and put it in the context of something more familiar. Ultimately, if you really have difficulties then look at the variables provided and cross-reference them to the merchandising formulas. Your goal is to find formulas in which only one variable is unknown. These formulas are solvable. Then ask yourself, "How does knowing that new variable help solve any other formula?"

You do not need to get frustrated. Just be systematic and relate the question to what you already know.

Paths To Success

The triangle method simplifies rearranging both Formulas 6.8 and 6.9 to solve for other unknown variables as illustrated in the figure below.

Formula 6.8: Markup on Cost

Formula 6.9: Markup on Selling Price

Sometimes you need to convert the markup on cost percentage to a markup on selling price percentage, or vice versa. Two shortcuts allow you to convert easily from one to the other:

$$MoC\% = \frac{MoS\%}{1 - MoS\%}$$

$$MoS\% = \frac{MoC\%}{1 + MoC\%}$$

Notice that these formulas are very similar. How do you remember whether to add or subtract in the denominator? In normal business situations, the MoC% is always larger than the MoS%. Therefore, if you are converting one to the other you need to identify whether you want the percentage to become larger or smaller.

Key Takeaways

- To calculate MoC%, you want a larger percentage. Therefore, make the denominator smaller by subtracting MoS% from 1.

- To calculate MoS%, you want a smaller percentage. Therefore, make the denominator larger by adding MoC% to 1.

GIVE IT SOME THOUGHT

Answer the following true/false questions.

1. The markup on selling price percentage can be higher than 100%.

2. The markup dollar amount can be more than the selling price.

3. The markup on cost percentage can be higher than 100%.

4. The markup on cost percentage in most business situations is higher than the markup on selling price percentage.

5. If you know the markup on cost percentage and the cost, you can calculate a selling price.

6. If you know the markup on selling price percentage and the cost, you can calculate a selling price.

Solutions

1. False. The markup amount is a portion of the selling price and therefore is less than 100%.
2. False. The markup amount plus the cost equals the selling price. It must be less than the selling price.
3. True. A cost can be doubled or tripled (or increased even more) to reach the price.
4. True. The base for markup on cost percentage is smaller, which produces a larger percentage.
5. True. You could combine Formulas 6.7 and 6.8 to arrive at the selling price.
6. True. You could convert the MoS% to a MoC% and solve as in the previous question.

EXAMPLE 3.2G: MARKUP AS A PERCENTAGE

A large national retailer wants to price a Texas Instruments BAII Plus calculator at the MSRP of $39.99. The retailer can acquire the calculator for $17.23.What is the markup on cost percentage?What is the markup on selling price percentage?

Plan

You have been asked to solve for the markup on cost percentage (MoC%) and the markup on selling price percentage (MoS%).

What You Already Know

Step 1: The regular unit selling price and the cost are provided:

- S = $39.99

- C = $17.23

How You Will Get There

Step 1 (continued): You need the markup dollars. Apply Formula 6.7, rearranging for M$.
Step 2: To calculate markup on cost percentage, apply Formula 6.8.

Step 2 (continued): To calculate markup on selling price percentage, apply Formula 6.9.

Perform

Step 1 (continued): $39.99 = $17.23 + M\$

$22.76 = M\$

Step 2: MoC% = $\$22.76/\$_{17.23} \times 100 = 132.0952\%$

Step 2 (continued): MoS% = $\$22.76/\$_{39.99} \times 100 = 56.9142\%$

Present

The markup on cost percentage is 132.0952%. The markup on selling price percentage is 56.9142%.

BREAK-EVEN PRICING

In running a business, you must never forget the "bottom line." In other words, if you fully understand how your products are priced, you will know when you are making or losing money. Remember, if you keep losing money you will not stay in business for long! 15% of new businesses will not make it past their first year, and 49% fail in their first five years. This number becomes even more staggering with an 80% failure rate within the first decade.[1] Do not be one of these statistics! With your understanding of markup, you now know what it takes to break even in your business.

B*reak-even* means that you are earning no profit, but you are not losing money either. Your profit is zero.

The Formula

If the regular unit selling price must cover three elements—cost, expenses, and profit—then the regular unit selling price must exactly cover your costs and expenses when the profit is zero. In other words, if Formula 6.5 is modified to calculate the selling price at the break-even point (SBE) with P=0, then

$$S_{BE} = C + E$$

This is not a new formula. It just summarizes that at break-even there is no profit or loss, so the profit (P) is eliminated from the formula.

How It Works

The steps you need to calculate the break-even point are no different from those you used to calculate the regular selling price. The only difference is that the profit is always set to zero.

1. Statistics Canada, "Failure Rates for New Firms," The Daily, February 16, 2000, www.statcan.gc.ca/daily- quotidien/000216/ dq000216b-eng.htm.

Recall Example 6.2D that the cost of the MP3 player is $26.15 and expenses are $7.84. The break-even price (SBE) is $26.15 + $7.84 = $33.99. This means that if the MP3 player is sold for anything more than $33.99, it is profitable; if it is sold for less, then the business does not cover its costs and expenses and takes a loss on the sale.

EXAMPLE 6.2: KNOWING YOUR BREAK-EVEN PRICE

John is trying to run an eBay business. His strategy has been to shop at local garage sales and find items of interest at a great price. He then resells these items on eBay. On John's last garage sale shopping spree, he only found one item—a Nintendo Wii that was sold to him for $100. John's vehicle expenses (for gas, oil, wear/tear, and time) amounted to $40. eBay charges a $2.00 insertion fee, a flat fee of $2.19, and a commission of 3.5% based on the selling price less $25. What is John's minimum list price for his Nintendo Wii to ensure that he at least covers his expenses?

Plan

You are trying to find John's break-even selling price (SBE).

What You Already Know

Step 1: John's cost for the Nintendo Wii and all of his associated expenses are as follows:

- C = $100.00
- E (vehicle) = $40.00
- E (insertion) = $2.00
- E (flat) = $2.19
- E (commission) = 3.5%(SBE − $25.00)

How You Will Get There

Step 1 (continued): You have four expenses to add together that make up the E in the formula.
Step 2: Formula 6.5 states S = C + E + P.
Since you are looking for the break-even point, then P is set to zero and SBE = C + E.

Perform

Step 1 (continued): E = $40.00 + $2.00 + $2.19 + 3.5%($S_{BE}$ − $25.00)
E = $44.19 + 0.035($S_{BE}$ − $25.00) = $44.19 + 0.035$S_{BE}$ − $0.875
E = $43.315 + 0.035$S_{BE}$
Step 2:
S_{BE} = $100.00 + $43.315 + 0.035$S_{BE}$

$S_{BE} = \$143.315 + 0.035S_B$

$S_{BE} - 0.035_{BE} = \$143.315$

$0.965S_{BE} = \$143.315$

$S_{BE} = \$148.51$

Present

At a price of $148.51 John would cover all of his costs and expenses but realize no profit or loss. Therefore, $148.51 is his minimum price.

EXERCISES

Round all money to two decimals and percentages to four decimals for each of the following exercises.

Mechanics

For questions 1–8, solve for the unknown variables (identified with a ?) based on the information provided

Question	Regular Unit Selling Price	Cost	Expenses	Profit	Markup Amount	Break-Even Price	Markup on Cost	Markup on Selling Price
1.	?	$188.42	$48.53	$85.00	?	?	?	?
2.	$999.99	?	30% of C	23% of C	?	?	?	?
3.	?	?	?	10% of S	$183.28	?	155%	?
4.	$274.99	?	20% of S	?	?	?	?	35%
5.	?	?	45% of C	?	$540.00	$1,080.00	?	?
6.	?	$200 less 40%	?	15%of S	?	?	68%	?
7.	?	?	$100.00	?	$275.00		?	19%
8.	?	?	15% of C	12% of S	?	$253.00	?	?

Applications

9. If a pair of sunglasses sells at a regular unit selling price of $249.99 and the markup is always 55% of the regular unit selling price, what is the cost of the sunglasses?

10. A transit company wants to establish an easy way to calculate its transit fares. It has determined that the cost of a transit ride is $1.00, with expenses of 50% of cost. It requires $0.75 profit per ride. What is its markup on cost percentage?

11. Daisy is trying to figure out how much negotiating room she has in purchasing a new car. The car has an MSRP of $34,995.99. She has learned from an industry insider that most car dealerships have a 20% markup on selling price. What does she estimate the dealership paid for the car?

12. The markup amount on an eMachines desktop computer is $131.64. If the machine regularly retails for $497.25 and expenses average 15% of the selling price, what profit will be earned?

13. Manitoba Telecom Services (MTS) purchases an iPhone for $749.99 less discounts of 25% and 15%. MTS's expenses are known to average 30% of the regular unit selling price.

   a. What is the regular unit selling price if a profit of $35 per iPhone is required?

   b. What are the expenses?

   c. What is the markup on cost percentage?

   d. What is the break-even selling price?

14. A snowboard has a cost of $79.10, expenses of $22.85, and profit of $18.00.

   a. What is the regular unit selling price?

   b. What is the markup amount?

   c. What is the markup on cost percentage?

   d. What is the markup on selling price percentage?

   e. What is the break-even selling price? What is the markup on cost percentage at this break-even price?

Challenge, Critical Thinking, & Other Applications

15. A waterpark wants to understand its pricing better. If the regular price of admission is $49.95, expenses are 20% of cost, and the profit is 30% of the regular unit selling price, what is the markup amount?

16. Sally works for a skateboard shop. The company just purchased a skateboard for $89.00 less discounts of 22%, 15%, and 5%. The company has standard expenses of 37% of cost and desires a profit of 25% of the regular unit selling price. What regular unit selling price should Sally set for the skateboard?

17. If an item has a 75% markup on cost, what is its markup on selling price percentage?

18. A product received discounts of 33%, 25%, and 5%. A markup on cost of 50% was then applied to arrive at the regular unit selling price of $349.50. What was the original list price for the product?

19. Mountain Equipment Co-op (MEC) wants to price a new backpack. The backpack can be purchased for a list price of $59.95 less a trade discount of 25% and a quantity discount of 10%. MEC estimates expenses to be 18% of cost and it must maintain a markup on selling price of 35%.

   a. What is the cost of backpack?

   b. What is the markup amount?

   c. What is the regular unit selling price for the backpack?

   d. What profit will Mountain Equipment Co-op realize?

   e. What happens to the profits if it sells the backpack at the MSRP instead?

20. Costco can purchase a bag of Starbucks coffee for $20.00 less discounts of 20%, 15%, and 7%. It then adds a 40% markup on cost. Expenses are known to be 25% of the regular unit selling price.

   a. What is the cost of the coffee?

b. What is the regular unit selling price?

c. How much profit will Costco make on a bag of Starbucks coffee?

d. What markup on selling price percentage does this represent?

e. Repeat questions (a) through (d) if the list price changes to $24.00.

3.3 Markdown: Setting the Sale Price

EVERYBODY LOVES A SALE

Flashy signs in a retail store announce, "40% off, today only!" Excitedly you purchase three tax-free products with regular price tags reading $100, $250, and $150. The cashier processing the transaction informs you that your total is $325. You are about to hand over your credit card when something about the total makes you pause. The regular total of all your items is $500. If they are 40% off, you should receive a $200 deduction and pay only $300. The cashier apologizes for the mistake and corrects your total.

Although most retail stores use automated checkout systems, these systems are ultimately programmed by human beings. A computer system is only as accurate as the person keying in the data. A study by the Competition Bureau revealed that 6.3% of items at various retail stores scanned incorrectly. The average error spread is up to 13% around the actual product's price![1] Clearly, it is important for you as a consumer to be able to calculate markdowns.

Businesses must also thoroughly understand markdowns so that customers are charged accurately for their purchases. Businesses must always comply with the Competition Act of Canada, which specifically defines legal pricing practices. If your business violates this law, it faces severe penalties.

THE IMPORTANCE OF MARKDOWNS

A **markdown** is a reduction from the regular selling price of a product resulting in a lower price. This lower price is called the **sale price** to distinguish it from the selling price.

Many people perceive markdowns as a sign of bad business management decisions. However, in most situations this is not true. Companies must always attempt to forecast the future. In order to stock products, a reseller must estimate the number of units that might sell in the near future for every product that it carries. This is both an art and a science. While businesses use statistical techniques that predict future sales with a relative degree of accuracy, consumers are fickle and regularly change shopping habits. Markdowns most commonly occur under four circumstances:

Clearing Out Excess or Unwanted Inventory

In these situations, the business thought it could sell 100 units; however, consumers purchased only 20 units. In the case of seasonal inventory, such as Christmas items on Boxing Day, the retailer wishes to avoid packing up and storing the inventory until the next season.

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1. Competition Bureau, Fair Business Practices Branch, *Price Scanning Report*, Table B, page 5, 1999,
   www.competitionbureau.gc.ca/epic/site/cb-bc.nsf/en/01288e.html.

Clearing Out Damaged or Discontinued Items

Selling a damaged product at a discount is better than not selling it at all. When products are discontinued, this leaves shelf space underused, so it is better to clear the item out altogether to make room for profitable items that can keep the shelves fully stocked.

Increasing Sales Volumes

Sales attract customers because almost everyone loves a deal. Though special marketing events such as a 48 hour sale reduce the profitability per unit, by increasing the volume sold these sales can lead to a greater profit overall.

Promoting Add-On Purchases

Having items on sale attracts customers to the store. Many times customers will not only purchase the item on sale but also, as long as they are on the premises, grab a few other items, which are regularly priced and very profitable. Like many others, you may have walked into Target to buy one item but left with five instead.

THE FORMULA

Markdowns are no different from offering a discount. Recall from Section 6.1 that one of the types of discounts is known as a sale discount. The only difference here lies in choice of language. Markdowns are common, so you will find it handy to adapt the discount formulas to the application of markdowns, replacing the symbols with ones that are meaningful in merchandising. Formula 6.1, introduced in Section 6.1, calculates the net price for a product after it receives a single discount:

$$N = L \times (1 - d)$$

Formula 6.10 adapts this formula for use in markdown situations.

S_{onsale} is Sale Price: The sale price is the price of the product after reduction by the markdown percent. Conceptually, the sale price is the same as the net price.

Formula 6.10 – The Sale Price Of A Product: $S_{onsale} = S \times (1 - d)$

S is Selling Price: The regular selling price of the product before any discounts. The higher price is the list price. In merchandising questions, this dollar amount may or may not be a known variable. If the selling price is unknown, you must calculate it using an appropriate formula or combination of formulas from either Section 6.1 or Section 6.2.

d is Markdown Rate: A markdown rate is the same as a sale discount rate. Therefore you use the same discount rate symbol from Section 6.1 to represent the percentage (in decimal format) by which you reduce the selling price. As in Formula 6.1, note that you are interested in calculating the sale price and not the amount saved. Thus, you take the markdown rate away from 1 to find out the rate owing.

In markdown situations, the selling price and the sale price are different variables. The sale price is always less than the selling price. In the event that a regular selling price has more than one markdown percent applied to it,

you can extend
Formula 6.10 in the same manner that Formula 6.3 calculated multiple discounts. If you are interested in the markdown amount in dollars, recall that Formula 6.2 calculates the discount amount in dollars. Depending on what information is known, the formula has two variations:

Formula 6.2a: D$ = L × d
Formula 6.2b: D$ = L − N

Formulas 6.11a and 6.11b adapt these formulas to markdown situations.

D$ is Markdown Amount: You determine the markdown amount using either formula depending on what information is known. If you know the selling price and markdown percent, apply Formula 6.11a. If you know the selling price and sale price, apply Formula 6.11b.

Formula 6.11a – Markdown Amount: $D\$ = S \times d$

Formula 6.11b – Markdown Amount: $D\$ = S - S_{onsale}$

S is Selling Price: The regular selling price before you apply any markdown percentages.

S_{onsale} is Sale Price: The price after you have deducted all markdown percentages from the regular selling price.

d is Markdown Rate: the percentage of the selling price to be deducted (in decimal format). In this case, because you are interested in figuring out how much the percentage is worth, you do not take it away from 1 as in Formula 6.10.

The final markdown formula reflects the tendency of businesses to express markdowns as percentages, facilitating easy comprehension and comparison. Recall Formula 6.9 from Section 6.2, which calculated a markup on selling price percent:

Formula 6.12 adapts this formula to markdown situations.

d is Markdown Percentage: You always deduct a markdown amount from the regular selling price of the product. Therefore, you always express the markdown percent as a percentage of the selling price. Use the same symbol for a discount rate, since markdown rates are synonymous with sale discounts.

Formula 6.12 – Markdown Percentage: $d = \dfrac{D\$}{S} \times 100$

S is Selling Price: The regular selling price of the product before any discounts.

×100 is Percent Conversion: The markdown is always expressed as a percentage.

D\$ is Markdown Amount: The total dollar amount deducted from the regular selling price.

HOW IT WORKS

Follow these steps to calculate a markdown:

Step 1: Across all three markdown formulas, the four variables consist of the selling price (S), sale price (Sonsale), markdown dollars (D\$), and markdown rate (d). Identify which variables are known. Depending on the known information, you may have to calculate the selling price using a combination of discount and markup formulas.

Step 2: Apply one or more of Formulas 6.10, 6.11a, 6.11b, and 6.12 to calculate the unknown variable(s). In the event that multiple markdown rates apply, extend Formula 6.10 to accommodate as many markdown rates as required.

Recall from Section 3.2 the example of the MP3 player with a regular selling price of $39.99. Assume the retailer has excess inventory and places the MP3 player on sale for 10% off. What is the sale price and markdown amount?

Step 1: The selling price and markdown percent are S = $39.99 and d = 0.10, respectively.

Step 2: Apply Formula 6.10 to calculate the sale price, resulting in Sonsale = $39.99 × (1 – 0.10) = $35.99.

Step 2 (continued): You could use either of Formulas 6.11a or 6.11b to calculate the markdown amount since the selling price, sale price, and markdown percent are all known. Arbitrarily choosing Formula 6.11a, you calculate a markdown amount of D$ = $39.99 × 0.10 = $4.00.

Therefore, if the retailer has a 10% off sale on the MP3 players, it marks down the product by $4.00 and retails it at a sale price of $35.99.

Things To Watch Out For

Just as in Section 3.2, avoid getting bogged down in formulas. Recall that the three formulas for markdowns are not new formulas, just adaptations of three previously introduced concepts. As a consumer, you are very experienced with endless examples of sales, bargains, discounts, blowouts, clearances, and the like.

Every day you read ads in the newspaper and watch television commercials advertising percent savings. This section simply crystallizes your existing knowledge. If you are puzzled by questions involving markdowns, make use of your shopping experiences at the mall!

Paths To Success

Three of the formulas introduced in this section can be solved for any variable through algebraic manipulation when any two variables are known. Recall that the triangle technique helps you remember how to rearrange these formulas, as illustrated here.

EXAMPLE 3.3A: DETERMINING THE SALE PRICE AND MARKDOWN AMOUNT

The MSRP for the "Guitar Hero: World Tour" video game is $189.99. Most retail stores sell this product at a price in linewith the MSRP. You have just learned that a local electronics retailer is selling the game for 45% off. What is the sale price for the video game and what dollar amount is saved?

Plan

There are two unknown variables. The first is the video game's sale price (S_{onsale}). The second is the markdown amount (D$) that is realized at that sale price.

What You Already Know

Step 1: The regular selling price for the video game and the markdown rate are known:S = $189.99d = 0.45

How You Will Get There

Step 2: Calculate the sale price by applying Formula 6.10.Step 2 (continued):
Calculate the markdown amount by applying Formula 6.11b.

Perform

Step 2: S_{onsale} = $189.99 × (1 − 0.45)= $189.99 × 0.55 = $104.49
Step 2 (continued): D$ = $189.99 − $104.49 = $85.50

Present

The sale price for the video game is $104.49. When purchased on sale, "Guitar Hero: World Tour" is $85.50 off of its regular price.

EXAMPLE 3.3B: MARKDOWN REQUIRING SELLING PRICE CALCULATION

A reseller acquires an Apple iPad for $650. Expenses are planned at 20% of the cost, and profits are set at 15% of the cost. During a special promotion, the iPad is advertised at $100 off. What is the sale price and markdown percent?

Plan

The unknown variables for the iPad are the sale price (Sonsale) and the markdown rate (d).

What You Already Know

Step 1: The pricing elements of the iPad along with the markdown dollars are known:C = $650E = 0.2CP = 0.15CD$ = $100

How You Will Get There

Step 1 (continued): Calculate the selling price of the product by applying Formula 6.5.
Step 2: Calculate the markdown percent by applying Formula 6.12:
Step 2 (continued): Calculate the sale price by applying Formula 6.11b,rearranging for S_{onsale}.

Perform

Step 1 (continued): S = $650 + 0.2($650) + 0.15($650) = $877.50Step 2: d = $100 × 100 = 11.396%$877.50

Step 2 (continued): $100 = $877.50 − S_{onsale}

S_{onsale} = $777.50

Present

When the iPad is advertised at $100 off, it receives an 11.396% markdown and it will retail at a sale price of $777.50.

Never-Ending Sales

Have you noticed that some companies always seem to have the same item on sale all of the time? This is a common marketing practice. Recall the third and fourth circumstances for markdowns. Everybody loves a sale, so markdowns increase sales volumes for both the marked-down product and other regularly priced items.For example, Michaels has a product line called the Lemax Village Collection, which has seasonal display villages for Christmas, Halloween, and other occasions. When these seasonal product lines come out, Michaels initially prices them at the regular unit selling price for a short period and then reduces their price. For Michaels, this markdown serves a strategic purpose. The company's weekly flyers advertising the Lemax Village Collection sale attract consumers who usually leave the store with other regularly priced items.

The Formula

If an item is on sale all the time, then businesses plan the pricing components with the sale price in mind. Companies using this technique determine the unit profitability of the product at the sale price and not the regular selling price. They adapt Formula 6.5 as follows:

$$S = C + E + P$$
$$S_{onsale} = C + E + P_{onsale}$$

where Ponsale represents the planned profit amount when the product is sold at the sale price. This is not a new formula, just a new application of Formula 6.5.

How It Works

Under normal circumstances, when businesses set their selling and sale prices they follow a three-step procedure:

- Determine the product's cost, expenses, and profit amount.
- Set the regular selling price of the product.
- If a markdown is to be applied, determine an appropriate markdown rate or amount and set the sale price.

However, when a product is planned to always be on sale, businesses follow these steps instead to set the sale price and selling price:

Step 1: Set the planned markdown rate or markdown dollars. Determine the pricing components such as cost and expenses. Set the profit so that when the product is marked down, the profit amount is achieved. Alternatively, a planned markup on cost, markup on selling price, or even markup dollars may be set for the sale price.

Step 2: Calculate the sale price of the product. If cost, expenses, and profit are known, apply the adapted version of Formula 6.5. Alternatively, adapt and apply any of the other markup formulas (Formulas 6.6 through 6.9) with the understanding that the result is the sale price of the product and not the regular selling price.

Step 3: Using the known markdown rate or markdown amount, set the regular selling price by applying any appropriate markdown formula (Formulas 6.10 through 6.12).

Example

  Assume for the Michael's Lemax Village Collection that most of the time these products are on sale for 40% off. A particular village item costs $29.99, expenses are $10.00, and a planned profit of $8.00 is achieved at the sale price. Calculate the sale price and the selling price.

  Step 1: The known variables at the sale price are C = $29.99, E = $10.00, P = $8.00, and d = 0.40.

  Step 2: Adapting Formula 6.5, the sale price is S_{onsale} = C + E + P_{onsale} = $29.99 + $10.00 + $8.00 = $47.99. This is the price at which Michael's plans to sell the product.

  Step 3: However, to be on sale there must be a regular selling price. Therefore, if the 40% off results in a price of $47.99, apply Formula 6.10 and rearrange to get the selling price: S = $47.99 ÷ (1 − 0.40) = $79.98. Therefore, the product's selling price is $79.98, which, always advertised at 40% off, results in a sale price of $47.99. At this sale price, Michael's earns the planned $8.00 profit.

Important Notes

You may ask, "If the product is always on sale, what is the importance of establishing the regular price?" While this textbook does not seek to explain the law in depth, it is worth mentioning that pricing decisions in Canada are regulated by the Competition Act. With respect to the discussion of never-ending sales, the Act does require that the product be sold at a regular selling price for a reasonable period of time or in reasonable quantity before it can be advertised as a sale price. If you revisit the Michael's example, note in the discussion that the village initially needs to be listed at the regular selling price before being lowered to the sale price.

Give It Some Thought

- If a product has a markup on cost of 40% and a markdown of 40%, will it sell above or below cost?
- What happens to the profit if a product that is always on sale actually sells at the regular selling price?
- Under normal circumstances, arrange from smallest to largest: regular selling price, cost, and sale price.

Solutions

1. Below cost, since the 40% markdown is off of the selling price, which is a larger value.
2. The profit will be increased by the markdown amount.
3. Cost, sale price, regular selling price.

EXAMPLE 3.3C: SETTING THE PRICE IN A NEVER-ENDING SALE

An electronics retailer has 16GB USB sticks on sale at 50% off. It initially priced these USB sticks for a short period of time at regular price, but it planned at the outset to sell them at the sale price. The company plans on earning a profit of 20% of the cost when the product is on sale. The unit cost of the USB stick is $22.21, and expenses are 15% of the cost.

1. At what price will the retailer sell the USB stick when it is on sale?

2. To place the USB stick on sale, it must have a regular selling price. Calculate this price.

3. If the USB stick is purchased at the regular selling price during the initial time period, how much profit is earned?

Plan

1. This company plans on always having the product on sale, so the pricing needs to be set for the sale price, or S_{onsale}.

2. You need the regular selling price, or S.

3. Solve for the profit at the regular selling price, or P.

What You Already Know

Step 1: You know the unit cost, the retailer's associated expenses, its planned profit at the sale price, and the markdown rate:

- $d = 0.50$

- $C = \$22.21$

- $E = 0.15C$

- $P_{onsale} = 0.2C$

How You Will Get There

Step 2: To solve part (a), apply the adapted version of Formula 6.5: $S_{onsale} = C + E + P_{onsale}$
Step 3: After you know the sale price, solve part (b) by applying Formula 6.10, rearranging for S.
Step 4: Solving part (c) requires applying Formula 6.5.

Perform

Step 2:

- $S_{onsale} = \$22.21 + 0.15(\$22.21) + 0.2(\$22.21)$
- $= \$22.21 + \$3.33 + \$4.44 = \$29.98$

Step 3:

- $\$29.98 = S \times (1 - 0.5)$
- $\$29.98 = S \times 0.5$
- $\$59.96 = S$

Step 4:

- $\$59.96 = \$22.21 + \$3.33 + P$
- $\$59.96 = \$25.54 + P$
- $\$34.42 = P$

Present

The USB stick is on sale for $29.98, letting the company achieve its profit of $4.44 per unit. During the initial pricing period, the USB stick sells for $59.96 (its regular selling price). If a consumer actually purchases a USB stick during the initial pricing period, the electronics store earns a profit of $34.42 per unit (which is a total of the $4.44 planned profit plus the planned markdown amount of $29.96).

EXERCISES

Round all money to two decimals and percentages to four decimals for each of the following exercises.

Mechanics

For questions 1–6, solve for the unknown variables (identified with a ?) based on the information provided.

Question	Regular Selling Price	Markdown Amount	Markdown Percent	Sale Price
1.	$439.85	?	35%	?
2.	?	$100.00	?	$199.95
3.	$1,050.00	?	?	$775.00
4.	$28,775.00	$3,250.00	?	?
5.	?	?	33%	$13,199.95
6.	?	$38.33	12%	?

Applications

7. A pair of Nike athletic shoes is listed at a regular selling price of $89.99. If the shoes go on sale for 40% off, what is the sale price?

8. During its special Bay Days, The Bay advertises a Timex watch for $39.99 with a regular price of $84.99. Calculate the markdown percent and markdown amount.

9. For spring break you are thinking about heading to Tulum, Mexico. In planning ahead, you notice that a one-week stay at the Gran Bahia Principe Tulum, regularly priced at $2,349 for air and six nights all inclusive, offers an early-bird booking discount of $350. What markdown percentage is being offered for booking early?

10. A Heritage Infusio deep frying pan is advertised at 70% off with a sale price of $39.99. What is the frying pan's regular selling price, and what markdown amount does this represent?

11. A mass merchandiser uses its Lagostina cookware product line as a marketing tool. The cookware is always on sale at an advertised price of 45% off. The cost of the cookware is $199.99, expenses are $75, and the planned profit at the sale price is $110. Calculate the sale price and selling price for the cookware.

12. Quicky Mart regularly sells its Red Bull sports drink for $2.99 per can. Quicky Mart noticed that one of its competitors down the street sells Red Bull for $1.89. What markdown percentage must Quicky Mart advertise if it wants to match its competitor.

13. A hardware store always advertises a Masterdesigner 75-piece screwdriver set at 80% off for a sale price of $17.99.

    a. If the cost of the set is $10 and expenses are 30% of the sale price, what is the planned profit when the product is on sale?

    b. What profit is earned if the product actually sells at its regular selling price?

14. A campus food outlet is advertising a "Buy one, get one 25% off" deal. The 25% off comes off the lower-priced item. If you purchase a chicken dinner for $8.99 and your friend gets the burger combo for $6.99, what is the markdown percentage on the total price?

15. Blast'em Stereos purchases a stereo system for $1,900 less two discounts of 40% and 18%. The store uses this product to draw customers to the store and always offers the stereo on sale at 25% off. When the stereo is on sale, it plans on expenses equalling 30% of the cost and a profit of 20% of the sale price.

    a. What is the sale price for the stereo?

    b. How much profit does Blast'em make when the stereo sells at the sale price?

    c. By law, this stereo must sell at the regular selling price for a period of time before going on sale. What is the regular selling price?

    d. What profit does Blast'em earn if a customer purchases the stereo during this initial period?

Challenge, Critical Thinking, & Other Applications

16. Frigid Boards purchases one of its snowboards for $395 less a retail trade discount of 15% and a loyalty discount of 4%. Its markup on selling price percentage on all snowboards is 21%. At the end of the season, any leftover snowboards are marked down by 10%. What is the sale price for the snowboard?

17. An HP LaserJet printer has an MSRP of $399.95. It is subject to trade discounts of 30% and 23%. The LaserJet is a featured item for a computer store and is always on sale. The store plans to sell the LaserJet for a sale price that allows it to cover expenses equalling 15% of cost and realize a profit of $35.00.

    a. What is the sale price?

    b. If the MSRP is the regular unit price of the printer, what rate of markdown can the computer store advertise?

    c. What markup on selling price percentage is realized at the sale price?

18. The Brick advertises that when you purchase a queen-size Tempur-Pedic mattress set for $2,499.97 it will give you a 51" 3-D plasma television with a 3-D starter kit included. The value of this gift is $1,199.99. What markdown percent does this represent?

19. A Maytag 27 cubic foot refrigerator retails for $2,400.00 at Landover Appliance Centre. The company, which is celebrating its 30th anniversary this coming weekend, features the fridge for 30% off. The markup on selling price percentage on the fridge at the regular unit selling price is 53%.

    a. What is the sale price?

    b. At the sale price, what is the markup on selling price percentage?

    c. If the expenses are 15% of the regular selling price, what is the profit when the fridge is on sale?

20. Dreger Jewellers is selling a diamond bracelet. It uses this bracelet in its promotions and almost always has it on sale. The cost of the bracelet is $2,135 less discounts of 20% and 30%. When the bracelet is on sale for 25% off, the expenses are 15% of cost and the profit is 20% of cost.

    a. What is the sale price?

    b. What is the bracelet's regular selling price?

    c. If the bracelet sells at the regular selling price, what are the markup amount and the markup on cost percent?

Credit: Adapted from unit 6.3 Business math: A Step-by-Step Handbook

CHAPTER 4: ACCOUNTING APPLICATIONS

BILLS, BILLS, AND MORE BILLS

The price tag on the product is not necessarily what you pay for it. In 1789, Benjamin Franklin might have said it best when he wrote, "In this world nothing can be said to be certain, except death and taxes."[1] Almost everything is taxed, whether a physical good, a service, or even property.

For instance, while shopping on eBay, you find a product listed in an eBay store located in your home province.

Another product you need is listed by a seller in the United States. You need both products and the prices including shipping costs are reasonable, so for each you click the "Buy It Now" option. The Canadian invoice lists the product's pre-tax price along with two more lines indicating GST and PST amounts, all totalled at the bottom. The US invoice lists the price of the item in US currency. When paying this seller, PayPal converts the price from US currency to Canadian currency using an exchange rate that does not quite match the rate you have seen published in the newspaper, and you wonder why. If you go with this US seller, you know that the Canada Border Services Agency (customs) will charge for GST.

This scenario highlights three of this chapter's key topics: taxes, currency exchange, and invoicing.

- If there is one thing you encounter in all aspects of your life, it is sales tax, which provides revenue to governments in Canada and abroad. In addition, anyone involved in the ownership of real estate must deal with property taxes. Corporate offices, production facilities, and warehouses sit on business-owned real estate, which is charged property taxes. Residential homeowners pay annual taxes to their municipalities. Even if you just rent property like an apartment, the apartment building owner is charged property taxes, which are passed on to tenants within the rental charges.
- Canadian businesses and consumers operate in a global economy in which they commonly complete international transactions. Any payment will incorporate an exchange rate ensuring that the amount paid in Canadian currency is equivalent to the amount charged in the foreign currency.
- Many transactions are completed through an invoicing procedure. Along with a list of the items purchased and their respective prices, invoices show terms of payment, consequences of failing to make the payment, and additional charges such as taxes and shipping.

There is no avoiding sales taxes, property taxes, currency exchange, and invoicing. As a consumer, you generally need to pay these costs. As a business, you not only pay these costs but you need to charge your customers

1. 1 Benjamin Franklin. Letter to Jean-Baptiste Leroy dated November 13, 1789.

appropriately and collect taxes on behalf of the government. This chapter explores the mathematics behind these concepts.

4.1 Sales Taxes

EVERYBODY WANTS A PIECE OF MY PIE

On your recent cross-Canada road trip, you purchased from many different Tim Hortons' stores. At each store, your products retailed for $6.99. When you review your credit card receipts after returning home from your trip, you notice that you paid different totals everywhere. In Alberta, they only added GST and your combo cost $7.34. In British Columbia, they added both PST and GST, resulting in total cost of $7.83. In Ontario, they added something called HST, resulting in a total cost of $7.90. You find it interesting that the same combo came to different totals as you travelled across Canada.

THREE SALES TAXES

A sales tax is a percent fee levied by a government on the supply of products. In Canada, there are three types of sales taxes: the goods and services tax (GST), provincial sales tax (PST), and the harmonized sales tax (HST). In this section you will learn the characteristics of each of these taxes and then the mathematics for calculating any sales tax.

Goods & Services Tax (GST)

The goods and services tax, better known as GST, is a national federal tax of 5% that applies to the purchase of most goods and services in Canada. Every province and territory has GST. The consumer ultimately bears the burden of this sales tax.

Businesses must collect GST on most of their sales and pay GST on most purchases in the daily course of operations. However, when remitting these taxes, businesses claim a credit with the federal government to recover the GST they paid on eligible purchases. The net result is that businesses do not pay the GST on these eligible purchases. While this may outrage some people, the logic is simple. If a business pays the GST, it becomes a cost of the business, which is then passed on to consumers as it is incorporated into retail prices. When the consumer purchases the product, the consumer would be charged the GST again! In essence, a consumer would be double-taxed on all purchases if businesses paid the GST.

Some goods and services are exempt from GST. While there are many complexities and nuances to the exemptions, generally items that are deemed necessities (such as basic groceries), essential services (such as health, legal aid, and childcare), and charitable activities are nontaxable. You can find a complete listing of exemptions on the Canada Revenue Agency website.

Provincial Sales Tax (PST)

Provincial sales taxes, or PST, are provincially administered sales taxes that are determined by each individual provincial or territorial government in Canada. The table here lists the current PST rates in Canada.

Province / Territory	PST
Alberta	0%
British Columbia	7%
Manitoba	8%
Northwest Territories	0%
Nunavut	0%
Quebec	9.975%
Saskatchewan	5%
Yukon Territory	0%

Similar to GST, PST applies to the purchase of most goods and services in the province, and consumers bear the burden. For the same reasons as with GST, businesses typically pay the PST on purchases for non-resale items (such as equipment and machinery) and do not pay the PST on resale items. Businesses are responsible for collecting PST on sales and remitting the tax to the provincial government. Individual provincial websites list the items and services that are exempt from PST.

Harmonized Sales Tax (HST)

The harmonized sales tax, or HST, is a combination of GST and PST into a single number. Since most goods and services are subjected to both taxes anyway, HST offers a simpler method of collecting and remitting the sales tax—a business has to collect and remit only one tax instead of two. Because there are pros and cons to HST, not all provinces use this method of collection, as summarized in the table below.

Pros	Cons
Items that are previously PST payable to a business are now refunded, lowering input costs and lowering consumer prices	Many items such as utilities, services, and children's clothing that are ineligible for PST become taxed at the full HST rate
Results in overall lower corporate taxes paid	Consumer cost of living increases
Increases the competitiveness of businesses and results in job creation	Tax-exempt items see prices rise because HST is being applied to services and goods such as transportation and gasoline
Businesses only remit one tax and not two, resulting in financial and auditing savings	

Province / Territory	HST
Ontario	13%
New Brunswick	13%
Nova Scotia	14%
Newfoundland and Labrador	13%
Prince Edward Island	14%

To understand HST you can separate it into its GST and PST components. For example, in the table to the left Ontario has an HST of 13%. If GST is 5% and the HST is 13%, then it is clear that the province has a PST of 8%. HST operates mostly in the same manner as GST, in that consumers generally bear the burden for paying this tax, and

businesses both collect and pay the HST but have the HST reimbursed on eligible purchases when remitting taxes to the government.

THE FORMULA

With respect to sales taxes, you usually calculate two things:

1. The dollar amount of the sales tax.
2. The price of a product including the sales tax.

Calculating the Sales Tax Amount

A sales tax is a percent rate calculated on the base selling price of the product. Therefore, if you are interested solely in the amount of the sales tax (the portion owing), apply Formula 2.2 on Rate, Portion, base:

$$Rate = \frac{Portion}{Base} becomes Tax Rate = \frac{Tax Amount}{Price before Taxes}$$

Rearranging this formula to solve for the tax amount gives the following:

Tax Amount = Tax Rate × Price before Taxes (which is the same as Portion = Rate × Base)

This is not a new formula. It applies the existing formula from Chapter 2.

Calculating a Price Including Tax

When calculating a selling price including the tax, you take the regular selling price and increase it by the sales tax percentage.

$$\triangle\% = \frac{New - Old}{Old} \times 100$$

The Old price is the price before taxes, the percent change is the sales tax percentage, and you need to calculate the New price including the tax. Rearranging the formula for New, you have

$$New = Old + Old(\frac{\triangle\%}{100})$$

which you factor and rewrite as

$$New = Old \times (1\frac{\triangle\%}{100})$$

or

$$New = Old + (Old \times \frac{\triangle\%}{100})$$

This is a widespread application of the percent change formula.

S_{tax} is Selling Price Including Tax: A subscript is added to the regular symbol of S for selling price to denote that this is the price that sums the selling price and sales tax amount together.

S is Selling Price Without Tax: The regular selling price before taxes.

Formula 7.1 - Selling Price Including Tax: $\mathbf{S_{tax} = S + (S \times Rate)}$

Rate is Tax Rate: This is the sales tax in any form of GST, HST, or PST expressed in its decimal format. Round the result of (S × Rate) to two decimals. If two taxes are involved in the tax-inclusive price (such as GST and PST), you cannot combine the rates together into a single rate. For example, Manitoba has 5% GST and 8% PST. This is not necessarily equivalent to 13% tax since each tax is rounded to two decimals separately and then summed. If you use a single rate of 13%, you may miscalculate by a penny. Instead, expand the formula to include two separate (S × Rate) calculations:

$$S_{tax} = S + (S \times GST\ Rate) + (S \times PST\ Rate)$$

Ensure that you round off each calculation in brackets to two decimals before adding.

How It Works

Follow these steps to perform calculations involving sales taxes:

Step 1: Identify the pricing information. In particular, pay careful attention to distinguish whether the price is before taxes (S)or inclusive of taxes (S_{tax}). Also identify all applicable sales taxes, including GST, PST, and HST.

Step 2: Apply Formula 7.1 to solve for the unknown variable.

Step 3: If you need to find sales tax amounts, apply Formula 2.2 and rearrange for portion. Ensure that for the base you use the price before taxes.

Assume a $549.99 product is sold in British Columbia. Calculate the amount of the sales taxes and the price including the sales taxes.

Step 1: The price before taxes is S = $549.99. In British Columbia, GST is 5% and PST is 7% (from the PST Table).

Step 2: To calculate the price including the sales taxes, apply Formula 7.1:

$$S_{tax} = \$549.99 + (\$549.99 \times 5\%) + (\$549.99 \times 7\%) = \$615.99$$

Step 3: Applying the rearranged Formula 2.2 for the GST, you calculate:

$$\text{GST Tax Amount} = 5\% \times \$549.99 = \$27.50$$

Applying the same formula for the PST, you calculate:

$$\text{PST Tax Amount} = 7\% \times \$549.99 = \$38.50.$$

> Note: You may notice that you could just pull these amounts from the interim calculations in step 2.

Therefore, on a $549.99 item in British Columbia, $27.50 in GST and $38.50 in PST are owing, resulting in a price including sales taxes of $615.99.

Important Notes

To calculate a price including a single sales tax rate, use the percent change function (Δ%) on the calculator. You can review the full instructions for this function at the end of Chapter 3. When using this function, OLD is the price before taxes, NEW is the price after taxes, and %CH is the single tax rate in its percentage format. You do not use the #PD variable, which therefore defaults to 1.

Note that if more than one tax rate applies on the same base, sum the tax rates together and enter as the percent change (or use as Rate in the formula). However, note that due to the penny rounding on the individual taxes, an error margin of a single penny may occur when trying to make S plus the taxes equal to S_{tax}. This may be an unavoidable error and it would remain impossible to exactly equal the price including the sales tax.

Paths To Success

You will often need to manipulate Formula 7.1. Most of the time, prices are advertised without taxes and you need

to calculate the price including the taxes. However, sometimes prices are advertised including the taxes and you must calculate the original price of the product before taxes. When only one tax is involved, this poses no problem, but when two taxes are involved (GST and PST), combine the taxes into a single amount before you solve for S.

Give It Some Thought

On any given product selling for the same price, put the following provinces in order from highest price to lowest price including taxes (GST and PST, or HST): Alberta, Saskatchewan, British Columbia, Ontario, Prince Edward Island.

Solutions

PEI (15% HST), Ontario (13% HST), British Columbia (5% GST + 7% PST), Saskatchewan (5% GST + 6% PST), Alberta (5% GST, no PST)

EXAMPLE 7.1 A: CALCULATING SALES TAX ACROSS CANADA

Dell Canada lists a complete computer system on its Canadian website for $1,999.99. Calculate the price including taxes if the Canadian buyer is located in: a. Alberta c. Quebec b. Ontario d. British Columbia (BC).

Plan

Four answers are required. For each of the provinces listed, calculate the appropriate GST/PST or HST to add onto the price and arrive at the selling price including taxes (S_{tax}).

What You Already Know

Step 1: The price of the computer and tax rates are known:

- S = $1,999.99
- Alberta sales tax = 5% GST
- Ontario sales tax = 13% HST
- Quebec sales tax = 5% GST & 9.975% PST
- BC sales tax = 5% GST & 7% PST

How Will You Get There

Step 2: For all provinces, apply Formula 7.1.

      a. For Alberta, the Rate is the 5% GST. Substitute the S and Rate to solve for S_{tax}.

      b. For Ontario, the Rate is the 13% HST. Substitute the S and Rate to solve for S_{tax}.

      c. For Quebec, both the 5% GST and 9.975% PST are based on the S. Expand the formula for each tax. Substitute these Rates with the S to arrive at S_{tax}.
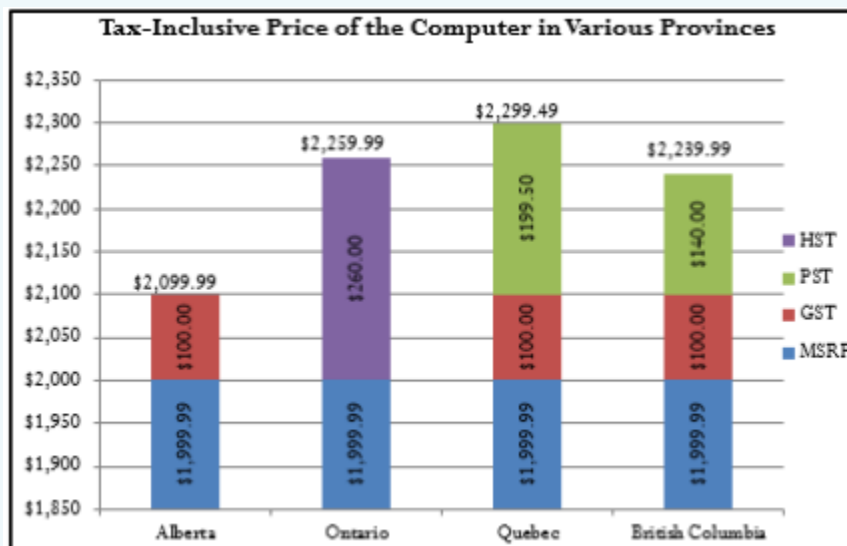
      d. d. For British Columbia, both the 5% GST and 7% PST are based on the S. Expand the formula for each tax. Substitute these Rates with the S to arrive at S_{tax}.

Perform

Step 2:

a. Alberta: S_{tax} = \$1,999.99 + (\$1,999.99 × 5%) = \$2,099.99

  b. Ontario: S_{tax} = \$1,999.99 + (\$1,999.99 × 13%) = \$2,259.99

  c. Quebec: S_{tax} = \$1,999.99 + (\$1,999.99 × 5%) + (\$1,999.99 × 9.975%) = \$2,299.49

  d. d. British Columbia: S_{tax} = \$1,999.99 + (\$1,999.99 × 5%) + (\$1,999.99 × 7%) =\$2,239.99

Present

Depending on which province you reside in, the price for the computer from lowest to highest is Alberta at \$2,099.99, British Columbia at \$2,239.99, Ontario at \$2,259.99, and Quebec at \$2,299.49.

Tax-Inclusive Price of the Computer in Various Provinces

EXAMPLE 7.1 B CALCULATING TAXES ON A TAX-INCLUSIVE PRICE

"The Brick is having its Midnight Madness sale! Pay no taxes on products purchased during this event!" While this is good marketing, it probably goes without saying that governments do not give up the sales taxes. Essentially The Brick is advertising a tax-inclusive price. Calculate GST and PST amounts for a product advertised at $729.95, including GST and PST, in Saskatchewan.

Plan

To calculate the PST and GST for Saskatchewan, you must calculate the price before taxes (S). Then calculate each tax amount based on the S.

What You Already Know

Step 1: The price after taxes and the tax rates are as follows:

- S_{tax} = $729.95
- Tax rates = 5% GST and 6% PST

How Will You Get There

Step 2: Apply Formula 7.1 using the combined PST and GST as the Rate to calculate the S.
Step 3: Apply Formula 2.2 rearranged for Portion to calculate the tax amounts.

Perform

Step 2: Remove the taxes:

- $729.95 = S + (S × 11%)
- $729.95 = 1.11S
- S = $657.61

Step 3:

- GST: 5% = GST Portion ÷ $657.61;
- GST Portion = $32.88 PST: 6% = PST Portion ÷ $657.61;
- PST Portion = $39.46

The Structure Of A Tax-Inclusive Price

$729.95
Including
Taxes

■ The regular selling price
(before taxes) --> $657.61

■ The 5% GST --> $32.88

■ The 6% PST --> $39.46

When paying a tax-inclusive price of $729.95 in Saskatchewan, the regular selling price of the item is $657.61 with $ $32.88 GST and $39.46 PST included.

THE GST/HST REMITTANCE

When a business collects sales taxes, it is a go-between in the transaction. These sales tax monies do not belong to the business. On a regular basis, the business must forward this money to the government. This payment is known as a tax remittance.

The Formula

Generally speaking, a business does not pay sales taxes. As a result, the government permits a business to take all eligible sales taxes that it paid through its acquisitions and net them against all sales taxes collected from sales. The end result is that the business is reimbursed for any eligible out-of -pocket sales tax that it paid. Formula 7.2 expresses this relationship.

Formula 7.2

Remit is GST/HST Remittance: This is the dollar amount of the remittance.
- If this amount is positive, it means that the business collected more tax than it paid out; the company must remit this balance to the government.
- If this amount is negative, it means that the business paid out more taxes than it collected; the government must refund this balance to the company.

Formula 7.2 – GST/HST Remittance: $$\text{Remit} = \text{Tax Collected} - \text{Tax Paid}$$

Tax Collected and Tax Paid: Both of these parts of the formula apply Formula 2.2, representing the total amount of sales tax (portion) from all taxable amounts (base) at the appropriate sales tax rate (rate). The taxes collected are based on total tax-eligible revenues. The taxes paid are based on the total tax-eligible acquisitions.

How It Works

Follow these steps to complete a GST/HST remittance:

Step 1: Identify the total amounts of tax-eligible revenues and acquisitions upon which the sales tax is collected or paid, respectively. Identify the applicable sales tax rate of the GST or HST.

Step 2: Calculate the total taxes collected by applying Formula 2.2, where the sales tax is the rate and the total revenue is the base. Solve for portion.

Step 3: Calculate the total taxes paid by applying Formula 2.2, where the sales tax is the rate and the total acquisitions are the base. Solve for portion.

Step 4: Apply Formula 7.2 to calculate the tax remittance.

Example

Assume a business has paid GST on purchases of $153,000. It has also collected GST on sales of $358,440. Calculate the GST remittance.

Step 1: Identifying the variables, you have:

$$\text{Total Revenue} = \$358,440$$
$$\text{Total Acquisitions} = \$153,000$$
$$\text{GST Tax Rate} = 5\%$$

Step 2: Calculate taxes collected by applying Formula 2.2, where:

$$\text{GST collected} = 5\% \times \$358,440 = \$17,922.$$

Step 3: Calculate taxes paid by applying Formula 2.2, where:

$$\text{GST paid} = 5\% \times \$153,000 = \$7,650.$$

Step 4: To calculate the remittance, apply Formula 7.2 and calculate Remit = $17,922 − $7,650 = $10,272. The business should remit a cheque for $10,272 to the government.

Paths To Success

A shortcut can help you calculate the GST/HST. Remittance using Formula 7.2. If you do not need to know the actual amounts of the tax paid and collected, you can net GST/HST–eligible revenues minus acquisitions and multiply the difference by the tax rate:

$$\text{Remit} = (\text{Revenues} - \text{Acquisitions}) \times \text{Rate}$$

In the example above, Remit = ($358,440 – $153,000) x 5% = $10,272. If this calculation produces a negative number, then the business receives a refund instead of making a remittance.

EXAMPLE 7.1 C CALCULATING A GST/HST REMITTANCE

An Albertan lumber company reported the following quarterly purchases and sales in its 2013 operating year:

Assuming all purchases and sales are eligible and subject to GST, calculate the GST remittance or refund for each quarter.

Plan

Calculate the GST tax remittance, or Tax Remit, for each of the quarters.

What You Already Know

Step 1: From the information provided, the total purchases and sales for each quarter along with the GST = 5%, which is the Tax Rate, are known.

How Will You Get There

Step 2: For each quarter, calculate the GST collected by rearranging and applying Formula 2.2.
Step 3: Calculate the tax paid.
Step 4: Apply Formula 7.2 for each quarter

Perform

	Step 2		Step 3		Step 4
Quar ter	Sales	GST Collected (Sales x 5%)	Purchases	GST Paid (Purchases x 5%)	Remittance (Tax Collected – Tax Paid)
Wint er	$911,500	$45,575	$1,316,000	$65,800	$45,575 – $65,800 = – $20,225
Sprin g	$1,475,300	$73,650	$1,266,200	$63,310	$73,650 – $63,310 = – $10,340
Sum mer	$1,381,700	$69,085	$1,053,700	$52,685	$69,085 – $52,685 = – $16,400
Fall	$1,124,600	$56,230	$878,100	$43,905	$56,230 – $43,905 = – $12,325

Present

In the Winter quarter, the company gets a GST refund of $20,225. In the other three quarters, the company remits to the government payments of $10,340, $16,400, and $12,325, respectively.

EXERCISES

Mechanics

1. You are purchasing a new BlackBerry at the MSRP of $649.99. Calculate the price including taxes in the following provinces or territories:
   a)Northwest Territories
   b)New Brunswick
   c)Nova Scotia
   d)British Columbia

2. The Brick is advertising a new Serta mattress nationally for a price of $899.99 including taxes. What is the price before taxes and the sales tax amounts in each of the following provinces?
   a)Ontario
   b)Saskatchewan

3. Audiophonic Electronics is calculating its HST remittance in Prince Edward Island. For each of the following months, calculate the HST remittance or refund on these HST-eligible amounts.

Month	Purchases	Sales
January	$48,693	$94,288
February	$71,997	$53,639

4. Airwaves Mobility is calculating its GST remittance in Alberta. For each of the following quarters, calculate the GST remittance or refund on these GST-eligible amounts.

Quarter	Purchases	Sales
Winter	$123,698	$267,122
Spring	$179,410	$158,905
Summer	$216,045	$412,111
Fall	$198,836	$175,003

Applications

1. Elena lives in Nova Scotia and has relatives in Alberta, Saskatchewan, and Quebec. She gets together with them often. She wants to purchase a new aerobic trainer and would like to pay the lowest price. If a family member buys the item, Elena can pick it up at one of their regular family gatherings. The price of the trainer for each province is listed below:

New Aerobic Trainer Prices per Province	
Province	**Selling Price**
Nova Scotia	$1,229.50
Alberta	$1,329.95
Saskatchewan	$1,274.25
Quebec	$1,219.75

  a. Where should Elena have the aerobic trainer purchased and how much would she pay?

  b. How much money would she save from her most expensive option?

2. Mary Lou just purchased a new digital camera in Nunavut for $556.49 including taxes. What was the price of the camera before taxes? What amount of sales tax is paid?

3. Marley is at Peoples Jewellers in New Brunswick wanting to purchase an engagement ring for his girlfriend. The price of the ring is $2,699.95. If the credit limit on his credit card is $3,000, will he be able to purchase the ring on his credit card? If not, what is the minimum amount of cash that he must put down to use his credit card?

4. In the IKEA store in Vancouver, British Columbia, you are considering the purchase of a set of kitchen cabinets priced at $3,997.59. Calculate the amount of GST and PST you must pay for the cabinets, along with the total price including taxes.

5. A company in Saskatchewan recorded the following GST-eligible purchases and sales throughout the year. Determine the GST remittance or refund per quarter.

Quarter	Purchases	Sales
1st	$2,164,700	$2,522,000
2nd	$1,571,300	$2,278,700
3rd	$1,816,100	$1,654,000
4th	$2,395,900	$1,911,700

6. A manufacturer in Nova Scotia recorded the following HST-eligible purchases and sales in its first three months of its fiscal year. Determine the HST remittance or refund per month.

Month	Purchases	Sales
March	$20,209	$26,550
April	$28,861	$20,480
May	$22,649	$42,340

CHALLENGE, CRITICAL THINKING, & OTHER APPLICATIONS

1. 15. If the selling price of an item is 6% higher in Yukon than in Ontario, will the price including taxes be higher in Yukon or Ontario? What percentage more?

2. Colin just travelled across the country on a road trip. He bought some skis in Alberta for $879.95 plus tax, a boombox in British Columbia for $145.58 including taxes, a Niagara Falls souvenir in Ontario for $99.97 plus tax, and some maple syrup in Quebec for $45.14 including tax. Overall, how much GST, PST, and HST did Colin pay on his trip?

3. 17. Cisco Enterprises in Ontario purchased the following in a single month:
   • 16,000 units of network routers at $79.25 each, priced at $97.97 each
   • 12,000 units of wireless LAN adapters at $129.95 each, priced at $189.55 each
   • 13,500 units of computer boards at $229.15 each, priced at $369.50 each.
   Assuming that all units purchased are sold during the same month and that all purchases and sales are taxable, calculate the tax remittance or refund for the month.

4. In Quebec, the PST used to be calculated on the price including GST. When the PST was calculated in this manner, what PST rate did Quebec set to arrive at the same price including taxes?

5. For each of the following situations, compute the selling price of the product before taxes in the other province/territory that would result in the same selling price including taxes as the item listed.

	Price Before Tax	Sold In	Find Equivalent Price
a.	$365.75	British Columbia	Prince Edward Island
b.	$1,795.00	Alberta	Manitoba
c.	$19,995.95	Saskatchewan	Ontario
d.	$4,819.35	New Brunswick	Quesbec

6. A company made the following taxable transactions in a single month. Compute the GST remittance on its operations assuming all sales and purchases are eligible for GST

Transaction Type	Unit Type	Quantity
Purchase	$168.70	5,430
Sale	$130.00	4,000
Sale	$148.39	3,600
Purchase	$93.47	2,950
Purchase	$24.23	3,325
Purchase	$121.20	2,770
Sale	$188.88	6,250

4.2 Invoicing: Terms of Payment and Cash Discounts

MAKE SURE YOU BILL THEM

What a way to start your Monday morning! With dread you face a great stack of envelopes in your in-basket. The first envelope contains an invoice from your major supplier indicating an outstanding balance of $3,600 with terms of 2/10, 1/20, net 30. In the second envelope, an office supplies bill totals $500 with terms of 3/15, net 45 EOM. The third envelope holds yet another invoice from your transportation company indicating an outstanding balance of $21,000 with terms of 2/15, 1/25, net 60 ROG. It also indicates that you have an overdue balance of $4,000 subject to a 3% monthly penalty. Before opening any more envelopes that probably contain still more bills, you pour yourself another cup of coffee and settle down to figuring out how much you need to pay and when.

In the world of business, most purchases are not paid for in cash. Instead, businesses tend to work through an invoicing system in which they send out bills to their clients on a regular basis. In accounting, this means that the purchase is placed into accounts receivables until such time as a cheque arrives and the purchase is converted to cash. Invoices provide detailed transaction information, listing the amount owed and also indicating the terms on which payment is expected. Companies may offer what are known as cash discounts as incentives for early invoice payment. The rationale for these discounts is simple—a sale is not a sale until you have the cash in hand. The longer an invoice remains in accounts receivable, the less likely that it will be paid. Thus, it might turn into bad debt, which the creditor cannot collect at all.

Invoicing is less common in consumer purchases because of the sheer volume of transactions involved and the higher risk of nonpayment. Imagine purchasing items at Walmart and receiving an invoice to pay your bill next month instead of paying cash. It is hard to fathom the number of invoices Walmart would have to distribute monthly. How much would it cost to collect those debts? How many of those invoices would go unpaid? That is why consumer purchases typically do not involve invoicing.

Invoicing does commonly occur at a consumer level on credit card transactions along with many services where the business may not be able to assess the exact amount of the bill at the time of the transaction or until the service is delivered.

Two examples illustrate this point:

1. Think of your MasterCard bill. You are able to make purchases, say, from March 9 to April 8. Then on April 9 the company sends out a statement saying you have until April 29 to pay your bill. If you do not, interest and late payment penalties are involved.

2. You have a dental visit for a regular cleaning. Before charging you, the dentist's office needs to determine how much your insurance will cover. It may not find out the answer for a day or two, so it sends you an invoice at a later date once it hears from the insurance company. The invoice terms indicate that payment is due upon receipt and you will incur late penalties if payment is not forthcoming.

This section explores the most common aspects of invoicing, including terms of payments and cash discounts. You will learn to calculate the amount required to pay invoices and how to reduce the outstanding balance of an invoice if a partial payment is received. Additionally, the calculation of late payments and penalties is introduced.

INVOICE TERMS AND INVOICE DATING

You must know how to read a business invoice. The figure on the next page[1] provides a sample invoice and highlights the following areas:

1. **Invoicing Company.** The invoice must identify who is sending and issuing the invoice.

2. **Invoice Date.** The date on which the invoice was printed, along with the invoice tracking number (in this case, the order date and shipping date are equivalent to the invoice date). When an invoice is paid, the cheque must reference the invoice number so that the invoicing company can identify which invoice to credit the payment to.

3. **Transaction Details**. The details of the transaction might include number of units, unit prices, and any discounts for which the item is eligible.

4. **Invoice Total.** The total amount owing is indicated, including any taxes or additional charges.

5. **Terms of Payment.** The terms of payment include any cash discounts and due dates. The date of commencement (as discussed below) is determined from this part of the invoice.

6. **Late Penalty.** A penalty, if any, for late payments is indicated on the invoice. Whether or not a company enforces these late penalties is up to the invoice-issuing organization.

1. Invoice provided by and courtesy of Kerry Mitchell Pharmacy Ltd. And Shoppers Drug Mart.

Three Dates of Commencement

All invoice terms are affected by what is known as the date of commencement, which is the first day from which all due dates stated on the invoice will stem. The date of commencement is determined in one of three ways as illustrated below.
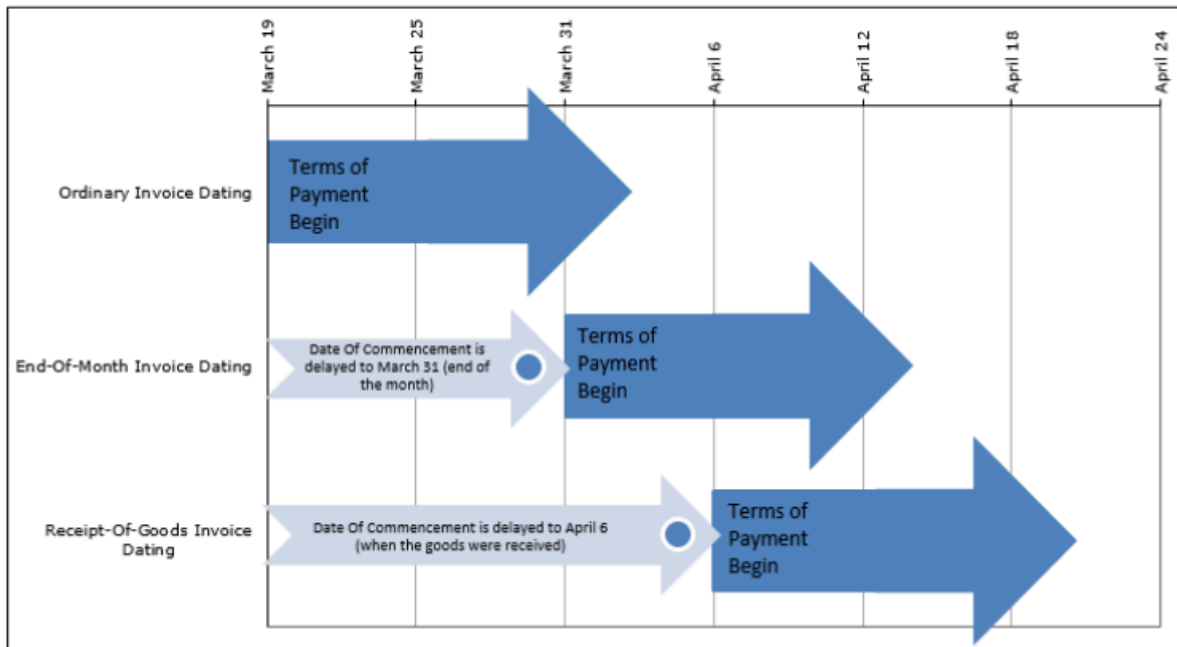
Ordinary Invoice Dating.

In ordinary invoice dating, or just invoice dating for short, the date of commencement is the same date as the invoice date. Therefore, if the invoice is printed on March 19, then all terms of payment commence on March 19. This is the default manner in which most companies issue their invoices, so if an invoice does not specify any other date of commencement you can safely assume it is using ordinary dating.
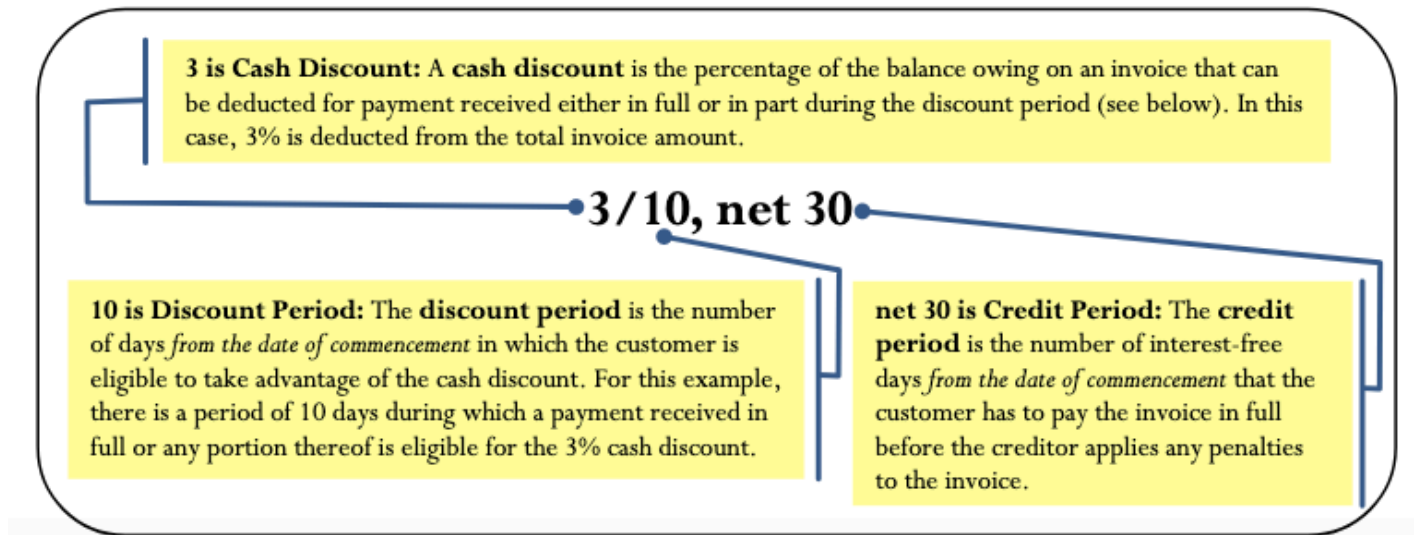
End-of -Month Invoice Dating.

End-of -month invoice dating applies when the terms of payment include the wording "end-of -month" or the abbreviation "EOM" appears after the terms of payment. In end-of -month dating, the date of commencement is the last day of the same month as indicated by the invoice date. Therefore, if the invoice is printed on March 19, then all terms of payment commence on the last day of March, or March 31. Many companies use this method of dating to simplify and standardize all of their due dates, in that if the date of commencement is the same forall invoices, then any terms of payment will also share the same dates.

Receipt-Of-Goods Invoice Dating.

Receipt-of-goods invoice dating applies when the terms of payment include the wording "receipt-of -goods" or the abbreviation "ROG" appears after the terms of payment. In receipt-of -goods dating, the date of commencement is the day on which the customer physically receives the goods. Therefore, if the invoice is printed on March 19 but the goods are not physically received until April 6, then all terms of payment commence on April 6. Companies with long shipping times involved in product distribution or long lead times in production commonly use this method of dating.

TERMS OF PAYMENT

The most common format for the terms of paying an invoice is shown on the next page and illustrated by an example.

3 is Cash Discount: A **cash discount** is the percentage of the balance owing on an invoice that can be deducted for payment received either in full or in part during the discount period (see below). In this case, 3% is deducted from the total invoice amount.

3/10, net 30

10 is Discount Period: The **discount period** is the number of days *from the date of commencement* in which the customer is eligible to take advantage of the cash discount. For this example, there is a period of 10 days during which a payment received in full or any portion thereof is eligible for the 3% cash discount.

net 30 is Credit Period: The **credit period** is the number of interest-free days *from the date of commencement* that the customer has to pay the invoice in full before the creditor applies any penalties to the invoice.

The figure below plots the invoice term of "3/10, net 30" along a time diagram to illustrate the terms of payment. This term is combined with various dates of commencement using the examples from the "Date of Commencement" section.
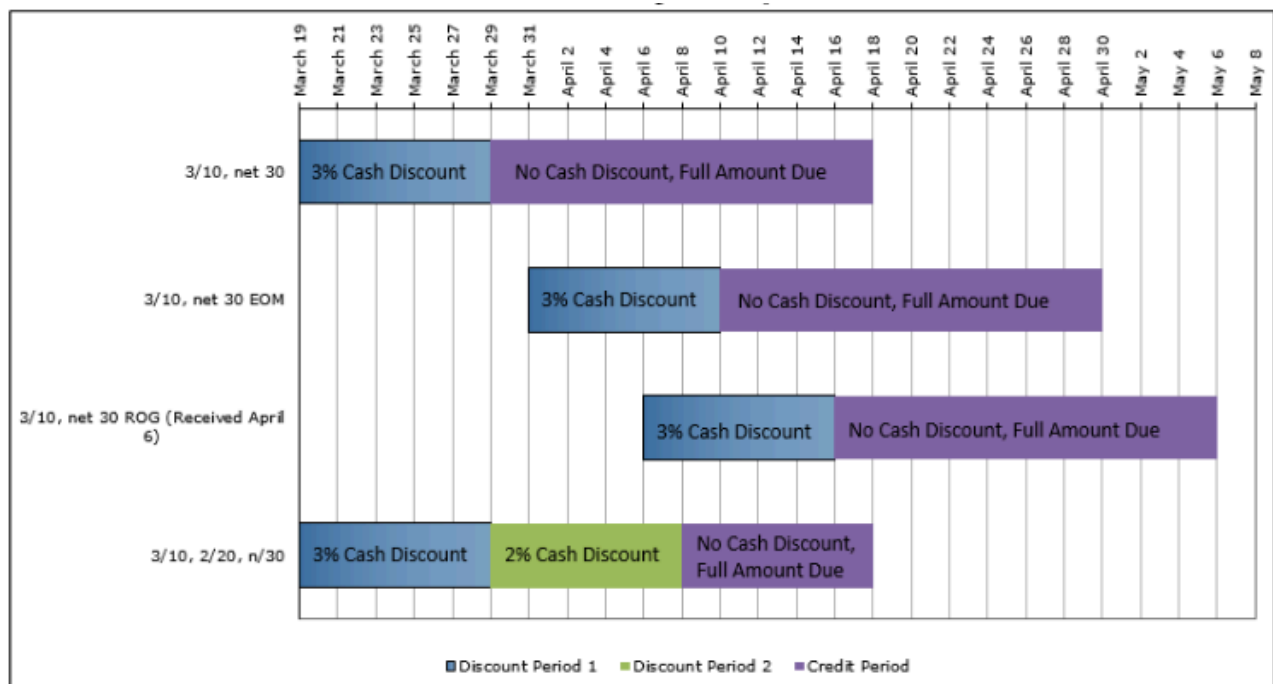
Note the following observations:

- In all four scenarios, the total length of all bars is the same, since the credit period is 30 days.
- Although all four invoices have the same invoice date of March 19, the date of commencement is

modified in the ROG and EOM scenarios, which shift the discount and credit periods into the future. In these cases, any payment before the end of the discount period qualifies for the discount. For example, in the EOM scenario a company could pay its bill early on March 27 and qualify for the 3% cash discount.

You may see invoice terms displayed in several other common formats:

1. 3/10, n/30: In this case, the word "net" has been abbreviated to "n/". This is illustrated in the fourth scenario in the figure.

2. 3/10, 2/20, n/30. In this case, multiple cash discounts are being offered, meaning 3% within 10 days from the date of commencement and 2% from the 11th to the 20th day from the date of commencement. This is also illustrated in the fourth scenario of the figure.

3. n/30. In this case, no cash discount is offered and only the credit period is identified.

RULES FOR INVOICE TERMS

Some common business practices are implemented across most industries. Note that you must always check with the invoicing organization to ensure it is applying these practices.

No Net Figure

If no net figure is stated in the terms of payment, you should assume that the credit period is 20 days after the last discount period. If there are no discount periods, then the credit period is 20 days from the date of commencement. For example, "3/10, 2/15" means that the credit period ends 20 days after the second discount period of 15 days. Hence, the credit period is 35 days after the date of commencement.

No Cash Discount

Not every invoice receives a cash discount. If no terms indicate a cash discount, then the invoicing company seeks full payment only. For example, "n/30" means that no cash discount applies and the credit period ends 30 days from the date of commencement.

Nonbusiness Days

Most businesses operate Monday through Friday and are closed on weekends and holidays. As such, any date falling on a nonworking day is moved to the next business day. For example, if an invoice is ordinary dated December 21 and lists a cash discount of 2/10, the end of the discount period falls on January 1. Since this is New Year's Day, the discount period extends to January 2. In this textbook, you should apply this practice to any of the five known statutory holidays discussed in Chapter 4 (New Year's Day, Good Friday or Easter Monday in Quebec only, Canada Day, Labour Day, and Christmas Day).

THREE TYPES OF PAYMENTS

When businesses pay invoices, three situations can occur:

Full Payment

A full payment means that the company wants to pay its invoice in full and reduce its balance owing to zero dollars. This is the most common practice.

Partial Payment

A partial payment means that the company wants to lower its balance owing but will not reduce that balance to zero. A company will generally employ this method when it wants to either lower its accounts payable or demonstrate good faith in paying its invoices. Partial payment may also occur if the company wants to take advantage of a cash discount but lacks the funds to clear the invoice in its entirety.

Late Payment

A late payment occurs when the company pays its invoice either in full or partially after the credit period has elapsed. Late payments occur for a variety of reasons, but the most common are either insufficient funds to pay the invoice or a simple administrative oversight.

The rest of this section shows you how to handle each of these types of payments mathematically, and it also explores the implications of cash discounts.

GIVE IT SOME THOUGHT

In each of the following cases, determine which term of payment results in the longest credit period extending from the invoice date.

Exercises

- 2/10, n/30 or 3/15
- 3/15, n/45 or 2/10, 1/20
- 2/10, n/60 or 3/10, n/30 EOM on an invoice dated January 7
- 3/15, n/45 or 2/10, n/35 ROG, or 2/20 EOM on an invoice dated September 22 and goods received on September 29
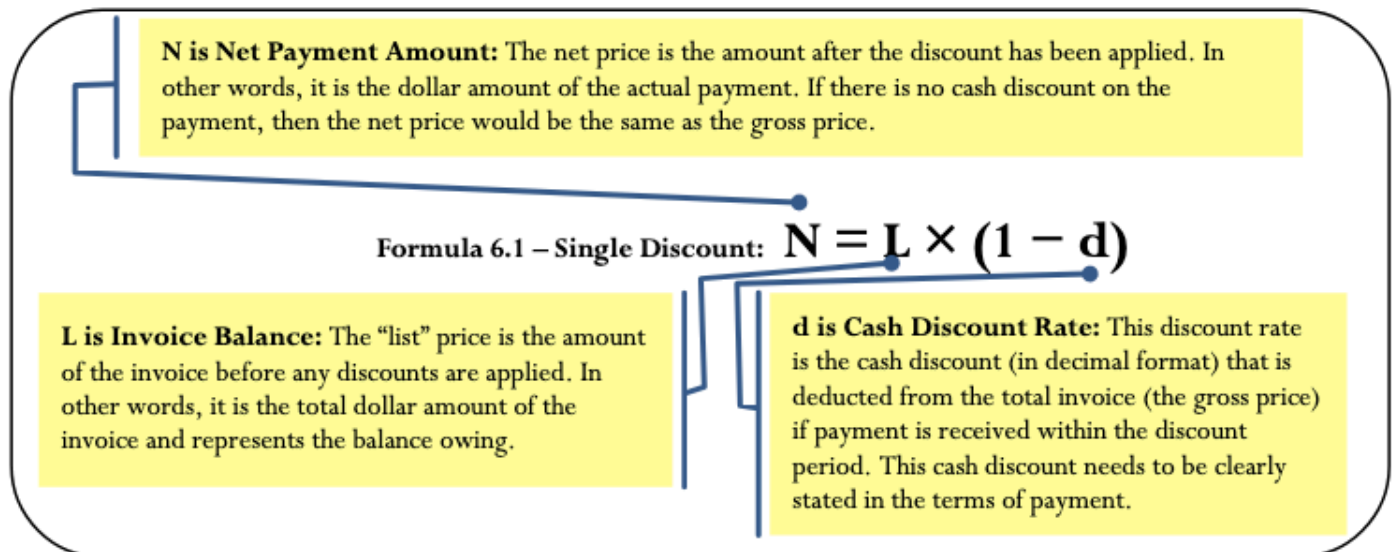

Solutions

1. 30 days or 35 days
2. 45 days or 40 days
3. 60 days or 54 days (24 days left in January plus 30 more)
4. 45 days or 42 days (7 days until received plus 35) or 48 days (8 left in month plus 40 more

FULL PAYMENTS

In the opening scenario in this section, the first invoice on your desk was for $3,600 with terms of 2/10, 1/20, net 30. Suppose that invoice was dated March 19. If you wanted to take advantage of the 2% cash discount, when is the last day that payment could be received, and in what amount would you need to write the cheque? In this section, we will look at how to clear an invoice in its entirety.

The Formula

The good news is that a cash discount is just another type of discount similar to what you encountered in Section 6.1,and you do the calculations using the exact same formula. Apply Formula 6.1 on single discounts, which is reprinted below.

N is Net Payment Amount: The net price is the amount after the discount has been applied. In other words, it is the dollar amount of the actual payment. If there is no cash discount on the payment, then the net price would be the same as the gross price.

Formula 6.1 – Single Discount: $N = L \times (1 - d)$

L is Invoice Balance: The "list" price is the amount of the invoice before any discounts are applied. In other words, it is the total dollar amount of the invoice and represents the balance owing.

d is Cash Discount Rate: This discount rate is the cash discount (in decimal format) that is deducted from the total invoice (the gross price) if payment is received within the discount period. This cash discount needs to be clearly stated in the terms of payment.

How It Works

Follow these steps when working with payments and invoices:

Step 1: Look for and identify key information such as the invoice date, date of commencement modifiers such as EOM or ROG, terms of payment, late penalty, and the invoice amount.

Step 2: Draw a timeline similar to the figure on the next page. On this timeline, chronologically arrange from left to right the invoice date and amount, the date of commencement, the end of any discount (if any) or credit periods, cash discounts (if any) that are being offered, and penalties (if any).

Step 3: Determine when payments are being made and in what amount. Locate them on the timeline.

Step 4: Apply the correct calculations for the payment, depending on whether the payment is a full payment, partial payment, or late payment.

Let's continue with the example of the first invoice on your desk. You mail in a cheque to be received by March 29. What amount is the cheque?

Step 1: The invoice amount is L = $3,600, invoice date is March 19, and terms of payment are 2/10, 1/20, net 30.

Step 2: The figure on the next page displays the invoice timeline.
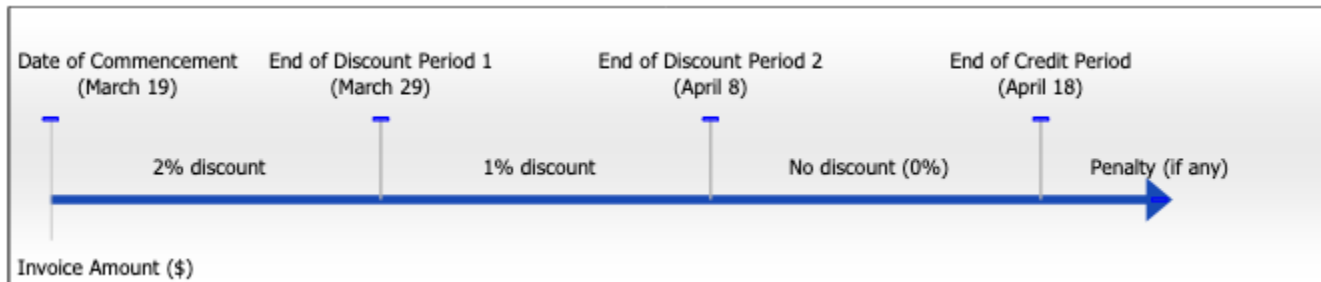
Step 3: Note on the timeline that a payment on March 29 is the last day of the 2% discount period.

Step 4: According to Formula 6.1, the amount to pay is N = $3,600 × (1 − 0.02) = $3,600 × .098 = $3,528. A cheque for $3,528 pays your invoice in full. By taking advantage of the cash discount, you reduce your payment by $72.

Things To Watch Out For

If there is one area of invoicing that causes the most confusion, it is assigning information to the net price and invoice amount variables. These are commonly assigned backwards. Remember these rules so that you always get the correct answer:

1.   The invoice amount or any reference to the balance owing on an invoice is always a list price.

List Price = Invoice Amount

2.   The payment of an invoice, whether in full, partial, or late, is always a net price.

Net Price = Payment Amount

Paths To Success

Who cares about 1% or 2% discounts when paying bills? While these percentages may not sound like a lot, remember that these discounts occur over a very short time frame. For example, assume you just received an invoice for $102.04 with terms of 2/10, n/30. When taking advantage of the 2% cash discount, you must pay the bill 20 days early, resulting in a $100 payment. That is a $2.04 savings over the course of 20 days. To understand the significance of that discount, imagine that you had $100 sitting in a savings account at your bank. Your

savings account must have a balance of $102.04 twenty days later. This requires your savings account to earn an annual interest rate of 44.56%! Therefore, a 20-day 2% discount is the same thing as earning interest at 44.56%. Outrageously good!

Give It Some Thought

In each of the following cases, determine the cash discount or penalty for which the payment qualifies.

Exercises

Question	Invoice Date	Terms of Payment	Payment Received On
1.	April 14	2/10, n /30	April 24
2.	July 7	3/10, 2/20, n/30 EOM, 1% per month penalty	August 12
3.	November 12 (goods received November 28)	2/20 ROG, 2% per month penalty	December 29
4.	February 27 (non-leap year)	4/10, 2/15, 1/25 EOM	March 25

Solutions

1. 2% discount
2. 2% discount
3. no discount (0%) or penalty
4. 1% discount

EXAMPLE 7.4A: PAYING YOUR BILLS IN FULL

You receive an invoice for $35,545.50 dated August 14. The terms of payment are listed as 3/10, 2/20, net 45 EOM. The accounting department is considering paying this debt on one of three days. Determine the full payment required if payment is received by the invoicing company on each of the following dates:

      a.   September 3

      b.   September 19

      c.   September 30

Plan

In each case, calculate the full payment (N) after applying any cash discount.

What You Already Know

Step 1: The invoice amount, terms of payment, and any payments are known:

- L = $35,545.50

- Invoice date = August 14

- Terms of payment = 3/10, 2/20, net 45 EOM

- Three payment date options are September 3, September 19, and September 30.

- Steps 2 & 3: The figure below illustrates the timeline for the invoice and identification of payments.

How You Will Get There

Step 4: Calculate the payment required by applying Formula 6.1. As noted on the timeline:

      a.   The first payment option qualifies for d = 3%.

      b.   The second payment option qualifies for d = 2%.

      c.   The third payment option qualifies for d = 0% (no cash discount).
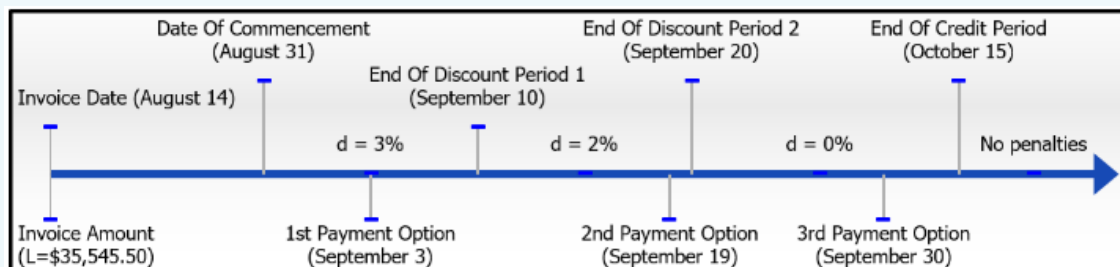
Perform

Step 4:

a. $N = \$35,545.50 \times (1 - 0.03) = \$35,545.50 \times 0.97 = \$34,479.14$

b. $N = \$35,545.50 \times (1 - 0.02) = \$35,545.50 \times 0.98 = \$34,834.59$

c. $N = \$35,545.50 \times (1 - 0) = \$35,545.50 \times 1 = \$35,545.50$
(Note that since there is no discount in part (c), you could have concluded tha N = L)

Present

If the invoicing company receives payment on September 3, a 3% discount is allowed and $34,479.14 clears the invoice. If the payment is received on September 19, a 2% discount is allowed and $34,834.59 pays it off. Finally, if the payment arrives on September 30, there is no discount so the full invoice amount of $35,545.50 is due.

PARTIAL PAYMENTS

In the section opener, your third invoice is for $21,000 with terms of 2/15, 1/25, net 60 ROG. After you paid the $3,528 to clear the first invoice, you realize that your company has insufficient funds to take full advantage of the 2% cash discount being offered by the transportation company. Not wanting to lose out entirely, you decide to submit a partial payment of $10,000 before the first discount period elapses. What is the balance remaining on the invoice?

Unless you pay attention to invoicing concepts, it is easy to get confused. Perhaps you think that $10,000 should be removed from the $21,000 invoice total, thereby leaving a balance owing of $11,000. Or maybe you think the payment should receive the discount of 2%, which would be $200. Are you credited with $9,800 off of your balance? Or maybe $10,200 off of your balance? In all of these scenarios, you would be committing a serious mistake and miscalculating your balance owing. Let's look at the correct way of handling this payment.

The Formula

Recall that invoice amounts are gross amounts (amounts before discounts), or G, and that payment amounts are net amounts (amounts after discounts), or N. Also recall that an algebraic equation requires all terms to be expressed in the same unit.

In the case of invoice payments, you can think of the balance owing as being in the unit of "pre-discount" and any payment being in the unit of "post-discount." Therefore, a payment cannot be directly deducted from the invoice balance since it is in the wrong unit. You must convert the payment from a "post-discount" amount into a "pre-discount" amount using a rearranged version of Formula 6.1. You can then deduct it from the invoice total to calculate any balance remaining.

N is Net Payment Amount: This is the actual payment amount made that must be converted into its value toward the invoice total. Since the payment represents an amount after the discount has been removed, you need to find out what it is worth before the cash discount was removed.

$$\textbf{Formula 6.1 – Single Discount Rearranged: } \mathbf{L} = \frac{\mathbf{N}}{\mathbf{(1-d)}}$$

L is Invoice Amount: This is the amount deducted from the invoice balance as a result of the payment. It is what the payment is worth once the cash discount has been factored into the calculation.

d is Cash Discount Rate: Any payment received within any discount period, whether in full or partial, is eligible for the specified cash discount indicated in the terms of payment.

How It Works

Follow the same invoice payment steps even when working with partial payments. Commonly, once you calculate the gross amount of the payment in step 4 you will also have to calculate the new invoice balance by deducting the gross payment amount. Let's continue working with the $21,000 invoice with terms of 2/15, 1/25, net 60 ROG. Assume the invoice is dated March 19 and the goods are received on April 6. If a $10,000 payment is made on April 21, what balance remains on the invoice?

Step 1: The invoice amount is $21,000, the invoice date is March 19, goods are received on April 6, and the terms of payment are 2/15, 1/25, net 60. A payment of $10,000 is made on April 21.

Step 2: The figure below shows the invoice timeline.
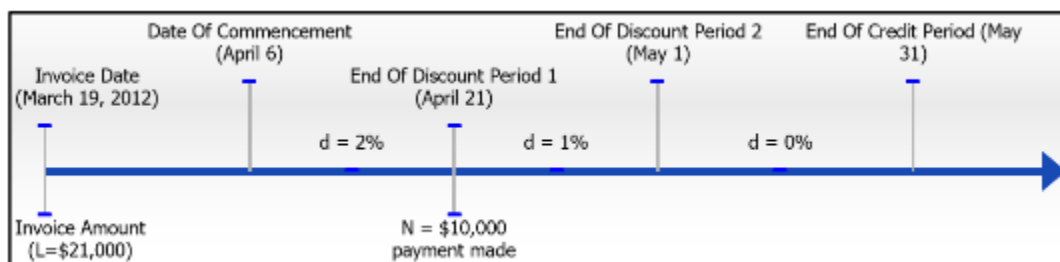
Step 3: The payment of $10,000 on April 21 falls at the end of the first discount period and qualifies for a 2% discount.

Step 4: To credit the invoice, apply the rearranged Formula 6.1:

$$L = \frac{\$10,000}{(100\% - 2\%)} = \frac{\$10,000}{98\%} = \frac{\$10,000}{0.98} =$$

10,204.08

This means that before any cash discounts, the payment is worth $10,204.08 toward the invoice total. The balance remaining is:

IMPORTANT NOTES

In the case where a payment falls within a cash discount period, the amount credited toward an invoice total is always larger than the actual payment amount. If the payment does not fall within any discount period, then the amount credited toward an invoice total is equal to the actual payment amount (since there is no cash discount).

To help you understand why a partial payment works in this manner, assume an invoice is received in the amount of $103.09 and the customer pays the invoice in full during a 3% cash discount period. What amount is paid? The answer is N = $103.09(100% − 3%) = $100. Therefore, any payment of $100 made during a 3% cash discount period is always equivalent to a pre-discount invoice credit of $103.09. If the balance owing is more than $103.09, that does not change the fact that the $100 payment during the discount period is worth $103.09 toward the invoice balance.

Give It Some Thought

In each of the following situations, determine for the partial payment whether you would credit the invoice for an amount that is larger than, equal to, or less than the partial payment amount.

1. An invoice dated April 7 with terms 4/20, 2/20, n/60 EOM. The goods are received on April 9 and a partial payment is made on May 21.

2. An invoice dated July 26 with terms 3/30, n/45 ROG. The goods are received on August 2 and a partial payment is made on September 3.

3. An invoice dated January 3 with terms 2½/10, 1/20. The goods are received on January 10 and a partial payment is made on January 24.

Solutions

1. Since the payment falls within a discount period, the credited amount is larger than the payment.
2. Since the payment does not fall within a discount period, the credited amount is equal to the payment.
3. Since the payment falls within a discount period, the credited amount is larger than the payments

EXAMPLE 7.4B: APPLYING YOUR PARTIAL PAYMENTS TOWARD YOUR BALANCE OWING

Heri just received an invoice from his supplier, R&B Foods. The invoice totalling $68,435.27 is dated June 5 with terms of 2½/10, 1/25, n/45. Heri sent two partial payments in the amounts of $20,000 and $30,000 that R&B Foods received on June 15 and June 29, respectively. He wants to clear his invoice by making a final payment to be received by R&B Foods on July 18. What is the amount of the final payment?

Plan

Determine the amount of the final payment (N) after the first two partial payments are credited toward the invoice total. To credit the two partial payments you must find the list amount before any cash discounts (L) such that the payment can be deducted from the invoice total.

What You Already Know

Step 1: The invoice amount, terms of payment, and payment amounts are known:

- L = $68,435.27 Invoice date = June 5
- Terms of payment = 2½/10, 1/25, n/45
- N1 = $20,000 on June 15
- N2 = $30,000 on June 29
- N3 = ? on July 18

Steps 2 & 3:The figure illustrates the timeline for the invoice and identifies the payments.

How You Will Get There

Step 4: The first two payments are partial. Apply the rearranged Formula 6.1. As noted on the timeline:

a.   The first partial payment qualifies for d = 2½%.

b.   The second partial payment qualifies for d = 1%.

c.   The last payment is a full payment, where d = 0% (no cash discount).
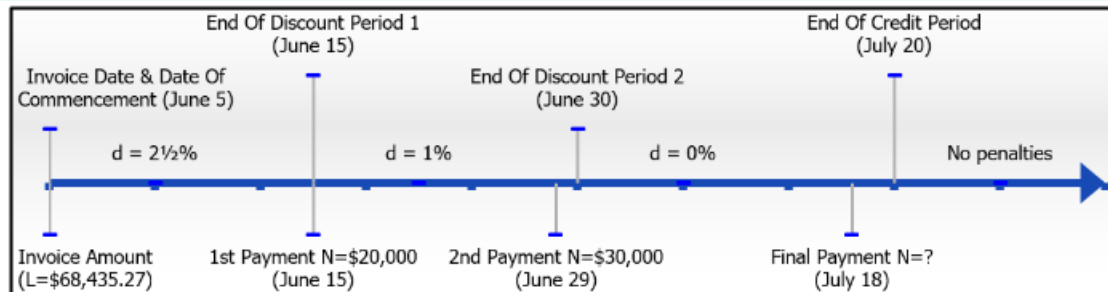
Perform

Step 4

$$a. \quad L = \frac{\$20,000}{(1-0.025)} = \frac{\$20,000}{0.975} = \$20,512.82$$

Balance Owing = $68,435.27 − $20,512.82 = $47,922.45

$$b. \quad L = \frac{\$30,000}{(1-0.01)} = \frac{\$30,000}{0.99} = \$30,303.03$$

Balance Owing = $47,922.45 − $30,303.03 = $17,619.42

c. Since there is no cash discount, N3 = L. The balance owing is L = $17,619.42.

Therefore, the payment is the same number and N3 = $17,619.42.

Present

The first two payments receive a credit of $20,512.82 and $30,303.03 toward the invoice total, respectively. This leaves a balance on the invoice of $17,619.42. Since the final payment is made during the credit period, but not within any cash discount period, the final payment is the exact amount of the balance owing and equals $17,619.42.

LATE PAYMENT PENALTIES

Among all of the invoices that you were paying in the section opener, you realize upon opening that third envelope that you somehow missed an invoice last month and failed to pay the $4,000 owing, which is now overdue. In looking at the invoice, you notice that it states on the bottom that late payments are subject to a 3% per month penalty. You urgently want to pay this invoice to maintain good relations with your supplier but wonder in what amount you must issue the cheque?

Why Do Late Penalties Exist?

Suppliers are doing their business customers a favour when they use invoicing to seek payment. In essence, suppliers provide the products to the customer but are not paid for those products up front. This means customers are granted a period during which they "borrow" the products for free without any associated interest costs that are usually tied to borrowing. If the invoice is not paid in full by the time the credit period elapses, then the supplier starts treating any remaining balance like a loan and charges interest, which is called a late payment penalty.

The Formula

When a payment arrives after the credit period has expired, you need to adjust the balance of the invoice upward to account for the penalty. Continue to use Formula 6.1 on Single Discounts for this calculation. In this unique case, the discount rate represents a penalty. Instead of deducting money from the balance, you add a penalty to the balance. As a result, the late penalty is a negative discount. For example, if the penalty is 3% then d= −3%.

How It Works

Follow the same steps to solve the invoice payment once again when working with late payments. As an example, work with the $4,000 overdue invoice that is subject to a 3% per month late penalty. If the invoice is paid within the first month of being overdue, what payment is made?Step 1: The invoice balance is L = $4,000, and the penalty percent is d= −0.03.

Steps 2 & 3: Since the focus is strictly on the late payment calculation, you do not need a timeline for the discount or credit periods. The late payment occurs within the first month past the expiry of the credit period.Step 4: To determine the new invoice balance, apply Formula 6.1 to calculate N = $4,000 × (1 − (−0.03)) = $4,000 × 1.03 = $4,120. Therefore, to clear the invoice you must issue a cheque in the amount of $4,120. This covers the $4,000 owing along with a late penalty of $120.

Important Notes

Not all invoices have penalties, nor are they always enforced by the supplier. There are many reasons for this:

1. Most businesses pay their invoices in a timely manner, which minimizes the need to institute penalties.

2. Waiving a penalty maintains good customer relations and reinforces a positive, cooperative business partnership.

3. Strict application of late penalties results in difficult situations. For example, if a payment was dropped in the mail on August 13 from your best customer and it arrives one day after the credit period expires on August 15, do you penalize that customer? Is it worth the hassle or the risk of upsetting the customer?

4. The application of late penalties generally involves smaller sums of money that incur many administrative costs. The financial gains from applying penalties may be completely wiped out by the administrative expenses incurred.

5. Businesses have other means at their disposal for dealing with delinquent accounts. For example, if a customer regularly pays its invoices in an untimely manner, the supplier may just to decide to withdraw the privilege of invoicing and have the customer always pay up front instead.In this textbook, all

penalties are strictly and rigidly applied. If a payment is late, the invoice balance has the appropriate late penalty applied.

Things To Watch Out For

When a late penalty is involved, you must resist the temptation to adjust the payment by the penalty percentage. The discussion and calculations in this section focus on adjusting the outstanding invoice balance to figure out the payment required. For example, assume there is a $500 outstanding balance subject to a 2% penalty.

- This means that with the penalty, the invoice total is $510, which determines that a payment of $510 is required.
- If the customer makes a $200 partial payment on this late debt, you cannot apply the partial payment procedure, which would give a credit of $204.08 for the payment, resulting in a balance owing of $295.92. If you make this mistake, you in fact reward the customer for being tardy!
- Nor can you deduct the 2% penalty from the payment and give credit for $200(1 – 0.02) = $196 (resulting in a balance owing of $304) because the 2% penalty applies to the whole invoice amount and not just the partial payment.
- Instead, the $200 payment must be deducted directly from the penalty-adjusted balance of $510. In this case, the customer still owes $310 to clear the invoice.

Paths To Success

An alternative method for applying a late penalty is to treat the penalty like a positive percent change (see Section 3.1). In this case, the penalty is how much you want to increase the balance owing.

Thus, if an invoice for $500 is subject to a 2% penalty, the Δ% = 2% and Old = $500.

The goal is to look for the New balance in Formula 3.1:

$$\triangle\% = \frac{New - Old}{Old} \times 100$$

$$2\% = \frac{new - \$500}{\$500} \times 100$$

$$New = \$510$$

EXAMPLE 7.4C: WHOOPS! I MISSED THE DUE DATE!

You received an invoice dated December 17 in the amount of $53,455.55 with terms of 4/15, 2/30,

n/60 ROG, 2.75% penalty per month for late payments. The merchandise is received on January 24. You sent in a partial payment of $40,000
on January 31 with intention to pay the remaining balance before the credit period expired. However, you forgot about the invoice and realized your mistake on March 30, when you submit payment in full for the invoice. What amount is the final payment? (Note: Assume this is not a leap year)

Plan

The final payment must cover the invoice balance owing on any remaining balance after the partial payment is
credited and the outstanding balance is penalized as per the late penalty.

What You Already Know

Step 1: The invoice amount, terms of payment, and payment information are known:

- L = $53,455.55
- Invoice date = December 17
- Goods received = January 24
- Terms of payment = 4/15, 2/30, n/60 ROG,
- 2.75% penalty per month for late payments
- N1 = $40,000 on January 31
- N2 = ? on March 30

Steps 2 & 3: The figure below illustrates the timeline for the invoice and identifies the payments.

How You Will Get There

Step 4 (Partial Payment): Credit the partial payment during the discount period (d = 4%) by applying the rearranged Formula 6.1. Step 4 (New Invoice Balance): Deduct the partial payment from the invoice total.
Step 4 (Late Payment): The second payment occurs after the credit period elapses. Therefore the outstanding invoice balance is penalized by d = −0.0275 as per the policy. Apply Formula 6.1. This calculates the outstanding balance including the penalty, which
equals the final payment.
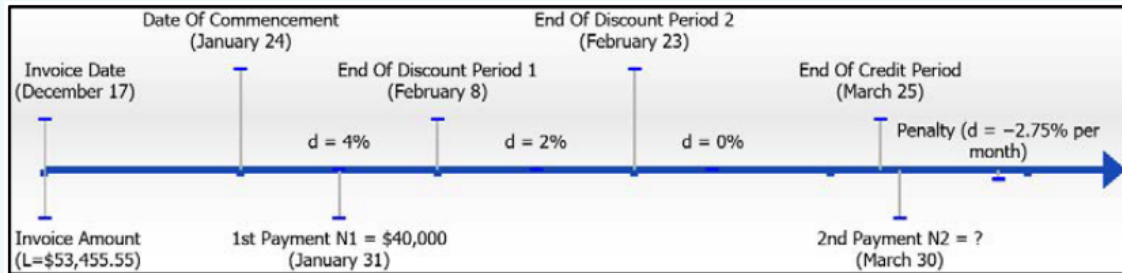
Perform

Ste
p 4
$$(partial Payment) : L = \frac{\$40,000}{(1 - 0.04)} = \$40,0000.96 = \$41,666.67$$

Ste
p 4
(New Invoice Balance): Balance Owing = $53,455.55 − $41,666.67 = $11,788.88

Ste
p 4
(Late Payment): N = $11,788.88 × (1 − (−0.0275))

N = $11,788.88 × 1.0275 = $12,113.07.

Present

The first payment resulted in a $41,666.67 deduction from the invoice. The overdue remaining balance of $11,788.88 has a penalty of $324.19 added to it, resulting in a final clearing payment of $12,113.07

EXERCISES

Round all money to two decimals and percentages to four decimals for each of the following exercises. In all questions, assume that the dates indicated are the dates upon which the payment is received.

Mechanics

For questions 1–3, calculate the full payment required on the payment date that reduces the balance on the invoice to zero. Assume this is not a leap year.

Question	Invoice Amount	Invoice Date	Invoice Terms	Receipt of Goods	Date of Full Payment
1.	$136,294.57	January 14	2/10, n/30	January 10	January 22
2.	$98,482.75	September 14	3/10, 2/20, 1/30, n/50 EOM	October 3	October 19
3.	$48,190.38	February 21		February 27	April 3

For questions 4–6, calculate the remaining invoice balance after crediting the invoice for the partial payments indicated.

Question	Invoice Amount	Invoice Date	Invoice Terms	Receipt of Goods	Partial Payments and Dates
4.	$57,775.00	June 16	3/15, $^{1/2}$/25	June 12	$20,000 on July 2; $15,000 on July 10
5.	$1,200,310.75	August 25	3/10, 2/20, 1/30, n/50 EOM	September 2	$500,000 on September 4; $250,000 on September 14
6.	$17,481.68	December 3	4/10, 1/30, 1/40, n/60	December 20	$10,000 on January 2

For questions 7–8, calculate the final payment required on the payment date that reduces the balance of the invoice to zero.

Question	Invoice Amount	Invoice Date	Invoice Terms	Late Payment Penalty	Receipt of Goods Date	Partial Payment and Dates	Final Payment date
7.	$23,694.50	April13	3/15, 2/25, 1/40, n/60	1 1/2% per month	April 18	$10,000 on April 30	June 15
8.	$332,053.85	October 4	4/10 EOM	2% per month	October 30	$100,000 on November 5	December 15

Applications

9. Hudson's Bay received an invoice dated July 13 from Nygard International in the amount of $206,731.75. The terms are 2/10, 1/20, n/30. If Nygard receives full payment on August 1, what amount is paid?

10. Time Bomb Traders Inc. in Burnaby just received an invoice dated February 17 (of a leap year) from UrbanEars headphones in the amount of $36,448.50 with terms of 1½/15, ½/30, n/45 ROG. The items on the invoice are received on March 3. What amount is the full payment if UrbanEars receives it on March 18?

11. Family Foods received an invoice dated July 2 from Kraft Canada in the amount of $13,002.96 with terms of 2/15, 1/30, n/45 EOM and a late penalty of 2% per month. What amount is paid in full if Kraft Canada receives a cheque from Family Foods on August 17?

12. An invoice dated November 6 in the amount of $38,993.65 with terms of 2½/10, 1/15 ROG, 2% penalty per month is received by Cargill Limited from Agricore United. The wheat shipment is received on December 2. Cargill Limited made a partial payment of $15,000 on December 10. What amount should Cargill pay to clear its invoice on December 13?

13. Yamaha Music received two invoices from the same vendor. The first is for $1,260 dated March 9 with terms 3/10, net 30. The second is for $2,450 dated March 12 with terms 2/10, 1/20, net 30. If a payment of $1,000 is made on March 19, what payment amount on March 31 settles both invoices? (Note: Payments are applied to the earlier invoice first.)

14. An invoice is dated July 26 for $5,345.50 with terms of 3¼/10, net 45, 2½% per month penalty. If the

invoice is paid on September 10, what payment amount is required to pay the entire balance owing?

15. Mohawk College received an invoice dated August 20 from Office Depot for office supplies totalling $10,235.97 with terms of 3¾/15, 1½/30, n/45, EOM, 2¾% per month penalty. The college made three payments of $2,000 dated September 4, September 29, and October 10. What payment on October 25 settles the invoice?

Challenge, Critical Thinking, & Other Applications

16. An invoice for $100,000 dated February 2 (of a non–leap year) with terms of 4/10, 3/20, 2/30, 1/40, net 60, ROG, 1¾% per month penalty. The merchandise is received on February 16. If four equal payments are made on February 20, March 17, April 1, and April 20 resulting in full payment of the invoice, calculate the amount of each payment.

17. An accounting department receives the following invoices from the same vendor and makes the indicated payments. The vendor always applies payments to the earliest invoices first and its late payment policy stands at 1½% per month for any outstanding balance.

Invoice #	Invoice Amount	Invoice Date	Invoice Terms	Receipt of Goods Date
3866	$47,690.11	June 18	$\frac{2}{10}, EOM$	July 3
2928	$123,691.82	June 26	$2^1/_2$/15 , $^1/_{25}$	July 6
4133	$96,004.21	June 30	$^2/_{20}$, ROG	July 7
6767	$16,927.50	July10	$^2/_{10}$, $^1/_{20}$	July 11

Payment Received by Vendor on	Payment Amount
July 2	$40,000
July 11	$75,000
July 21	$100,000
July 30	$25,000

Calculate the amount of the final payment on August 17 that reduces the total balance owing to zero.

18. John's Home Hardware was invoiced on May 29 for the following non-taxable items with terms of 2/20, 1/40, 2% per month penalty.
If a partial payment of $20,000 is made on July 7, what payment amount on August 30 reduces the balance owing to zero? What is the total dollar amount of the late penalty?

Item	Quantity	Unit Price
2" x 4" x 8" framing studs	13,000	$0.96
4" x 4" x 8' fence posts	2,400	$2.87
2" x 8" x 16' wood planks	480	$8.47
1" x 6" x 5' fence boards	8,000	$1.22

19. Your company receives a $138,175.00 invoice dated April 12 with terms of 4/10, 3/20, 2/30, 1/40, n/60. Due to the large amount, the accounting department proposes two different ways to pay this invoice, depending on company income and financing alternatives. These plans are listed in the table below.

Payment Date	Plan #1	Plan #2
April 21	$35,000	$50,000
May 2	$20,000	$10,000
May 10	$25,000	$50,000
May 13	$30,000	$20,000
June 11	Balance Owing	Balance Owing

    a. Which alternative do you recommend?

    b. If your recommendation is followed, how much money is saved over the other option?

20. You receive an invoice dated July 29 with terms of 3/15, 1¾/30, n/50 EOM, 2% penalty per month. If the payments in the following table are made and you pay the invoice in full, what is the total invoice amount?

Payment Date	Payment Amount
August 4	$67,000.00
August 28	$83,000.00
September 28	$15,000.00
September 30	$83,297.16

Credit: Adapted from unit 7.4 in Business Math: A Step-by-Step Handbook

APPENDIXES

Appendix 1: Rounding Rules

How and when to round always is a source of confusion. This Appendix provides you with some rules for rounding your answers. We start with some general guidelines that apply globally throughout the entire textbook, then comment at the rules that apply to some specific sections of text.

GLOBAL ROUNDING RULES

1. All interim solutions never get rounded unless there is a logical reason or business process forcing it to be rounded.

2. When writing nonterminating decimals in this textbook, up to six decimals are written. The horizontal line format is used
    for repeating decimals. If the number is not a final solution, then it is assumed that all decimals or as many as possible aren being carried forward.

3. All final numbers are rounded to four decimals in decimal format and two decimals in percent format unless instructions indicate otherwise.

4. Final solutions are rounded according to common business practices, practical limitations, or specific instructions.

5. Zeroes not required at the end of decimals are generally not written unless required to meet a rounding standard or to visually line up a sequence of numbers.

TOPIC SPECIFIC ROUNDING RULES

Some specific topics have their own rounding standards. Until you learn about these topics, it does not make any sense to put those standards here. However, the standards are introduced with the relevant topic. They are also summarized into Appendix C: Rounding Rules.

Appendix 2: Exercise Solutions

SECTION 2.1

1. 15

2. 10

3. 40

4. 100

5. 55

6. 10

7. $21.70

8. $7,645.52

9. $19,215.37

10. 400

11. $25,967.38

12. $1,345.87

13. $5,978.08

14. $1,056,767.41

15. $2,156,764.06

16. $41,271.46

17. $83,228.60

18. $2,339.07

19. $26,450.25

20. $14,561.43

SECTION 2.2

1a. proper

1b.compound

1c.improper

1d. proper

1e. complex

1f. improper

1g. complex

1h.proper

2a. 3/4=27/36

2b. 2/8=16/64

2c. 2/5=18/45

2d. 5/6=75/90

3a. $$\frac{5 \times 2}{10 \times 2} = \frac{10}{20} \frac{5 \div 5}{10 \div 5} = \frac{1}{2}$$

3b. $$\frac{6 \times 5}{8 \times 5} = \frac{30}{40} \frac{6 \div 2}{8 \div 2} = \frac{3}{4}$$

4a. 0.875

4b. 16.25

4c. 2.6

4d. 135.5

5a. 0.875

5b. 16.250

5c. 1.111

5d. 0.469

6a. $0.08\overline{3}$

6b. $5.2\overline{4}$

6c. $1.\overline{3}$

6d. $-0.30\overline{9}$

7a. 5.95

7b. 15.3125

8a. 1.04

8b. 0.96

8c. 9.64

9a. $3.\overline{20}$

9b. 0.5

10. $1,182.14

11. $14,005.26

12. $525,745.68

13. $2,693.13

14. $149,513.74

15. $4,450.29

16. $9,579.23

17. $138,934.38

18. $12,252.25

19. $267,952.30

20. $1,041.58

SECTION 2.3

1a. 46.38%

1b. 315.79%

1c. 0.0138%

1d. 1.5%

2a. 37.5%

2b. 53.125%

2c. 350%

2d. 280%

3a.	$383.\overline{3}\%$
3b.	$22.\overline{2}\%$
3c.	$27.\overline{27}\%$
3d.	51.6129%

4a. 0.153

4b. 0.0003

4c. 1.53987

4d. 0.140005

5. 67.5%

6. $2,500

7. $0.003061

8. 3,862,105

9. $608,600

10. 8.2358%

11. 7,325

12. $147.8 billion

13. $8.50

14. 1.48%

15. 13.0235%

16. $124,722.22

17. $20,000,000

18. $128,000.03

19. share price = $12.36

Money made = $2,360

20. 116. 6%

SECTION 3.1

1. D$ = $411.60; N = $568.40

2.L = $800; D$ = $200

3. d = 42.625%; N = $1,133.16; D$ = $841.84

4. d = 26.7904%; L=$500; D$ = $133.95

5. $259.97

6. $24.32

7. 31.985%

8. $19.99

9. d = 43.972%; L = $51.76

10. L = $24; N = $13.92

11. d = 62.8%; L = $700; D$ = $439.60

12. d = 26%; N = $303.99; D$ = $96

13. $1,026

14. d = 63.14%; N = $29.49

15. $1,194,553.06

16. 2 years 216 days

17. Reduced by 17.4154%

18a. 3%

18b. $2.50

19a. 52.2328%

19b. $3,073.82

20a. $74.99

20b. $60.91

20c. L = $78.74; N = $63.96; D$ = $14.78

SECTION 3.2

1. S=$321.95; S_{BE} =$236.95; M$=$133.53; MoC%=70.8683%; MoS%=41.4754%

2. C=$653.59; S_{BE} =$849.67; M$=$346.41; MoC%=53.0011%; MoS%=34.6413%

3. C=$118.25; S=$301.53; E=$153.13; MoS%=60.7833%; S_{BE} =$271.38

4. M$=$96.25; P=$41.25; C=$178.74; S_{BE} =$223.74

5. C=$744.83; S=$1,284.83; P=$204.83; MoC%=72.5%; MoS%=42.0289%

6. M$=$81.60; S=$201.60; E=$51.36; MoS%=40.4762%; S_{BE} =$171.36

7. P=$175; S=$1,447.37; C=$1,172.37; MoC%=23.4568%; S_{BE} =$1,272.37

8. C=$220; S=$287.50; M$=$67.50; MoS%=23.4783%; MoC%=30.6818%

9. $112.50

10. 125%

11. $27,996.79

12. $57.05

13a. $733.03

13b. $219.91

13c. 53.3151%

13d. $698.03

14a. $119.95

14b. $40.85

14c. 51.6435%

14d. 34.0559%

14e. 28.8887%

15. $20.82

16. $102.40

17. 42.8571%

18. $488.09

19a. $40.47

19b. $21.79

19c. $62.26

19d. $14.51

19e. $12.20; $2.31 reduction

20a. $12.65

20b. $17.71

20c. $0.63

20d. MoS% = 28.5714%

20e. C = $15.18; S = $21.25; P = $0.76; MoS% = 28.5647%

SECTION 3.3

1. D$ = $153.95; S_{onsale} = $285.90

2. S = $299.95; d = 33.3389%

3. D$ = $275; d = 26.1905%

4. S_{onsale} = $25,525; d = 11.2945%

5. S = $19,701.42; D$ = $6,501.47

6. S = $319.42; S_{onsale} = $281.09

7. S_{onsale} = $53.99

8. D$ = $45; d = 52.9474%

9. d = 14.9%

10. S = $133.30; D$ = $93.31

11. S_{onsale} = $384.99; S = $699.98

12. d = 36.7893

13a. P_{onsale} = $2.59

13b. P = $74.55

14. d = 10.9512%

15a. S_{onsale} = $1,519.05

15b. P_{onsale} = $303.81

15c. S = $2,025.40

15d. P = $810.16

16. S$_{onsale}$ = $367.20

17a. S$_{onsale}$ = $282.91

17b. d = 29.2637%

17c. MoS% = 23.8026%

18. D = 32.4325%

19a. S$_{onsale}$ = $1,680

19b. MoS% when on sale = 32.8571%

19c. P$_{onsale}$ = $192.00

20a. S$_{onsale}$ = $1,614.06

20b. S = $2,152.08

20c. M$ = $956.48; MoC% = 80%

SECTION 4.1

1. S$_{tax}$ = $682.49

2. S$_{tax}$ = $750.74

3. S$_{tax}$ = $747.49

4. S$_{tax}$ = $727.99

5. S = $796.45; HST Tax Amount = $103.54

6. S = $818.17; GST Tax Amount = $40.91; PST Tax Amount = $40.91

7. January Remit $5,471.40; February Refund $2,202.96

8. Winter Remit $7,171.20; Spring Refund $1,025.25; Summer Remit $9,803.30; Fall Refund $1,191.65

9a.Alberta is best = $1,396.45

9b. Savings = $17.48

10. S = $529.99; GST Tax Amount = $26.50

11. Can't purchase at S$_{tax}$ = $3,050.94; Down Payment = $45.08

12. S$_{tax}$ = $4,477.30; GST Tax Amount = $199.88; PST Tax Amount = $279.83

13. Q1 Remit $17,865; Q2 Remit $35,370; Q3 Refund $8,105; Q4 Refund $24,210

14. March Remit $951.15; April Refund; $1,257.15; May Remit $2,953.65

15. Higher in Ontario by 1.5274%.

16. Total PST = $13.02; Total GST = $52.46; Total HST = $13

17. Remit $378,227.85

18. PST Rate = 9.505%

19a. S = $352.72

19b. S = $1,682.81

19c. S = $19,465.09

19d. S = $4,736.57

20. Remit $31,331.89

SECTION 4.2

1. $133,568.68

2. $96,513.10

3. $46,744.67

4. Balance Remaining = $21,928.01

5. Balance Remaining = $434,964.99

6. Balance Remaining = $7,065.01

7. $13,692.78

8. $232,444.92

9. $204,664.43

10. $35,901.77

11. $12,872.93

12. $23,372.94

13. $2,654.57

14. $5,479.14

15. $4,241.10

16. $24,772.64

17. $40,608.80

18. N = $13,511.24; Late penalty amount = $519.66

19a. Choose Plan #2

19b. Savings = $724.92

20. $250,214.41

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CONTENT MAPPING FOR NSCC ADAPTATION

NSCC Math for Bookkeeping	Business Math: A Step-by-Step Handbook
Chapter 1	Chapter 1 1. & 1.2
Chapter 2	Chapter 2 – 2.1 – Order of Operations 2.2 – Fractions, Decimals, and Rounding 2.3 – Percentages and Secret Path to Success – include verbage and two charts of language and math symbols
Chapter 3	Chapter 6 6.1 – Figuring out the cost: Discounts; 6.2 – Markup: Setting the Regular price; 6.3 – Markdown, setting the sale price
Chapter 4	Chapter 7 7.1 – Sales Tax & 7.4 – Invoicing: Terms of Payment and Cash Discounts