

# The Math of Money



# THE MATH OF MONEY

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## ABOUT THIS BOOK

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# MODULE 1: THE MATH OF MONEY

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## 1.1 THE MACROECONOMY

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### A BRIEF INTRODUCTION TO PERSONAL FINANCE



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### PERSONAL FINANCE<sup>1</sup>

The circumstances or characteristics of your life influence your financial concerns and plans. What you want and need—and how and to what extent you want to protect the satisfaction of your wants and needs—all depend on how you live and how you'd like to live in the future. While everyone is different, there are common circumstances of life that affect personal financial concerns and thus affect everyone's financial planning. Factors that affect personal financial concerns are family structure, health, career choices, and age.

1. Adapted from [1.1 Individual or “Micro” Factors That Affect Financial Thinking](#) in [Personal Finance](#) by Lumen learning. [CC BY-NC-SA 3.0](#)

## FAMILY STRUCTURE

Marital status and dependents, such as children, parents, or siblings, determine whether you are planning only for yourself or for others as well. If you have a spouse or dependents, you have a financial responsibility to someone else, and that includes a responsibility to include them in your financial thinking. You may expect the dependence of a family member to end at some point, as with children or elderly parents, or you may have lifelong responsibilities to and for another person.

Partners and dependents affect your financial planning as you seek to provide for them, such as paying for children's education. Parents typically want to protect or improve the quality of life for their children and may choose to limit their own fulfillment to achieve that end.

Providing for others increases income needs. Being responsible for others also affects your attitudes toward and tolerance of risk. Typically, both the willingness and ability to assume risk diminishes with dependents, and a desire for more financial protection grows. People often seek protection for their income or assets even past their own lifetimes to ensure the continued well-being of partners and dependents. An example is a life insurance policy naming a spouse or dependents as beneficiaries.

## HEALTH

Your health is another defining circumstance that will affect your expected income needs and risk tolerance and thus your personal financial planning. Personal financial planning should include some protection against the risk of chronic illness, accident, or long-term disability and some provision for short-term events, such as pregnancy and birth. If your health limits your earnings or ability to work or adds significantly to your expenditures, your income needs may increase. The need to protect yourself against further limitations or increased costs may also increase. At the same time your tolerance for risk may decrease, further affecting your financial decisions.

## CAREER CHOICE

Your career choices affect your financial planning, especially through educational requirements, income potential, and characteristics of the occupation or profession you choose. Careers have different hours, pay, benefits, risk factors, and patterns of advancement over time. Thus, your financial planning will reflect the realities of being a postal worker, professional athlete, commissioned sales representative, corporate lawyer, freelance photographer, librarian, building contractor,

tax preparer, professor, Web site designer, and so on. For example, the careers of most athletes end before middle age, have higher risk of injury, and command steady, higher-than-average incomes, while the careers of most sales representatives last longer with greater risk of unpredictable income fluctuations.

Most people begin their independent financial lives by selling their labor to create an income by working. Over time they may choose to change careers, develop additional sources of concurrent income, move between employment and self-employment, or become unemployed or reemployed. Along with career choices, all these changes affect personal financial management and planning.

## AGE

Needs, desires, values, and priorities all change over a lifetime, and financial concerns change accordingly. Ideally, personal finance is a process of management and planning that anticipates or keeps abreast with changes. Although everyone is different, some financial concerns are common to or typical of the different stages of adult life. Analysis of **life stages** is part of financial planning.

At the beginning of your adult life, you are more likely to have no dependents, little if any accumulated wealth,



and few **assets**. (Assets are resources that can be used to create income, decrease expenses, or store wealth as an investment.) As a young adult you also are likely to have comparatively small income needs, especially if you are providing only for yourself. Your employment income is probably your primary or sole source of income. Having no one and almost nothing to protect, your willingness to assume risk is usually high. At this point in your life, you are focused on developing your career and increasing your earned income. Any investments you may have are geared toward growth.

As your career progresses, income increases but so does spending. Lifestyle expectations increase. If you now have a spouse and dependents and elderly parents to look after, you have additional needs to manage. In middle adulthood you may also be acquiring more assets, such as a house, a retirement account, or an inheritance.

As income, spending, and asset base grow, ability to assume risk grows, but willingness to do so typically decreases. Now you have things that need protection: dependents and assets. As you age, you realize that *you* require more protection. You may want to stop working one day, or you may suffer a decline in health. As an older adult you may want to create alternative sources of income, perhaps a retirement fund, as insurance against a loss of employment or income. Figure 1.1.1 suggests the effects of life stages on financial decision making.

	Young Adulthood	Middle Adulthood	Older Adulthood	Retirement
Source of Income	Wages	Wages/ Investment	Wages/ Investment	Investment
Asset Base	None	Accumulating	Growing	Using up
Expenses	Low	Growing	Growing	Low
Risk: Ability	Low	Higher	Higher	High
Risk: Willingness	High	Lower	Lower	Low

*Figure 1.1.1: Life Stages and Finances*

Early and middle adulthoods are periods of building up: building a family, building a career, increasing earned income, and accumulating assets. Spending needs increase, but so do investments and alternative sources of income.

Later adulthood is a period of spending down. There is less reliance on earned income and more on the accumulated wealth of assets and investments. You are likely to be without dependents, as your children have grown up or your parents passed on, and so without the responsibility of providing for them, your expenses are lower. You are likely to have more leisure time, especially after retirement.

Without dependents, spending needs decrease. On the other hand, you may feel free to finally indulge in those things that you've "always wanted." There are no longer dependents to protect, but assets demand even more protection as, without employment, they are your only source of income. Typically, your ability to assume risk is high because of your accumulated assets, but your willingness to assume risk is low, as you are now dependent on those assets for income. As a result, risk

tolerance decreases: you are less concerned with increasing wealth than you are with protecting it.

Effective financial planning depends largely on an awareness of how your current and future stages in life may influence your financial decisions.

*Khan Academy: Easy Tips to Save Money Every Day*



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## THE MACROECONOMY<sup>2</sup>

Financial planning has to take into account conditions in the wider economy and in the markets that make up the economy. The **labor market**, for example, is where labor is traded through hiring or employment. Workers compete for jobs and employers compete for workers. In the **capital market**, capital (cash or assets) is traded, most commonly in the form of stocks and bonds (along with other ways to package capital). In the **credit market**, a part of the capital market, capital is loaned and borrowed rather than bought and sold. These and other markets exist in a dynamic economic environment, and those

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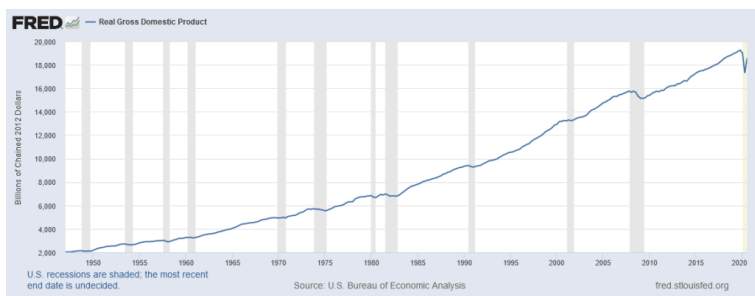
environmental realities are part of sound financial planning.

In the long term, history has proven that an economy can grow over time, that investments can earn returns, and that the value of currency can remain relatively stable. In the short term, however, that is not continuously true. Contrary or unsettled periods can upset financial plans, especially if they last long enough or happen at just the wrong time in your life. Understanding large-scale economic patterns and factors that indicate the health of an economy can help you make better financial decisions. These systemic factors include, for example, business cycles and employment rates.

## BUSINESS CYCLES

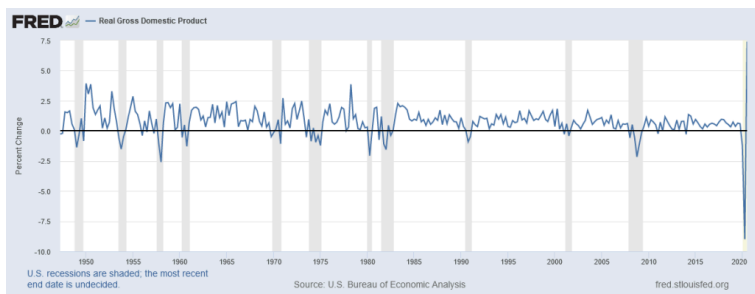
An economy tends to be productive enough to provide for the wants of its members. Normally, economic output increases as population increases or as people's expectations grow. An economy's output or productivity is measured by its **gross domestic product** or GDP, the value of what is produced in a period. When the GDP is increasing, the economy is in an expansion, and when it is decreasing, the economy is in a contraction. An economy that contracts for half a year is said to be in **recession**; a prolonged recession is a **depression**. The GDP is a closely

watched barometer of the economy. The GDP since 1947 is shown in Figure 1.1.2 below.



*Figure 1.1.2: The GDP of the United States, 1948-2020 (Source: US Bureau of Economic Analysis/St. Louis Federal Reserve)*

Over time, the economy tends to be cyclical, usually expanding but sometimes contracting. This is called the **business cycle**. Periods of contraction are generally seen as market corrections, or the market regaining its equilibrium, after periods of growth. Growth is never perfectly smooth, so sometimes certain markets become unbalanced and need to correct themselves. Over time, the periods of contraction seem to have become less frequent, as you can see in Figure 1.1.3, the growth rate of the real GDP fluctuates from one year to the next. The business cycles still occur nevertheless.



*Figure 1.1.3: Percent Change in the Real GDP of the United States, 1948-2020 (Source: US Bureau of Economic Analysis/St. Louis Federal Reserve)*

There are many metaphors to describe the cyclical nature of market economies: “peaks and troughs,” “boom and bust,” “growth and contraction,” “expansion and correction,” and so on. While each cycle is born in a unique combination of circumstances, cycles occur because things change and upset economic equilibrium. That is, events change the balance between supply and demand in the economy overall. Sometimes demand grows too fast and supply can’t keep up, and sometimes supply grows too fast for demand. There are many reasons that this could happen, but whatever the reasons, buyers and sellers react to this imbalance, which then creates a change.

*One Minute Economics: The Gross Domestic Product and Government Revenue Explained (all rights reserved)*



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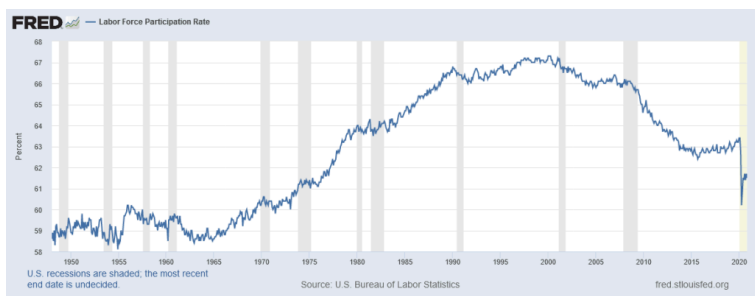
## EMPLOYMENT

An economy produces not just goods and services to satisfy its members but also jobs, because most people participate in the market economy by trading their labor, and most rely on wages as their primary source of income. The economy therefore must provide opportunity to earn wages so more people can participate in the economy through the market. Otherwise, more people must be provided for in some other way, such as a private or public subsidy (charity or welfare).

The **unemployment rate** is a measure of an economy's shortcomings, because it shows the proportion of people who want to work but don't because the economy cannot provide them jobs. There is always some so-called natural rate of unemployment as people move in and out of the workforce as the circumstances of their lives change—for example, as they retrain for a new career or take time out for family. But natural unemployment should be consistently low and not affect the productivity of the economy. Figure 1.1.4 shows the unemployment rate in the United States since 1947.

Unemployment also shows that the economy is not efficient, because it is not able to put all its productive human resources to work.

The labor force participation rate shows how successful an economy is at creating opportunities to sell labor and efficiently using its human resources. A healthy market economy uses its labor productively, is productive, and provides employment opportunities as well as consumer satisfaction through its markets. This is shown below in Figure 1.1.5.



*Figure 1.1.5: The Labor Force Participation Rate in the United States, 1948-2020 (Source: US Bureau of Economic Analysis/St. Louis Federal Reserve)*

At either end of this scale of growth, the economy is in an unsustainable position: either growing too fast, with too much demand for labor, or shrinking, with too little demand for labor.

If there is too much demand for labor—more jobs than workers to fill them—then wages will rise, pushing up the cost of everything and causing prices to rise. Prices usually rise faster than wages, for many reasons, which would discourage consumption that would eventually discourage production and cause the economy to slow down from its “boom” condition into a more manageable rate of growth.

If there is too little demand for labor—more workers than jobs—then wages will fall or, more typically, there will be people without jobs, or unemployment. If wages become low enough, employers theoretically will be encouraged to hire more labor, which would bring employment levels back up. However, it doesn’t always work that way, because people have job mobility—they are willing and able to move between economies to seek employment.

If unemployment is high and prolonged, then too many



people are without wages for too long, and they are not able to participate in the economy because they have nothing to trade. In that case, the market economy is just not working for too many people, and they will eventually demand a change (which is how most revolutions have started).

*One Minute Economics: The Unemployment Rate and the Labor Force Participation Rate Compared (all rights reserved)*



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## OTHER INDICATORS OF ECONOMIC HEALTH

Other economic indicators give us clues as to how “successful” our economy is, how well it is growing, or how well positioned it is for future growth. These indicators include statistics, such as the number of houses being built or existing home sales, orders for durable

goods (e.g., appliances and automobiles), consumer confidence, producer prices, and so on. However, GDP growth and unemployment are the two most closely watched indicators, because they get at the heart of what our economy is supposed to accomplish: to provide diverse opportunities for the most people to participate in the economy, to create jobs, and to satisfy the consumption needs of the most people by enabling them to get what they want.

An expanding and healthy economy will offer more choices to participants: more choices for trading labor and for trading capital. It offers more opportunities to earn a return or an income and therefore also offers more diversification and less risk.

Naturally, everyone would rather operate in a healthier economy at all times, but this is not always possible. Financial planning must include planning for the risk that economic factors will affect financial realities. A recession may increase unemployment, lowering the return on labor—wages—or making it harder to anticipate an increase in income. Wage income could be lost altogether. Such temporary involuntary loss of wage income probably will happen to you during your lifetime, as you inevitably will endure economic cycles.

A hedge against lost wages is investment to create other forms of income. In a period of economic contraction, however, the usefulness of capital, and thus its value, may decline as well. Some businesses and industries are considered immune to economic cycles (e.g., public education and health care), but overall, investment returns may suffer. Thus, during your lifetime business cycles will likely affect your participation in the capital markets as well.

## CURRENCY VALUE

Stable currency value is another important indicator of a healthy economy and a critical element in financial planning. Like anything else, the value of a currency is based on its usefulness. We use currency as a medium of exchange, so the value of a currency is based on how it can be used in trade, which in turn is based on what is produced in the economy. If an economy produces little that anyone wants, then its currency has little value relative to other currencies, because there is little use for it in trade. So a currency's value is an indicator of how productive an economy is.

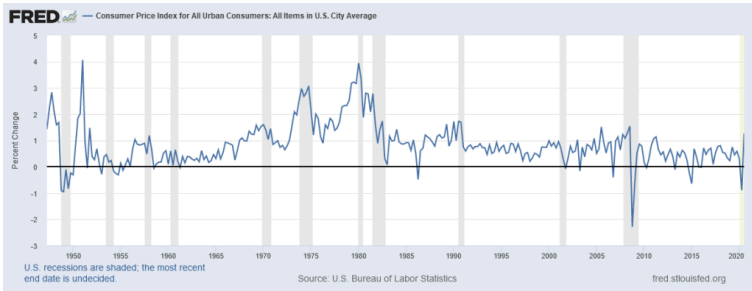
A currency's usefulness is based on what it can buy, or its **purchasing power**. The more a currency can buy, the more useful and valuable it is. When prices rise or when things cost more, purchasing power decreases; the currency buys less and its value decreases.

When the value of a currency decreases, an economy has **inflation**. Its currency has less value because it is less useful; that is, less can be bought with it. Prices are rising. It takes more units of currency to buy the same amount of goods. When the value of a currency increases, on the other hand, an economy has **deflation**. Prices are falling; the currency is worth more and buys more.

For example, say you can buy five video games for \$20. Each game is worth \$4, or each dollar buys  $\frac{1}{4}$  of a game. Then we have inflation, and prices—including the price of video games—rise. A year later you want to buy games, but now your \$20 only buys two games. Each one costs \$10, or each dollar only buys one-tenth of a game. Rising prices have eroded the purchasing power of your dollars.

If there is deflation, prices fall, so maybe a year later

you could buy ten video games with your same \$20. Now each game costs only \$2, and each dollar buys half a game. The same amount of currency buys more games: its purchasing power has increased, as has its usefulness and its value. Figure 1.1.6 shows the inflation rate for the United States.



*Figure 1.1.6: Inflation Rate in the United States, 1948-2020 (Source: US Bureau of Economic Analysis/St. Louis Federal Reserve)*

Currency instabilities can also affect investment values, because the dollars that investments return don't have the same value as the dollars that the investment was expected to return. Say you lend \$100 to your sister, who is supposed to pay you back one year from now. There is inflation, so over the next year, the value of the dollar decreases (it buys less as prices rise). Your sister does indeed pay you back on time, but now the \$100 that she gives back to you is worth less (because it buys less) than the \$100 you gave her. Your investment, although nominally returned, has lost value: you have your \$100 back, but you can't do as much with it; it is less useful.

If the value of currency—the units in which wealth is measured and stored—is unstable, then investment returns are harder to predict. In those circumstances, investment involves more risk. Both inflation and

deflation are currency instabilities that are troublesome for an economy and also for the financial planning process. An unstable currency affects the value or purchasing power of income. Price changes affect consumption decisions, and changes in currency value affect investing decisions.

It is human nature to assume that things will stay the same, but financial planning must include the assumption that over a lifetime you will encounter and endure economic cycles. You should try to anticipate the risks of an economic downturn and the possible loss of wage income and/or investment income. At the same time, you should not assume or rely on the windfalls of an economic expansion.

*One Minute Economics: Inflation Explained in One Minute.*



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## 1.2 TERMINOLOGY

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### INCOME AND EXPENSES<sup>1</sup>

Personal finance is the process of paying for or financing a life and a way of living. Just as a business must be financed—its buildings, equipment, use of labor and materials, and operating costs must be paid for—so must a person's possessions and living expenses. Just as a business relies on its revenues from selling goods or services to finance its costs, so a person relies on income earned from selling labor or capital to finance costs. You need to understand this financing process and the terms used to describe it. In the next chapter, you'll look at how to account for it.

### WHERE DOES INCOME COME FROM?

**Income** is what is earned or received in a given period.

1. Adapted from [2.1 Income and Expenses](#) in [Personal Finance](#) by Lumen Learning shared under a [CC BY-NC-SA license](#).

There are various terms for income because there are various ways of earning income. Income from employment or self-employment is wages or salary. Deposit accounts, like savings accounts, earn interest, which could also come from lending. Owning stock entitles the shareholder to a dividend, if there is one. Owning a piece of a partnership or a privately held corporation entitles one to a draw.

The two fundamental ways of earning income in a market-based economy are by selling labor or selling capital. Selling labor means working, either for someone else or for yourself. Income comes in the form of a paycheck. Total compensation may include other benefits, such as retirement contributions, health insurance, or life insurance. Labor is sold in the labor market.

Selling capital means investing: taking excess cash and selling it or renting it to someone who needs **liquidity** (access to cash). Lending is renting out capital; the interest is the rent. You can lend privately by direct arrangement with a borrower, or you can lend through a public debt exchange by buying corporate, government, or government agency bonds. Investing in or buying corporate stock is an example of selling capital in exchange for a share of the company's future value.

You can invest in many other kinds of assets, like antiques, art, coins, land, or commodities such as soybeans, live cattle, platinum, or light crude oil. The principle is the same: investing is renting capital or selling it for an asset that can be resold later, or that can create future income, or both. Capital is sold in the capital market and lent in the credit market—a specific part of the capital market (just like the dairy section is a specific

part of the supermarket). Figure 1.3.1 shows different sources of income.

	Work	Invest	Lend
Trade	Sell Labor	Sell Capital	Rent Capital
Return/ Income	Wages or Salary	Profit or Dividend Capital Gain (Loss)	Interest
Market	Labor Market	Capital Market	Credit Market

Figure 1.2.1: Sources of Income

In the labor market, the price of labor is the wage that an employer (buyer of labor) is willing to pay to the employee (seller of labor). For any given job, that price is determined by many factors. The nature of the work defines the education and skills required, and the price may reflect other factors as well, such as the status or desirability of the job.

In turn, the skills needed and the attractiveness of the work determine the supply of labor for that particular job—the number of people who could and would want to do the job. If the supply of labor is greater than the demand, if there are more people to work at a job than are needed, then employers will have more hiring choices. That labor market is a buyers’ market, and the buyers can hire labor at lower prices. If there are fewer people willing and able to do a job than there are jobs, then that labor market is a sellers’ market, and workers can sell their labor at higher prices.

Similarly, the fewer skills required for the job, the more people there will be who are able to do it, creating a



buyers' market. The more skills required for a job, the fewer people there will be to do it, and the more leverage or advantage the seller has in negotiating a price. People pursue education to make themselves more highly skilled and therefore able to compete in a sellers' labor market.

When you are starting your career, you are usually in a buyers' market (unless you have some unusual gift or talent), if only because of your lack of experience. As your career progresses, you have more, and perhaps more varied, experience and presumably more skills, and so can sell your labor in more of a sellers' market. You may change careers or jobs more than once, but you would hope to be doing so to your advantage, that is, always to be gaining bargaining power in the labor market.

Many people love their work for many reasons other than the pay, however, and choose it for those rewards. Labor is more than a source of income; it is also a source of many intellectual, social, and other personal gratifications. Your labor nevertheless is also a tradable commodity and has a market value. The personal rewards of your work may ultimately determine your choices, but you should be aware of the market value of those choices as you make them.

Your ability to sell labor and earn income reflects your situation in your labor market. Earlier in your career, you can expect to earn less than you will as your career progresses. Most people would like to reach a point where they don't have to sell labor at all. They hope to retire someday and pursue other hobbies or interests. They can retire if they have alternative sources of income—if they can earn income from savings and from selling capital.

*Khan Academy: Anatomy of a Paycheck*



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Capital markets exist so that buyers can buy capital. Businesses always need capital and have limited ways of raising it. Sellers and lenders (investors), on the other hand, have many more choices of how to invest their excess cash in the capital and credit markets, so those markets are much more like sellers' markets. The following are examples of ways to invest in the capital and credit markets:

- Buying stocks
- Buying government or corporate bonds
- Lending a mortgage

The market for any particular investment or asset may be a sellers' or buyers' market at any particular time, depending on economic conditions. For example, the market for real estate, modern art, sports memorabilia, or vintage cars can be a buyers' market if there are more sellers than buyers. Typically, however, there is as much or more demand for capital as there is supply. The more capital you have to sell, the more ways you can sell it to more kinds of buyers, and the more those buyers may be willing to pay. At first, however, for most people, selling labor is their only practical source of income.

## WHERE DOES INCOME GO?

**Expenses** are costs for items or resources that are used up or consumed in the course of daily living. Expenses recur (i.e., they happen over and over again) because food, housing, clothing, energy, and so on are used up on a daily basis.

When income is less than expenses, you have a **budget deficit**—too little cash to provide for your wants or needs. A budget deficit is not sustainable; it is not financially viable. The only choices are to eliminate the deficit by (1) increasing income, (2) reducing expenses, or (3) borrowing to make up the difference. Borrowing may seem like the easiest and quickest solution, but borrowing also increases expenses, because it creates an additional expense: interest. Unless income can also be increased, borrowing to cover a deficit will only increase it.

Better, although usually harder, choices are to increase income or decrease expenses.

When income for a period is greater than expenses, there is a **budget surplus**. That situation is sustainable and remains financially viable. You could choose to decrease income by, say, working less. More likely, you would use the surplus in one of two ways: consume more or save it. If consumed, the income is gone, although presumably you enjoyed it.

If saved, however, the income can be stored, perhaps in a piggy bank or cookie jar, and used later. A more profitable way to save is to invest it in some way—deposit in a bank account, lend it with interest, or trade it for an asset, such as a stock or a bond or real estate. Those ways of saving are ways of selling your excess capital in the

capital markets to increase your wealth. The following are examples of savings:

1. Depositing into a statement savings account at a bank
2. Contributing to a retirement account
3. Purchasing a certificate of deposit (CD)
4. Purchasing a government savings bond
5. Depositing into a money market account

## OPPORTUNITY COSTS AND SUNK COSTS

There are two other important kinds of costs aside from expenses that affect your financial life. Suppose you can afford a new jacket or new boots, but not both, because your resources—the income you can use to buy clothing—are limited. If you buy the jacket, you cannot also buy the boots. Not getting the boots is an **opportunity cost** of buying the jacket; it is cost of sacrificing your next best choice.

In personal finance, there is always an opportunity cost. You always want to make a choice that will create more value than cost, and so you always want the opportunity cost to be less than the benefit from trade.

You bought the jacket instead of the boots because you decided that having the jacket would bring more benefit than the cost of not having the boots. You believed your benefit would be greater than your opportunity cost.

In personal finance, opportunity costs affect not only consumption decisions but also financing decisions, such as whether to borrow or to pay cash. Borrowing has obvious costs, whereas paying with your own cash or savings seems costless. Using your cash does have an opportunity cost, however. You lose whatever interest you may have had on your savings, and you lose liquidity—that is, if you need cash for something else, like a better choice or an emergency, you no longer have it and may even have to borrow it at a higher cost.

When buyers and sellers make choices, they weigh opportunity costs, and sometimes regret them, especially when the benefits from trade are disappointing. Regret can color future choices. Sometimes regret can keep us from recognizing **sunk costs**.

Sunk costs are costs that have already been spent; that is, whatever resources you traded are gone, and there is no way to recover them. Decisions, by definition, can be made only about the future, not about the past. A trade, when it's over, is over and done, so recognizing that sunk costs are truly sunk can help you make better decisions.

For example, the money you spent on your jacket is a sunk cost. If it snows next week and you decide you really do need boots, too, that money is gone, and you cannot use it to buy boots. If you really want the boots, you will have to find another way to pay for them.

Unlike a price tag, opportunity cost is not obvious. You tend to focus on what you are getting in the trade, not on what you are *not* getting. This tendency is a cheerful

aspect of human nature, but it can be a weakness in the kind of strategic decision making that is so essential in financial planning. Human nature also may make you focus too much on sunk costs, but all the relish or regret in the world cannot change past decisions. Learning to recognize sunk costs is important in making good financial decisions.

*Mindful Thinks: Sunk Cost Fallacy and Why You Should Quit*



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## ASSETS<sup>2</sup>

As defined earlier in this chapter, an asset is any item with economic value that can be converted to cash. Assets are

2. Adapted from [2.2 Assets](#) in [Personal Finance](#) by Lumen Learning shared under a [CC BY-NC-SA](#) license.

resources that can be used to create income or reduce expenses and to store value. The following are examples of tangible (material) assets:

- Car
- Savings account
- Wind-up toy collection
- Money market account
- Shares of stock
- Forty acres of farmland
- Home

When you sell excess capital in the capital markets in exchange for an asset, it is a way of storing wealth, and hopefully of generating income as well. The asset is your investment—a use of your liquidity. Some assets are more liquid than others. For example, you can probably sell your car more quickly than you can sell your house. As an investor, you assume that when you want your liquidity back, you can sell the asset. This assumes that it has some liquidity and market value (some use and value to someone else) and that it trades in a reasonably efficient market. Otherwise, the asset is not an investment, but merely a possession, which may bring great happiness but will not serve as a store of wealth.

Assets may be used to store wealth, create income, and reduce future expenses.

## ASSETS STORE WEALTH

If the asset is worth more when it is resold than it was when it was bought, then you have earned a **capital gain**: the investment has not only stored wealth but also increased it. Of course, things can go the other way too: the investment can decrease in value while owned and be worth less when resold than it was when bought. In that case, you have a **capital loss**. The investment not only did not store wealth, it lost some.

The better investment asset is the one that increases in value—creates a capital gain—during the time you are storing it.

## ASSETS CREATE INCOME

Some assets not only store wealth but also create income. An investment in an apartment house stores wealth and creates rental income, for example. An investment in a share of stock stores wealth and also perhaps creates dividend income. A deposit in a savings account stores wealth and creates interest income.

Some investors care more about increasing asset value than about income. For example, an investment in a share of corporate stock may produce a dividend, which is a share of the corporation's profit, or the company may



keep all its profit rather than pay dividends to shareholders. Reinvesting that profit in the company may help the company to increase in value. If the company increases in value, the stock increases in value, increasing investors' wealth. Further, increases in wealth through capital gains are taxed differently than income, making capital gains more valuable than an increase in income for some investors.

On the other hand, other investors care more about receiving income from their investments. For example, retirees who no longer have employment income may be relying on investments to provide income for living expenses. Being older and having a shorter horizon, retirees may be less concerned with growing wealth than with creating income.

## ASSETS REDUCE EXPENSES

Some assets are used to reduce living expenses. Purchasing an asset and using it may be cheaper than arranging for an alternative. For example, buying a car to drive to work may be cheaper, in the long run, than renting one or using public transportation. The car typically will not increase in value, so it cannot be expected to be a store of wealth; its only role is to reduce future expenses.

Sometimes an asset may be expected to both store wealth and reduce future expenses. For example, buying a house to live in may be cheaper, in the long run, than renting one. In addition, real estate may appreciate in value, allowing you to realize a gain when you sell the asset. In this case, the house has effectively stored wealth. Appreciation in value depends on the real estate market and demand for housing when the asset is sold, however, so you cannot count on it. Still, a house usually can reduce living expenses and be a potential store of wealth. Figure 1.2.2 summarizes different types of assets that households typically hold.

Asset	Reduce Expenses	Increase Income	Store Wealth
Car	Yes	No	No
Savings Account	No	Yes	Yes
Money Market Account	No	Yes	Yes
Home	Yes	No	Yes
Rental Property	No	Yes	Yes
Investment in Bonds	No	Yes	Yes
Investment in Stocks	No	Yes	Yes

Figure 1.2.2: Uses of Assets

The choice of investment asset, then, depends on your belief in its ability to store and increase wealth, create income, or reduce expenses. Ideally, your assets will store and increase wealth while increasing income or reducing expenses. Otherwise, acquiring the asset will not be a productive use of liquidity. Also, in that case the opportunity cost will be greater than the benefit from the investment, since there are many assets to choose from.

*Khan Academy: Should I Spend More Money on Experiences or Things?*



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## DEBT AND EQUITY<sup>3</sup>

Buying capital, that is, borrowing enables you to invest without first owning capital. By using other people's money to finance the investment, you get to use an asset before actually owning it, free and clear, assuming you can repay out of future earnings.

Borrowing capital has costs, however, so the asset will have to increase wealth, increase earnings, or decrease expenses enough to compensate for its costs. In other words, the asset will have to be more productive to earn enough to cover its financing costs—the cost of buying or borrowing capital to buy the asset.

3. Personal Finance. **Provided by:** Saylor Academy. **Located at:** [https://saylordotorg.github.io/text\\_personal-finance](https://saylordotorg.github.io/text_personal-finance). **License:** [CC BY-NC-SA: Attribution-NonCommercial-ShareAlike](#)

Buying capital gives you equity, borrowing capital gives you debt, and both kinds of financing have costs and benefits. When you buy or borrow liquidity or cash, you become a buyer in the capital market.

## THE COSTS OF DEBT AND EQUITY

You can buy capital from other investors in exchange for an ownership share or **equity**, which represents your claim on any future gains or future income. If the asset is productive in storing wealth, generating income, or reducing expenses, the equity holder or shareholder or owner enjoys that benefit in proportion to the share of the asset owned. If the asset actually loses value, the owner bears a portion of the loss in proportion to the share of the asset owned. The **cost of equity** is in having to share the benefits from the investment.

For example, in 2004 Google, a company that produced a very successful Internet search engine, decided to buy capital by selling shares of the company (shares of stock or equity securities) in exchange for cash. Google sold over 19 million shares for a total of \$1.67 billion. Those who bought the shares were then owners or shareholders of Google, Inc. Each shareholder has equity in Google, and as long as they own the shares they will share in the

profits and value of Google, Inc. The original founders and owners of Google, Larry Page and Sergey Brin, have since had to share their company's gains (or income) or losses with all those shareholders. In this case, the cost of equity is the minimum rate of return Google must offer its shareholders to compensate them for waiting for their returns and for bearing some **risk** that the company might not do as well in the future.

Borrowing is renting someone else's money for a period of time, and the result is **debt**. During that period of time, rent or **interest** must be paid, which is a **cost of debt**. When that period of time expires, all the capital (the **principal** amount borrowed) must be given back. The investment's earnings must be enough to cover the interest, and its growth in value must be enough to return the principal. Thus, debt is a liability, an obligation for which the borrower is liable.

In contrast, the cost of equity may need to be paid only if there is an increase in income or wealth, and even then can be deferred. So, from the buyer's point of view, purchasing liquidity by borrowing (debt) has a more immediate effect on income and expenses. Interest must be added as an expense, and repayment must be anticipated.

## THE USES OF DEBT AND

## EQUITY

Debt is a way to make an investment that could not otherwise be made, to buy an asset (e.g., house, car, corporate stock) that you couldn't buy without borrowing. If that asset is expected to provide enough benefit (i.e., increase value or create income or reduce expense) to compensate for its additional costs, then the debt is worth it. However, if debt creates additional expense without enough additional benefit, then it is not worth it. The trouble is, while the costs are usually known up front, the benefits are not. That adds a dimension of risk to debt, which is another factor in assessing whether it's desirable.

For example, after the housing boom began to go bust in 2008, homeowners began losing value in their homes as housing prices dropped. Some homeowners are in the unfortunate position of owing more on their mortgage than their house is currently worth. The costs of their debt were knowable upfront, but the consequences—the house losing value and becoming worth less than the debt—were not.

Debt may also be used to cover a budget deficit, or the excess of expenses over income. As mentioned previously, however, in the long run the cost of the debt will increase expenses that are already too big, which is what created the deficit in the first place. Unless income can also be increased, debt can only aggravate a deficit.

## THE VALUE OF DEBT

The value of debt includes the benefits of having the asset sooner rather than later, something that debt financing enables. For example, many people want to buy a house when they have children, perhaps because they want bedrooms and bathrooms and maybe a yard for their children. Not far into adulthood, would-be homebuyers may not have had enough time to save enough to buy the house outright, so they borrow to make up the difference. Over the length of their mortgage (real estate loan), they pay the interest.

The alternative would be to rent a living space. If the rent on a comparable home were more than the mortgage interest (which it often is, because a landlord usually wants the rent to cover the mortgage *and* create a profit), it would make more sense, if possible, to borrow and buy a home and be able to live in it. And, extra bedrooms and bathrooms and a yard are valuable while children are young and live at home. If you wait until you have saved enough to buy a home, you may be much older, and your children may be off on their own.

Another example of the value of debt is using debt to finance an education. Education is valuable because it has many benefits that can be enjoyed over a lifetime. One benefit is an increase in potential earnings in wages and salaries. Demand for the educated or more skilled employee is generally greater than for the uneducated or less-skilled employee. So education creates a more valuable and thus higher-priced employee.

It makes sense to be able to maximize value by becoming educated as soon as possible so that you have as long as possible to benefit from increased income. It



even makes sense to invest in an education before you sell your labor because your opportunity cost of going to school—in this case, the “lost” wages of not working—is lowest. Without income or savings (or very little) to finance your education, typically, you borrow. Debt enables you to use the value of the education to enhance your income, out of which you can pay back the debt.

The alternative would be to work and save and then get an education, but you would be earning income less efficiently until you completed your education, and then you would have less time to earn your return. Waiting decreases the value of your education, that is, its usefulness, over your lifetime.

In these examples seen in Figure 1.2.3, debt creates a cost, but it reduces expenses or increases income to offset that cost. Debt allows this to happen sooner than it otherwise could, which allows you to realize the maximum benefit for the investment. In such cases, debt is “worth” it.

Debt	Debt Used to Finance	Value	Cost Paid from
Credit Cards	Living Expenses	Convenience	Income
Auto Loan	Car	Reduce Expenses	Income
Mortgage	Home	Reduce Expenses	Income
College Loan	Education	Increase (Future) Income	Future Income

Figure 1.2.3: Types of Debt

*Khan Academy: The Economics Behind Living Paycheck to Paycheck*



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## INCOME AND RISK<sup>4</sup>

Personal finance is not just about getting what you want; it is also about protecting what you have. Since the way to accumulate assets is to create surplus capital by having an income larger than expenses, and since you rely on income to provide for living expenses, you also need to think about protecting your income. One way to do so is through **diversification**, or spreading the risk.

You already know not to put all your eggs in one basket, because if something happens to that basket, all the eggs are gone. If the eggs are in many baskets, on the other hand, the loss of any one basket would mean the loss of just a fraction of the eggs. The more baskets, the smaller

4. Adapted from [2.4 Income and Risk](#) in [Personal Finance](#) by Lumen Learning shared under a [CC BY-NC-SA](#) license.

your proportional loss would be. Then if you put many different baskets in many different places, your eggs are diversified even more effectively, because all the baskets aren't exposed to the same environmental or systematic risks.

Diversification is more often discussed in terms of investment decisions, but diversification of sources of income works the same way and makes the same kind of sense for the same reasons. If sources of income are diverse—in number and kind—and one source of income ceases to be productive, then you still have others to rely on.

If you sell your labor to only one buyer, then you are exposed to more risk than if you can generate income by selling your labor to more than one buyer. You have only so much time you can devote to working, however. Having more than one employer could be exhausting and perhaps impossible. Selling your labor to more than one buyer also means that you are still dependent on the labor market, which could suffer from an economic cycle such as a recession affecting many buyers (employers).

Mark, for example, works as a school counselor, tutors on the side, paints houses in the summers, and buys and sells sports memorabilia on the Internet. If he got laid off from his counseling job, he would lose his paycheck but still be able to create income by tutoring, painting, and trading memorabilia.

Similarly, if you sell your capital to only one buyer—invest in only one asset—then you are exposed to more risk than if you generate income by investing in a variety of assets. Diversifying investments means you are dependent on trade in the capital markets, however,

which likewise could suffer from unfavorable economic conditions.

Mark has a checking account, an online money market account, and a balanced portfolio of stocks. If his stock portfolio lost value, he would still have the value in his money market account.

A better way to diversify sources of income is to sell both labor *and* capital. Then you are trading in different markets, and are not totally exposed to risks in either one. In Mark's case, if all his incomes dried up, he would still have his investments, and if all his investments lost value, he would still have his paycheck and other incomes. To diversify to that extent, you need surplus capital to trade. This brings us full circle to Adam Smith, quoted at the beginning of this chapter, who said, essentially, "It takes money to make money."

## 1.3 BALANCE SHEETS

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### BALANCE SHEET<sup>1</sup>

In business or in personal finance, a critical piece in assessing the current situation is the balance sheet. Often referred to as the “statement of financial condition,” the **balance sheet** is a snapshot of what you have and what you owe at a given point in time. Unlike the income or cash flow statements, it is not a record of performance over a period of time, but simply a statement of where things stand at a certain moment.

The balance sheet is a list of assets, debts or liabilities, and equity or net worth, with their values. In business, assets are resources that can be used to create income, while debt and equity are the capital that financed those assets. Thus, the value of the assets must equal the value of the debt and the equity. In other words, the value of the business’s resources must equal the value of the capital it borrowed or bought in order to get those resources.

**assets = liabilities + equity**

In business, the **accounting equation** is as absolute as the law of gravity. It simply must always be true, because if there are assets, they must have been financed somehow—either through debt or equity. The value of that debt and equity financing must equal or balance the value of the assets it bought. Thus, it is called the

1. Adapted from 3.1 Accounting and Financial Statements in Personal Finance by Lumen Learning shared under a CC BY-NC-SA license.

“balance” sheet because it *always* balances the debt and equity with the value of the assets.

In personal finance, assets are also things that can be sold to create liquidity. Liquidity is needed to satisfy or repay debts. Because your assets are what you use to satisfy your debts when they become due, the assets’ value should be greater than the value of your debts. That is, you should have more to work with to meet your obligations than you owe.

The difference between what you have and what you owe is your **net worth**. Literally, net worth is the share that you own of everything that you have. It is the value of what you have *net of* (less) what you owe to others. Whatever asset value is left over after you meet your debt obligations is your own worth. It is the value of what you have that you can claim free and clear.

$$\text{assets} - \text{debt} = \text{net worth}$$

Your net worth is really your equity or financial ownership in your own life. Here, too, the personal balance sheet must balance, because if

$$\text{assets} - \text{debts} = \text{net worth},$$

then it should also be

$$\text{assets} = \text{debts} + \text{net worth}.$$

Alice could write a simple balance sheet to see her current financial condition. She has two assets (her car and her savings account), and she has two debts (her car and student loans). We show this below in Table 1.3.1.

**Table 1.3.1: Alice's Balance Sheet**

Asset	Amount	Liability	Amount
Car	5,000	Car Loan	2,700
Savings	250	Student Loan	53,000
Total	5,250	Total	55,700
		Net Worth	(50,450)

Alice's balance sheet presents her with a much clearer picture of her financial situation, but also with a dismaying prospect: she seems to have negative net worth. **Negative net worth** results whenever the value of debts or liabilities is actually greater than the assets' value. If

**liabilities < assets** then **assets – liabilities > 0; net worth > 0** (net worth is positive)

If

**liabilities > assets** then **assets – liabilities < 0; net worth < 0** (net worth is negative)

Negative net worth implies that the assets don't have enough value to satisfy the debts. Since debts are obligations, this would cause some concern.

## NET WORTH AND BANKRUPTCY

In business, when liabilities are greater than the assets

to meet them, the business has negative equity and is literally bankrupt. In that case, it may go out of business, selling all its assets and giving whatever it can to its **creditors** or lenders, who will have to settle for less than what they are owed. More usually, the business continues to operate in bankruptcy, if possible, and must still repay its creditors, although perhaps under somewhat easier terms. Creditors (and the laws) allow these terms because creditors would rather get paid in full later than get paid less now or not at all.

In personal finance, personal **bankruptcy** may occur when debts are greater than the value of assets. But because creditors would rather be paid eventually than never, the bankrupt is usually allowed to continue to earn income in the hopes of repaying the debt later or with easier terms. Often, the bankrupt is forced to liquidate (sell) some or all of its assets.

Because debt is a legal as well as an economic obligation, there are laws governing bankruptcies that differ from state to state in the United States and from country to country. Although debt forgiveness was discussed in the Old Testament, throughout history it was not uncommon for bankrupts in many cultures to be put to death, maimed, enslaved, or imprisoned. The use of another's property or wealth is a serious responsibility, so debt is a serious obligation.

However, Alice's case is actually not as dismal as it looks, because Alice has an "asset" that is not listed on her balance sheet, that is, her education. It is not listed on her balance sheet because the value of her education, like the value of any asset, comes from how useful it is, and its usefulness has not happened yet, but will happen over her lifetime. It will happen in her future, based on how



she chooses to use her education to increase her income and wealth. It is difficult to assign a monetary value to her education now. Alice knows what she paid for her education, but, sensibly, its real value is not its cost but its potential return, or what it can earn for her as she puts it to use in the future.

Current studies show that a college education has economic value, because a college graduate earns more over a lifetime than a high school graduate. Recent estimates put that difference at about \$1,000,000<sup>2</sup> So, if Alice assumes that her education will be worth \$1,000,000 in extra income over her lifetime, and she includes that asset value on her balance sheet, then her net worth is positive, just unrealized.

This looks much better, but it's not sound accounting practice to include an asset—and its value—on the balance sheet before it really exists. After all, education generally pays off, but until it does, it hasn't yet and there is a chance, however slim, that it won't for Alice. A balance sheet is a snapshot of one's financial situation at one particular time. At this particular time, Alice's education has value, but its amount is unknown.

It is easy to see, however, that the only thing that creates negative net worth for Alice is her student loan. The student loan causes her liabilities to be greater than her assets—and if that were paid off, her net worth would be positive. Given that Alice is just starting her adult earning years, her situation seems quite reasonable.

Let's try one together...

Ron is taking stock of his assets and debts and has come up with the following information. Ron has a car which could be sold for \$15,000. He still owes \$12,000 on the car. He has \$2,000 in savings and has \$3,000 in an investment account. He also still owes \$20,000 in student loan debt. He has decided not to include the value of the college degree as an asset. Create a balance sheet for Ron and determine his net worth.

Answer: See video for table; net worth =  $-\$12,000$



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## Let's try one together...

Jeremy has just graduated college and currently has \$10,000 in student loan debt. After graduating, he purchased a new Ford F150 Super Cab for \$35,000. The truck could be sold today for \$25,000 and Jeremy still owes \$31,000 on the truck. Additionally, he purchased a Polaris Sportsman 850 with low, low monthly payments. The quad could be sold today for \$6,500. He still owes \$9,000 for the remaining quad payments. Finally, he has purchased a variety of things on his credit card. These things have no significant resale value but currently owes \$7,000 in credit card debt.

Create a balance sheet for Jeremy and calculate his net worth.

Answer: See video for balance sheet; net worth = -\$25,500



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Some software/programs that may be useful for tracking your finances:

- Quicken
- Mint
- YNAB
- Mvelopes

*The Dave Ramsey Show: Good versus Bad Financial Decisions*



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## 1.4 MATH REVIEW

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### *Silly School Songs: The Order of Operations*



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For the purpose of this course, we will need to learn some basic when dealing with logarithms. We will not, however, delve too deeply into theory and will instead focus on the skills we need.

### EXPONENTIAL FUNCTIONS<sup>1</sup>

India is the second most populous country in the world with a population of about 1.25 billion people in 2013. The population is growing at a rate of about 1.2% each year[17]. If this rate continues, the population of India will exceed China's population by the year 2031. When populations grow rapidly, we often say that the growth is "exponential," meaning that something is growing very rapidly. To a mathematician, however, the term

1. Adapted from 12.1 exponential-functions in [Personal Finance](#) by lumen learning shared under a [CC BY-NC-SA 3.0](#) license

exponential growth has a very specific meaning. In this section, we will take a look at exponential functions, which model this kind of rapid growth.

When exploring linear growth, we observed a constant rate of change—a constant number by which the output increased for each unit increase in input. For example, in the equation  $f(x) = 3x + 4$ , the slope tells us the output increases by 3 each time the input increases by 1. The scenario in the India population example is different because we have a percent change per unit time (rather than a constant change) in the number of people.

What exactly does it mean to grow exponentially? What does the word double have in common with percent increase? People toss these words around errantly. Are these words used correctly? The words certainly appear frequently in the media.

- Percent change refers to a change based on a percent of the original amount.
- Exponential growth refers to an increase based on a constant multiplicative rate of change over equal increments of time, that is, a percent increase of the original amount over time.
- Exponential decay refers to a decrease based on a constant multiplicative rate of change over equal increments of time, that is, a percent decrease of the original amount over time.

For us to gain a clear understanding of exponential growth, let us contrast exponential growth with linear growth. We will construct two functions. The first function is exponential. We will start with 1 and then we will double the corresponding consecutive outputs. The

second function is linear. We will start with an input of 0 and then we will add 2 to the corresponding consecutive outputs. See Table 1.4.1.

**Table 1.4.1:**  
**Exponential versus**  
**Linear Growth**

x	$y=2x$	$y=2^x$
Start	1	0
1	2	2
2	4	4
3	8	6
4	16	8
5	32	10
6	64	12
7	128	14
8	256	16

From Table 1.8.1 we can infer that for these two functions, exponential growth dwarfs linear growth.

- Exponential growth refers to the original value from the range increases by the same percentage over equal increments found in the domain.
- Linear growth refers to the original value from the range increases by the same amount over equal increments found in the domain.

Apparently, the difference between “the same percentage” and “the same amount” is quite significant. For exponential growth, over equal increments, the constant multiplicative rate of change resulted in doubling the output whenever the input increased by one. For linear

growth, the constant additive rate of change over equal increments resulted in adding 2 to the output whenever the input was increased by one.

The general form of the exponential function is  $f(x) = ab^x$  where  $a$  is any nonzero number,  $b$  is a positive real number not equal to 1. For our class,  $b$  will always be greater than one.

Suppose that we have the following exponential equation:

$$y = 3 \times 1.05^x.$$

If I want to see what the value of  $y$  will be when  $x = 7$ , we simply plug 7 in for  $x$ . When we do that, we get:

$$y = 3 \times 1.05^7 \approx 3 \times 1.407 \approx 4.221.$$

As long as we are careful about our order of operations, this is nothing more than a calculator exercise. But what if I want to know what  $x$  will be equal to when  $y = 12$ . How can we get the  $x$  down from the exponent? For that, we will need to use the inverse of the exponential...the logarithm.

### Let's try one together...

The elk population of an area is given by the equation below where  $t$  is given in number of years:

$$P(t) = 150 \times 1.05^t.$$

Calculate the number of elk after 5 years. 10 years. 100 years.

Answers: 191; 244; 19,725





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<https://pressbooks.nsc.ca/mathofmoney/?p=35#oembed-4>

College Algebra. Provided by: [OpenStax](https://openstax.org/books/college-algebra/pages/6-3-logarithmic-functions). Located at: <https://openstax.org/books/college-algebra/pages/6-3-logarithmic-functions>

## LOGARITHMS

In order to analyze the magnitude of earthquakes or compare the magnitudes of two different earthquakes, we need to be able to convert between logarithmic and exponential form. For example, suppose the amount of energy released from one earthquake were 500 times greater than the amount of energy released from another. We want to calculate the difference in magnitude. The equation that represents this problem is  $10^x = 500$ , where  $x$  represents the difference in magnitudes on the Richter Scale. How would we solve for  $x$ ?

We have not yet learned a method for solving exponential equations. None of the algebraic tools discussed so far is sufficient to solve something like  $10^x = 500$ . We can estimate what  $x$  needs to be since  $10^2 = 100$  and  $10^3 = 1,000$  so the value of  $x$  has to fall between 2 and 3.

The exponential function  $y = b^x$  is one-to-one, so its inverse,  $x = b^y$  is also a function. As is the case with all inverse functions, we simply interchange  $x$  and  $y$  and solve for  $y$  to find the inverse function. To represent  $y$  as a function of  $x$ , we use a logarithmic function of the form  $y = \log_b(x)$ .

The base  $b$  logarithm of a number is the exponent by which we must raise  $b$  to get that number.

We read a logarithmic expression as, “The logarithm with base  $b$  of  $x$  is equal to  $y$ ,” or, simplified, “log base  $b$  of  $x$  is  $y$ .” We can also say, “ $b$  raised to the power of  $y$  is  $x$ ,” because logs are exponents. For example, the base 2 logarithm of 32 is 5, because 5 is the exponent we must apply to 2 to get 32. Since  $2^5 = 32$ , we can write  $\log_2(32) = 5$ . We read this as “log base 2 of 32 is 5.”

We can express the relationship between logarithmic form and its corresponding exponential form as follows:

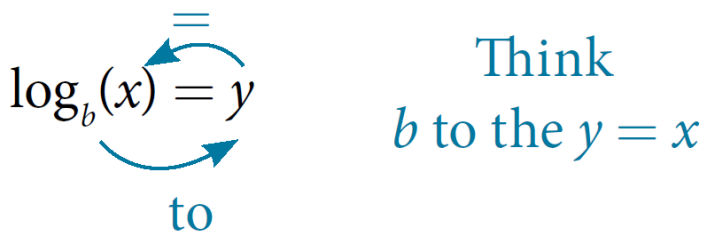


Figure 1.4.1: Explanation of the Logarithm (OpenStax: College Algebra)

### Using Your Calculator to Calculate Logarithms

College Algebra. Provided by: [OpenStax](https://openstax.org/books/college-algebra/pages/6-5-logarithmic-properties). Located at: <https://openstax.org/books/college-algebra/pages/6-5-logarithmic-properties>

Unfortunately, most calculators cannot calculate every single logarithm because there are an infinite number of

bases. However, we can use one simple rule to make it possible to calculate the value of any logarithm. Again, we will be focusing on the “how” here and not the “why”.

The most common “base” for the logarithm is base 10 which is also called the common log. On your calculator you should see some “log” button. This is (most likely) to calculate the log base 10. So for instance, if you had the following problem:

$$\log_{10}(33)$$

we would want to know what we could raise 10 to in order to get 33. If you enter  $\log(33)$  (or however your calculator has you enter it — if you are not sure, ask me!), you should get 1.5185... which means if you raise 10 to the 1.5185... power you will get 33.

But what if we want to calculate  $\log_7(41)$ ?

(Most) calculators are not equipped to handle a log of a base other than 10 (and one other that we will talk about soon), but we can use a property to help us.

An important logarithm property is the change-of-base property. This allows us to change the base of the log from one value to another. We will use this to our advantage. The change of base property is as follows:

$$\log_b(M) = \frac{\log_n M}{\log_n b}.$$

We will generally use this to convert to the common log (log base 10) so we can run it on a calculator.

When converting to base 10 (the common log), we use the following:

$$\log_b M = \frac{\log M}{\log b}.$$

Let us try a few examples!

Let's try one together...

Evaluate the following logarithms to two decimal places:

- $\log_6 31$
- $\log_9 150$
- $\log_3 81$

Answers: 1.92, 2.28, 4



One or more interactive elements has been excluded from this version of the text. You can view them online here:

<https://pressbooks.nsc.ca/mathofmoney/?p=35#oembed-5>

## SOLVING EXPONENTIAL EQUATIONS<sup>2</sup>

Now let us get back to the problem we were initially looking at. Remember we had explored the following problem:

$$y = 3 \times 1.05^x.$$

I had asked what the value of  $x$  is when  $y$  is equal to 12

. So let us plug that in for  $y$ :

$$12 = 3 \times 1.05^x.$$

Next, we divide both sides by 3:

$$\frac{12}{3} = 4 = \frac{3 \times 1.05^x}{3} = 1.05^x.$$

So, now we have

$$1.05^x = 4.$$

Where do we go next? The logarithm is the inverse

operation for the exponential. So for this problem, we need to take the log of base 1.05 of each side. On the left, we get:

$$\log_{1.05}(1.05^x) = x.$$

On the right, after using change of base, we get

$$\log_{1.05} 4 = \frac{\log 4}{\log 1.05} \approx 28.41.$$

Therefore,  $x \approx 28.41$ .

If we check our work and plug in 28.41 for  $x$  back into our original equation, we get  $y = 3 \times 1.05^{28.41} \approx 12$ .

Let us try a few examples.

Let's try one together...

Solve for  $t$  when  $y = 30$ :  $y = 7 \times 1.17^t$ .

Answer:  $t=9.269$



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<https://pressbooks.nsc.ca/mathofmoney/?p=35#oembed-1>

Let's try one together...

Solve for  $t$  when  $y = 200$ :  $y = 15 \times 1.04^t$ .

Answer:  $t=66.043$



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<https://pressbooks.nsc.ca/mathofmoney/?p=35#oembed-2>

## EXPONENTIAL GROWTH<sup>3</sup>

We now take a look at what the exponential means. Suppose that we have the equation  $y = 3 \times 1.06^t$ . This can be rewritten as  $y = 3 \times (1 + .06)^t$ . When we add something to 1, then the quantity is increasing by that amount as a percentage. For example, in our equation we can see that we have  $1 + .06$  which means that the quantity is growing by .06 or 6% per year (or time period.) A larger value means faster growth and vice versa.

We will see shortly once we start the financial math that this is where the interest rate will come into play.

If the value is less than 1, then we have decay. For example, if we have  $y = 3 \times 0.9^t$  then each year we only have 0.9 or 90 of what we had the previous year. This means that our quantity is decreasing by  $1 - 0.9$  or 0.1 or 10% each year (or time period.)

We will not use this very often (if ever) in this class.

3. New content added to *Math of Money* edition by J. Z. Klingensmith[

## 1.4 OPTIONAL VIDEOS

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### *Khan Academy: Introduction to Exponential Functions*



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<https://pressbooks.nscc.ca/mathofmoney/?p=37#oembed-1>

### *Khan Academy: Exponential versus Linear Growth*



One or more interactive elements has been excluded from this version of the text. You can view them online here:

<https://pressbooks.nscc.ca/mathofmoney/?p=37#oembed-2>

### *Khan Academy: Exponential Function Graph*



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excluded from this version of the text. You can view them online here: <https://pressbooks.nsc.ca/mathofmoney/?p=37#oembed-3>

### *Khan Academy: Intro to Logarithms*



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<https://pressbooks.nsc.ca/mathofmoney/?p=37#oembed-4>

### *Khan Academy: Relationship between Exponentials and Logarithms*



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<https://pressbooks.nsc.ca/mathofmoney/?p=37#oembed-5>

### *Khan Academy: Change of Base*





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<https://pressbooks.nsc.ca/mathofmoney/?p=37#oembed-6>

### *Khan Academy: Solving Exponential Equations*



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<https://pressbooks.nsc.ca/mathofmoney/?p=37#oembed-7>

## 1.4 PRACTICE PROBLEMS

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**Problem 1.4.1:** The number of fish in a pond is measured by the following exponential equation:

$$P(t) = 300 \times 1.04^t.$$

How many fish will be in the pond after 1 year? 5 years? 10 years? 50 years? Is this reasonable?

Answer: 312; 365; 444; 2,132



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<https://pressbooks.nsc.ca/mathofmoney/?p=39#oembed-1>

**Problem 1.4.2:** Evaluate the following logarithms. Round to two decimal places.

$$\log_2 64$$

$$\log_5 125$$

$$\log_6 100$$

$$\log_4 70$$

$$\log_8 200$$

Answers: 6; 3; 2.57; 3.07; 2.55



One or more interactive elements has been excluded from this version of the text. You can view them online here:

<https://pressbooks.nsc.ca/mathofmoney/?p=39#oembed-2>

**Problem 1.4.3:** Solve for  $t$  when  $y = 50$ :

$$y = 10 \times 1.04^t$$

Answer: 41.035



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<https://pressbooks.nsc.ca/mathofmoney/?p=39#oembed-3>

**Problem 1.4.4:** Solve for  $t$  when  $y = 200$ :

$$y = 20 \times 1.08^t$$

Answer: 29.919



One or more interactive elements has been excluded from this version of the text. You can view them online here:

<https://pressbooks.nsc.ca/mathofmoney/?p=39#oembed-4>

**Problem 1.4.5:** Solve for  $t$  when  $y = 80$ :

$$y = 5 \times 1.1^t$$

Answer: 29.090



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<https://pressbooks.nsc.ca/mathofmoney/?p=39#oembed-5>

## 1.5 THE TIME VALUE OF MONEY I: SINGLE DEPOSITS

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### THE CONCEPT OF THE TIME VALUE OF MONEY<sup>1</sup>

Part of the planning process is evaluating the possible future results of a decision. Since those results will occur some time from now (i.e., in the future), it is critical to understand how time passing may affect those benefits and costs—not only the probability of their occurrence, but also their value when they do. Time affects value because time affects liquidity.

Liquidity is valuable, and the liquidity of an asset affects its value: all things being equal, the more liquid an asset is, the better. This relationship—how the passage of time affects the liquidity of money and thus its value—is commonly referred to as the **time value of money**, which can actually be calculated concretely as well as understood abstractly.

In general, transforming not-so-liquid wealth into liquid wealth creates transaction costs, opportunity costs, and risk, all of which take away from the value of wealth. Liquidity has value because it can be used without any additional costs.

One dimension of difference between not-so-liquid wealth and liquidity is time. Cash flows (CF) in the past

1. Adapted from [4.1 The Time Value of Money](#) in [Personal Finance](#) by Lumen Learning shared under a [CC BY-NC-SA](#) license

are sunk, cash flows in the present are liquid, and cash flows in the future are not yet liquid. You can only make choices with liquid wealth, not with cash that you don't have yet or that has already been spent. Separated from your liquidity and your choices by time, there is an opportunity cost: if you had liquidity now, you could use it for consumption or investment and benefit from it now. There is also risk, as there is always some uncertainty about the future: whether or not you will actually get your cash flows and just how much they'll be worth when you do.

The further in the future cash flows are, the farther away you are from your liquidity, the more opportunity cost and risk you have, and the more that takes away from the present value (PV) of your wealth, which is not yet liquid. In other words, time puts distance between you and your liquidity, and that creates costs that take away from value. The more time there is, the larger its effect on the value of wealth.

Financial plans are expected to happen in the future, so financial decisions are based on values some distance away in time. You could be trying to project an amount at some point in the future—perhaps an investment payout or college tuition payment. Or perhaps you are thinking about a series of cash flows that happen over time—for example, annual deposits into and then withdrawals from a retirement account. To really understand the time value of those cash flows, or to compare them in any reasonable way, you have to understand the relationships between the nominal or face values in the future and their equivalent, present values (i.e., what their values would be if they were liquid today). The equivalent present values today will be less than the nominal or face values in the

future because that distance over time, that separation from liquidity, costs us by discounting those values.

Financial calculation is not often a necessary skill since it is easier to use calculators, spreadsheets, and software. However, understanding the calculations is important in understanding the relationships between time, risk, opportunity cost, and value.

To do the math, you need to know

- what the future cash flows (CF) will be,
- when the future cash flows will be,
- the rate at which time affects value (e.g., the costs per time period, or the magnitude [the size or amount] of the effect of time on value).

At times, the alternatives are clear: you could be putting the liquidity in an account earning 3 percent, so that's your opportunity cost of not having it. Or you are paying 6.5 percent on a loan, which you wouldn't be paying if you had enough liquidity to avoid having to borrow; so that's your opportunity cost. Sometimes, however, your opportunity cost is not so clear.

Say that today is your twentieth birthday. Your grandparents have promised to give you \$1,000 for your twenty-first birthday, one year from today. If you had the money today, what would it be worth? That is, how much would \$1,000 worth of liquidity one year from now be worth today?

That depends on the cost of its not being liquid today, or on the opportunity costs and risks created by not having liquidity today. If you had \$1,000 today, you could buy things and enjoy them, or you could deposit it in an interest-bearing account. So on your twenty-first

birthday, you would have more than \$1,000. You would have the \$1,000 plus whatever interest it had earned. If your bank pays 4 percent per year (interest rates are always stated as annual rates) on your account, then you would earn \$40 of interest in the next year, or  $\$1,000 \times .04$ . So on your twenty-first birthday you would have \$1,040.

$$\$1,000 + (1,000 \times .04) = \$1,000 + \$40 = \$1,040.$$

If you left that amount in the bank until your twenty-second birthday, you would have

$$\$1,040 + (1,040 \times .04) = \$1,040 + \$41.60 = \$1,081.60.$$

To generalize the computation, if your **present value**, or PV, is your value today,  $r$  is the rate at which time affects value or discount rate (in this case, your interest rate), and if  $t$  is the number of time periods between you and your liquidity, then the **future value**, or FV, for each year is shown below in Table 1.5.1.

Today	Interest Rate	Time (years)	One Year from Now
1,000	0.04	1	$1,040 = 1,000 \times (1 + 0.04)^1$
1,000	0.04	2	$1,081.60 = 1,000 \times (1 + 0.04)^2$
PV	$r$	$t$	$FV = PV \times (1 + r)^t$

Table 1.5.1: Future Value

## CALCULATING THE FUTURE VALUE OF MONEY<sup>2</sup>

Savings instruments in which earnings are continually reinvested, such as mutual funds and retirement accounts, use compound interest. The term

2. College Algebra. Provided by: [OpenStax](https://openstax.org/books/college-algebra/pages/6-1-exponential-functions). Located at: <https://openstax.org/books/college-algebra/pages/6-1-exponential-functions>



compounding refers to interest earned not only on the original value, but on the accumulated value of the account.

The annual percentage rate (APR) of an account, also called the nominal rate, is the yearly interest rate earned by an investment account. The term nominal is used when the compounding occurs a number of times other than once per year. In fact, when interest is compounded more than once a year, the effective interest rate ends up being greater than the nominal rate! This is a powerful tool for investing.

We can calculate the future value using the following formula:

$$FV = PV \left(1 + \frac{r}{n}\right)^{nt}.$$

In this equation,  $PV$  is the present value of money. We generally view this as the “starting” amount or initial investment though it does not always have to represent the starting value.  $FV$  is the future value of the money. In other words, how much is the  $PV$  worth after  $t$  years. The variable  $r$  is the annual interest rate given as a decimal. So, for instance, if the annual interest rate is 7 we would use 0.07. Finally,  $n$  is the number of compounding periods per year; that is, how many times to we add interest to the account each year? I discuss the concept of compounding periods below.

### Compounding Periods<sup>3</sup>

While the other variables are relatively straightforward, the concept of compounding periods tends to be a new concept for many students. Therefore, I discuss it more here.

3. New content added to Math of Money edition by J. Z. Klingensmith[

If you have an account that pays 12% interest per year, do you receive a single 12% interest payment each year? Maybe. Maybe not. That would be part of the agreement that you have with the bank. What if you are paid quarterly (once every three months?) Instead of getting a single 12% interest payment, you would instead get four 3% interest payments. Or, what if you get interest once per month? In that case, you would get 12 1% interest payments. We account for this in the future value using the variable  $n$ . The table below gives the compounding periods for different time periods. Simply put...how many times is interest accounted for each year?

**Table 1.5.2: Compounding Periods**

Compounding Periods	$n$
Annual	1
Quarterly	4
Monthly	12
Daily	365

We see the term in the equation is  $\frac{r}{n}$ . So essentially what we calculating the interest *per period*. Table 1.5.3 shows the interest paid per period if the annual interest rate is 12%.

**Table 1.5.3: Interest Paid per Period (12% rate)**

Compounding Periods	$n$	$r/n$
Annual	1	$12/1=12\%$
Quarterly	4	$12/4=3\%$
Monthly	12	$12/12=1\%$
Daily	365	$12/365=.033\%$

The final part of the equation is the exponent  $nt$ . While it is fine to just plug the value in, we can see that we are actually doing a per period calculation and not a per year calculation. For example, if we have a five-year period we are examining and interest compounds monthly, then there are  $(12)(5)$  or 60 periods (since there are sixty months in five years.)

While we could easily talk more about the concepts, it is easiest to just jump into the problems!

## SOLVING TIME VALUE OF MONEY QUESTIONS<sup>4</sup>

### Finding Future Value

For these, we simply plug in the required values and solve. Remember you order of operations! Do the inside of the parentheses first, then the exponent, then multiply by the value out in front.

Let's try one together...

You deposit \$25,000 into a savings account that pays 8% per year compounded monthly. Calculate the balance after 15 years assuming no additional deposits are made.

Answer: 82,673



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## Let's try one together...

You borrow \$50,000 at a rate of 4% per year compounded monthly. As part of the agreement, you do not have to make any payments for 3 years but interest will accrue during that time. How much will you owe after 3 years assuming you make no payments on the money you borrowed.

Answer: 56,364



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<https://pressbooks.nsc.ca/mathofmoney/?p=42#oembed-5>

## Finding Present Value

This is also algebraic. Because present value is only multiplied by the larger term, all we need to do is plug in

future value and the numbers into the larger multiplier term and then divide.

### Let's try one together...

Bob has a savings account currently worth \$35,440.63. He made one initial deposit five years ago. The account pays 7% interest compounded monthly. How much did Bob initially deposit?

Answer: 25,000



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### Let's try one together...

Tianyue currently owes \$53,933.94 from a loan she took out 10 years ago. She has yet to make a payment on the loan. She received the proceeds of the loan in one single disbursement. The loan comes at a rate of 3% and is compounded quarterly. What is the principal of the loan?

Answer: 40,000



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<https://pressbooks.nsc.ca/mathofmoney/?p=42#oembed-6>

## Finding Interest Rate

To find the number of years, we will need to use fractional exponents. We will not go over this other than how to input it into our calculators. For these problems, we have three steps:

1. Divide both sides by the future value.
2. Take the reciprocal root. For example, if the parentheses are raised to the 10th power, you will raise both sides to the  $1/10$  power.
3. Subtract by 1.
4. Multiply by  $n$ .

Again, it is much easier to see how to do this through an example.

Let's try one together...

Farouq has a savings account. He initially deposited \$

10,000 and today it is worth \$13,770.79. He made the initial (and only) deposit 4 years ago. He is not quite sure what the interest rate is, but he does know that the interest is compounded daily. What is the annual interest rate on the account?

Answer: 8%



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## Let's try one together...

You deposit \$7,500 into an account. After 10 years, the account is worth \$44,187.02. If the interest compounds quarterly, calculate the annual interest rate.

Answer: 18.13%



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## Finding the Number of Years

If we recall the formula from earlier, we see that the  $t$  variable which represents the number of years is in the exponent. As we saw in the previous section, we need to use logs to solve for a variable in the exponent. The steps for this process are:

1. Divide both sides by the present value.
2. Take the log of both sides with the base being the exponent.
3. Evaluate the innards of the multiplier term.
4. Divide both sides by the result of the innards.

Again, it is much easier to see how to do this through an example.

### Let's try one together...

Francisca deposits \$15,000 into an account. After a certain number of years, the account is worth \$220,959. If the account pays 9% compounded monthly, how long has the money been in the account?

Answer: 30 years



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<https://pressbooks.nsc.ca/mathofmoney/?p=42#oembed-8>

## Let's try one together...

You took out \$50,000 of debt several years ago. The debt was issued at a rate of 6% per year compounded annually. You have not made any payments since you borrowed the money and now owe \$79,692.40. How long has it been since you borrowed the money?

Answer: 8 years

<https://youtu.be/3q5Da45Ip18>



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<https://pressbooks.nsc.ca/mathofmoney/?p=42#oembed-3>

## Finding the Number of Compounding Periods

Good news...we will not solve for these!

## DISCRETE VERSUS CONTINUOUS GROWTH

In the previous sections we examined discrete growth. Discrete periods are countable periods such as years, quarters, months, days, or even minutes or seconds. But what if we split time into smaller and smaller units like milliseconds, nanoseconds, or the newly measured zeptosecond (measured by the Germans this year) which is 0.00000000000000000001 second. Eventually we move out of the world of discrete math and into the world of continuous math where there are no longer separate time units and instead one single continuum of time. If you have had calculus, this is where the concept of the infinitesimal comes from!

Due to its limited use in real world personal finance, we will not go any further with the continuous space. If we did, we would talk about the number “e”. You may have seen this discussed in math classes, but we will not cover it here.

Let us imagine that we have a single dollar that we place into a savings account. The account pays 100% (remember, as a decimal, 100%=1.0) annual interest. We want to know how much the account will be worth after 1 year. Let us take a look at the value of the account using different compounding periods:

*Compounds Annually ( $n = 1$ )*

$$FV = 1 \left(1 + \frac{1}{1}\right)^{(1)(1)} = \$2$$

*Compounds Semi-Annually ( $n = 2$ )*

$$FV = 1 \left(1 + \frac{1}{2}\right)^{(2)(1)} = \$2.25$$

*Compounds Quarterly ( $n = 4$ )*

$$FV = 1 \left(1 + \frac{1}{4}\right)^{(4)(1)} \approx \$2.44$$

*Compounds Monthly ( $n = 12$ )*

$$FV = 1 \left(1 + \frac{1}{12}\right)^{(12)(1)} \approx \$2.61$$

*Compounds Daily* ( $n = 365$ )

$$FV = 1 \left(1 + \frac{1}{365}\right)^{(365)(1)} \approx \$2.7146$$

*Compounds Hourly* ( $n = 8760$ )

$$FV = 1 \left(1 + \frac{1}{8760}\right)^{(8760)(1)} \approx \$2.7181$$

*Compounds Once per Minute* ( $n = 525600$ )

$$FV = 1 \left(1 + \frac{1}{525600}\right)^{(525600)(1)} \approx \$2.7183$$

*Compounds Once per Second* ( $n = 31536000$ )

$$FV = 1 \left(1 + \frac{1}{31536000}\right)^{(31536000)(1)} \approx \$2.7183$$

Even though the annual interest rate is the same, money grows faster the more frequently it is compounded. This again gets back to the idea of interest earning interest. The earlier interest is added to an account, the sooner it can start to earn interest and have that interest start to earn money, etc. This is great for savings but bad for debt. As we will soon find out, savings accounts are generally compounded monthly whereas credit cards are compounded daily!

## THE RULE OF 72<sup>5</sup>

In [finance](#), the **rule of 72**, the rule of 70<sup>6</sup> and the **rule of 69.3** are methods for estimating an [investment](#)'s doubling time. The rule number (e.g., 72) is divided by the interest percentage per period (usually years) to obtain the approximate number of periods required for doubling. Although [scientific calculators](#) and [spreadsheet](#) programs have functions to find the accurate doubling time, the

5. From: [https://en.wikipedia.org/wiki/Rule\\_of\\_72](https://en.wikipedia.org/wiki/Rule_of_72) (CC-BY)

6. Donella Meadows, *Thinking in Systems: A Primer*, Chelsea Green Publishing, 2008, page 33 (box "Hint on reinforcing feedback loops and doubling time").

rules are useful for [mental calculations](#) and when only a basic [calculator](#) is available.<sup>[2]</sup>

These rules apply to [exponential growth](#) and are therefore used for [compound interest](#) as opposed to [simple interest](#) calculations. They can also be used for [decay](#) to obtain a halving time. The choice of number is mostly a matter of preference: 69 is more accurate for continuous compounding, while 72 works well in common interest situations and is more easily divisible. There is a number of variations to the rules that improve accuracy.

Let us use the continuous growth model to determine how long a quantity takes to double. In this case, the future value will just be double the present value, so  $FV = 2PV$ . Therefore, we have:

$$2PV = PVe^{rt}.$$

Dividing both sides by  $PV$ , we get

$$e^{rt} = \frac{2PV}{PV} = 2.$$

We then take the natural log of both sides to get

$$\ln(e^{rt}) = rt = \ln(2) \approx 0.69.$$

To solve for  $t$ , we can divide both sides by  $r$  to get

$$t \approx \frac{0.69}{r}.$$

Since  $r$  is given a decimal, we can multiply it by 100 to use it as a whole number. Since we are multiplying the denominator by 100, we must also multiply the numerator by 100:

$$t \approx 100 \times \frac{0.69}{r} = \frac{69}{100r} = \frac{69}{R}.$$

As mentioned earlier, we use 72 because this method is only an approximation and 72 divides much better (72 is divisible by 2, 3, 4, 6, 8, 9, 12,...). Let us see how it works.

Let us try one together...

Mel has \$500 in an account that pays 4% interest annually. Use the Rule of 72 to approximate how long will it take to have \$1,000. \$2,000. \$4,000.

Answers: 18 years, 36 years, 54 years



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<https://pressbooks.nsc.ca/mathofmoney/?p=42#oembed-9>

## EFFECTIVE INTEREST RATE<sup>7</sup>

The **effective interest rate (EIR)**, **effective annual interest rate**, **annual equivalent rate (AER)** or simply **effective rate** is the [interest rate](#) on a loan or financial product restated from the [nominal interest rate](#) and expressed as the equivalent interest rate if [compound interest](#) was payable annually in arrears.

It is used to compare the interest rates between loans with different compounding periods, such as weekly, monthly, half-yearly or yearly. The effective interest rate sometimes differs in one important respect from the [annual percentage rate \(APR\)](#): the APR method converts this weekly or monthly interest rate into what would be

7. From: [https://en.wikipedia.org/wiki/Effective\\_interest\\_rate](https://en.wikipedia.org/wiki/Effective_interest_rate) (CC BY)

called an annual rate that (in some parts of the world) **doesn't take into account the effect of compounding.**<sup>[1]</sup>

By contrast, in the EIR, the periodic rate is annualized using compounding. It is the standard in the European Union and many other countries around the world.

The EIR is precise in financial terms, because it allows for the effects of compounding, i.e. the fact that for each period, interest is not calculated on the principal, but on the amount accumulated at the end of the previous period, including capital and interest. This reasoning is easily understandable when looking at savings: if interest is capitalized every month, then in every month the saver earns interest on the entire sum, including interest from the previous period. Thus if one starts with \$1000 and earns interest at 2% every month, the accumulated sum at the end of the year is \$1268.24, giving an effective interest rate of about 26.8%, not 24%.

The term nominal EIR or nominal APR can (subject to legislation) be used to refer to an annualized rate that does not take into account front-fees and other costs can be included.

[Annual percentage yield](#) or effective annual [yield](#) is the analogous concept used for savings or investment products, such as a [certificate of deposit](#). Since any loan is an investment product for the lender, the terms may be used to apply to the same transaction, depending on the point of view.

Effective annual interest or yield may be calculated or applied differently depending on the circumstances, and the definition should be studied carefully. For example, a [bank](#) may refer to the yield on a loan portfolio after expected losses as its effective yield and include income from other fees, meaning that the interest paid by each

borrower may differ substantially from the bank's effective yield.

The effective interest rate is calculated as if compounded annually. The effective rate is calculated in the following way, where  $r$  is the effective annual rate,  $i$  the nominal rate, and  $n$  the number of compounding periods per year (for example, 12 for monthly compounding):

$$EIR = \left(1 + \frac{r}{n}\right)^n - 1.$$

For continuously compounding interest, we have

$$EIR \approx e^r - 1.$$

Let's try one together...

PNC offers you a loan with an 8.55% annual interest rate compounded daily. Calculate the effective interest rate. Interpret the result.

Answer: 8.93%



One or more interactive elements has been excluded from this version of the text. You can view them online here:

<https://pressbooks.nsc.ca/mathofmoney/?p=42#oembed-10>

Let's Try One Together

You are looking to take out a \$15,000 loan for home renovations. If you want to minimize the interest paid, which bank should you get the loan from?

- Ally quotes you a rate of 4.77% compounded annually
- Bank of America quotes you a rate of 4.68% compounded quarterly.
- Capital One quotes you a rate of 4.61% compounded daily.

Answers: EIRs = 4.77%, 4.76%, 4.71%; Choose Capital One



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<https://pressbooks.nsc.ca/mathofmoney/?p=42#oembed-11>



# 1.5 PRACTICE PROBLEMS

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**Problem 1.5.1:** You deposit \$1,000 into a savings account that pays 6% per year compounded annually. Create a 5-year table with the following columns:

Year	Starting Balance	Interest Earned	Ending Balance
------	------------------	-----------------	----------------

Answer: See video.



One or more interactive elements has been excluded from this version of the text. You can view them online here:


<https://pressbooks.nsc.ca/mathofmoney/?p=44#oembed-1>

**Problem 1.5.2:** Jevonte deposits \$25,000 into a savings account that pays 8% per year compounded monthly. Calculate the balance after 15 years assuming no additional deposits are made.

Answer: \$82,673



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**Problem 1.5.3:** Imani needs to have \$50,000 in 10 years. She has the ability to deposit money into an account that pays 5% compounded annually today. How much should she deposit in order to have the \$50,000 in 10 years?

Answer: \$30,696



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<https://pressbooks.nsc.ca/mathofmoney/?p=44#oembed-3>

**Problem 1.5.4:** You deposit \$5,500 into an account that pays 7% per year. How much will the account be worth after 20 years if...

1. The interest compounds annually.
2. The interest compounds quarterly.
3. The interest compounds monthly.
4. The interest compounds daily.

Answers: \$21,284; \$22,035; \$22,213; \$22,301



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<https://pressbooks.nsc.ca/mathofmoney/?p=44#oembed-4>

**Problem 1.5.5:** Paval deposits \$30,000 into an account. After 15 years, the account is worth \$98,430. If the interest compounds quarterly, calculate the annual interest rate.

Answer: 8%



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<https://pressbooks.nsc.ca/mathofmoney/?p=44#oembed-5>

**Problem 1.5.6:** Seo-yun deposits \$1,800 into an account that pays 7.5% compounded daily. How much will the account be worth after 8 years?

Answer: \$3,280



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view them online here: <https://pressbooks.nsc.ca/mathofmoney/?p=44#oembed-6>

**Problem 1.5.7:** At some point in the past, you deposited \$800 into an account which is now worth \$2,121. If the account pays 6.5% compounded continuously, how long ago did you make your initial deposit?

Answer: 15.5 years



One or more interactive elements has been excluded from this version of the text. You can view them online here:

<https://pressbooks.nsc.ca/mathofmoney/?p=44#oembed-7>

**Problem 1.5.8:** In 10 years, Amy knows that she will need to have \$35,000. She has the ability to deposit money into an account today, which will pay 4.5% interest compounded annually. How much would she need to deposit in order to have \$35,000 in ten years?

Answer: \$22,537



One or more interactive elements has been excluded from this version of the text. You can view them online here:

<https://pressbooks.nsc.ca/mathofmoney/?p=44#oembed-8>

**Problem 1.5.9:** Razan deposits \$10,000 into an account. After a certain number of years, the account is worth \$25,450. If the account pays 6% compounded monthly, how long has the money been in the account?

Answer: 15.6 years



One or more interactive elements has been excluded from this version of the text. You can view them online here:

<https://pressbooks.nsc.ca/mathofmoney/?p=44#oembed-9>

**Problem 1.5.10:** When she was born, Andrea had \$10,000 deposited into an account for when she turned 21. Now she is 21 and the account is worth \$35,143. She is unsure of the interest rate that was being paid on the account but she does know that interest was compounded monthly. What interest rate did the account pay?

Answer: 6%



One or more interactive elements has been excluded from this version of the text. You can

view them online here:

<https://pressbooks.nsc.ca/mathofmoney/?p=44#oembed-10>

**Problem 1.5.11:** If an account pays an annual rate of 8%, use the Rule of 72 to approximate how long it will take for the value of the account to double. Quadruple. Octuple.

Answers: 9 years, 18 years, 27 years



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<https://pressbooks.nsc.ca/mathofmoney/?p=44#oembed-11>

**Problem 1.5.12:** A standard student credit card charges an annual rate of 24.99% compounded daily. Calculate the effective interest rate for the credit card.

Answers: 28.38%



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<https://pressbooks.nsc.ca/mathofmoney/?p=44#oembed-12>

## 1.6 THE TIME VALUE OF MONEY II: MULTIPLE DEPOSITS

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### ANNUITIES<sup>1</sup>

An **annuity** is a series of payments made at equal intervals.<sup>2</sup> Examples of annuities are regular deposits to a **savings account**, monthly **home mortgage** payments, monthly **insurance** payments and **pension** payments. Annuities can be classified by the frequency of payment dates. The payments (deposits) may be made weekly, monthly, quarterly, yearly, or at any other regular interval of time. Annuities may be calculated by **mathematical functions** known as “annuity functions”.

An annuity which provides for payments for the remainder of a person’s lifetime is a **life annuity**.

Payments of an *annuity-immediate* are made at the end of payment periods, so that interest accrues between the issue of the annuity and the first payment. These are also called ordinary annuities. Payments of an *annuity-due* are made at the beginning of payment periods, so a payment is made immediately on issuer. In our class we will only use ordinary (annuity-immediate) annuities.

Annuities can be founded in many ways. While I give three options below, we will only use fixed annuities in this class.

1. Adapted from Annuity. (2022, October 26), In *Wikipedia*.  
<https://en.wikipedia.org/wiki/Annuity>. CC BY-SA 3.0

2. Kellison, S. G. (1970). *The Theory of Interest*. Richard D. Irwin, 45.

- **Fixed annuities** – These are annuities with fixed payments. If provided by an insurance company, the company guarantees a fixed return on the initial investment. Fixed annuities are not regulated by the **Securities and Exchange Commission**.
- **Variable annuities** – Registered products that are regulated by the **SEC** in the United States of America. They allow direct investment into various funds that are specially created for Variable annuities. Typically, the insurance company guarantees a certain death benefit or lifetime withdrawal benefits.
- **Equity-indexed annuities** – Annuities with payments linked to an index. Typically, the minimum payment will be 0% and the maximum will be predetermined. The performance of an index determines whether the minimum, the maximum or something in between is credited to the customer.

## FUTURE VALUE OF ANNUITIES

If the payments are made at the end of the time periods, so that interest is accumulated before the payment, the annuity is called an *annuity-immediate*, or *ordinary annuity*. Mortgage payments are annuity-immediate, interest is earned before being paid.

The *future value* of an annuity is the accumulated amount, including payments and interest, of a stream of payments made to an interest-bearing account. For an annuity-immediate, it is the value immediately after the  $n$ -th payment.



Let us begin by working through an example where you put \$500 into an account each year for five years. The account pays 5% compounded annually. Because we are only using ordinary annuities, we will see that year 1 is sort of a wash. This is because the contributions are made at the end of the year. The annual breakdown is as follows:

Year	Starting Balance	Interest Earned	Contribution	Ending Balance
1	\$0.00	\$0.00	\$500.00	\$500.00
2	\$500.00	\$25.00	\$500.00	\$1,025.00
3	\$1,025.00	\$51.25	\$500.00	\$1,576.25
4	\$1,576.25	\$78.81	\$500.00	\$2,155.06
5	\$2,155.06	\$107.75	\$500.00	\$2,762.82

So we see that we contributed \$2,500 over the five years but the account is worth \$2,762.82 due to the interest payments. What if we want to do something more complicated? A table quickly becomes far too cumbersome. We need a method which allows us to find the future value using some of equation(s). Note: For these problems, the payment frequency must match the interest compounding frequency. For instance, we will not do any problems where interest is compounded annually but payments are made monthly.

There are two steps to finding the future value. The first is to calculate the *future value annuity factor*. Think of this as a type of multiplier that takes into account the interest rate, compounding/payment frequency, and the number of years (or periods) for which payments will be made. The calculation for the FVAF is

$$s_{n,i} = \frac{\left(1 + \frac{r}{n}\right)^{nt} - 1}{\frac{r}{n}}.$$

We pronounce the term as “s-nigh”.

Then, to calculate the future value, we take the FVAF and multiply it by the regular payment. This gives us

$$FV = PMT \times s_{n,i}.$$

To give you some idea as to why we need these equations and what we can do with them:

### Let's try one together...

You deposit \$50 per week into a savings account that pays 4% interest. Calculate the balance of the account after 25 years.

Answer: 111,620



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<https://pressbooks.nsc.ca/mathofmoney/?p=46#oembed-1>

### Let's try one together...

Suppose that you have a child and want to put money into an account for their college fund. You want the money to be worth \$250,000 when they attend college in

18 years. If you can expect a 7% return, calculate the amount of money you need to deposit into the account **each quarter**.

Answer: 1,759/quarter



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<https://pressbooks.nsc.ca/mathofmoney/?p=46#oembed-2>

## PRESENT VALUE OF ANNUITIES

The *present value* of an annuity is the value of a stream of payments, discounted by the interest rate to account for the fact that payments are being made at various moments in the future.

We generally use present value calculations for loans and situations where you owe money and will pay it back. When you take out a loan, you are essentially getting a lump-sum payment upfront and then paying it back over time. The bank pays for your car, house, etc. and then you pay the bank. The methodology is very similar, but slightly different from future value. We will again only use ordinary annuities.

The major difference is that this is a three-step process. The first step is to calculate the FVAF just like we did earlier. Then, we calculate the *present value annuity factor*

or PVAF. Again, this of this as a sort of multiplier. This is calculated as:

$$a_{n,i} = \frac{s_{n,i}}{\left(1 + \frac{r}{n}\right)^{nt}}.$$

This is pronounced “Annie.”

Then, to determine the present value, we use the following formula:

$$PV = PMT \times a_{n,i}.$$

To get a better understanding as to why we need this framework, let us look at a few examples.

Let's try one together...

You decide to purchase a car. You agree to pay \$325/month for 5 years at an annual rate of 3.5%. Calculate **both** the price of the car and the total interest you will pay.

Answers: 17,865; 1,635 of interest



One or more interactive elements has been excluded from this version of the text. You can view them online here:

<https://pressbooks.nsc.ca/mathofmoney/?p=46#oembed-3>

Let's try one together...

Heinz takes out a \$50,000 home renovation loan that

will be repaid monthly over 10 years. If his loan was approved at a rate of 4.18%, how much will Heinz have to repay each month?

Answer: 510.51/month



One or more interactive elements has been excluded from this version of the text. You can view them online here:

<https://pressbooks.nsc.ca/mathofmoney/?p=46#oembed-4>

## Amortization Tables

When discussing loans, we can create a table that shows how payments are being applied each month. If you are paying \$ 300 per month, your loan balance is not being reduced by that amount each month because some of your payment goes towards interest.

So let us consider this example: You borrow \$10,000 which you will repay with five annual payments. The interest rate is 3%. Therefore, our calculations are:

$$s_{n,i} = \frac{\left(1 + \frac{.03}{1}\right)^{(1)(5)}}{.03} = 5.3091$$

$$a_{n,i} = \frac{3.185}{\left(1 + \frac{.03}{1}\right)^{(1)(5)}} = 4.5797$$

$$PMT = \frac{PV}{a_{n,i}} = \frac{10,000}{4.5797} = \$2,183.55.$$

Because this is an ordinary annuity, we start the month with a balance, accrue interest on the starting balance, then make a payment and pay-off the interest with the

remainder going towards the balance. Let us take a look below.

Year	Starting Balance	Payment	Interest	Principal	Ending Balance
1	10,000.00	2,183.55	300.00	1,883.55	8,116.45
2	8,116.45	2,183.55	243.49	1,940.05	6,176.40
3	6,176.40	2,183.55	185.29	1,998.25	4,178.15
4	4,178.15	2,183.55	125.34	2,058.20	2,119.95
5	2,119.95	2,183.55	63.60	2,119.95	0.00

To help look at the process, let us examine the first row. We begin with \$10,000 of debt. We previously found the payment would be \$2,183.55. So how will our payment be split between interest and principal? To answer that we need to calculate the interest which we do by multiplying our starting balance for the year by the interest rate. Here we get  $(10,000.00)(.03) = 300$ . That means that the remainder of the \$2,183.55 payment, or \$1,883.55 will go towards paying down the loan balance. When we subtract the \$1,883.55 from \$10,000, we get a remaining balance of \$8,116.45. The process then continues over and over.

### Let's try one together...

You take out a \$7,500 loan which will be repaid yearly with four equal annual payments. If the loan carries an annual interest rate of 6%, create the amortization schedule for the loan.

Answer: The payment is 2,164.44/year. See video for amortization schedule.



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<https://pressbooks.nsc.ca/mathofmoney/?p=46#oembed-5>

## 1.7 PRACTICE PROBLEMS

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**Problem 1.6.1:** Chen deposits \$200 per month into a retirement account. He earns an annual interest rate of 6%. How much will be in his account after 30 years?

Answer: \$200,903



One or more interactive elements has been excluded from this version of the text. You can view them online here:

<https://pressbooks.nsc.ca/mathofmoney/?p=48#oembed-1>

**Problem 1.6.2:** Ami wants to start an emergency fund. She deposits \$50 per month into an account that pays 2% interest. How much will she have in 5 years?

Answer: \$3,152



One or more interactive elements has been excluded from this version of the text. You can view them online here:

<https://pressbooks.nsc.ca/mathofmoney/?p=48#oembed-2>



**Problem 1.6.3:** Greg is planning to take an extravagant vacation in 10 years. He will need \$15,000 to take the trip. He wants to make weekly contributions. If he has an account that pays 5% per year, how much must he deposit each week?

Answer: \$22.25/wk



One or more interactive elements has been excluded from this version of the text. You can view them online here:

<https://pressbooks.nsc.ca/mathofmoney/?p=48#oembed-3>

**Problem 1.6.4:** You purchase a car for \$30,000. The bank has approved you for a car loan at a rate of 6.5% which will be repaid over 60 months (5 years). How much will you pay per month? How much interest will you pay on the loan?

Answer: \$586.98/month; \$5,250 of interest



One or more interactive elements has been excluded from this version of the text. You can view them online here:

<https://pressbooks.nsc.ca/mathofmoney/?p=48#oembed-4>

**Problem 1.6.5:** You purchase a house for \$350,000. Assume there is no downpayment (we will talk about

these later.) If the bank approves you for a 30-year mortgage at a rate of 4.1%, how much will you pay each month? How much interest will you pay over the life of the loan?

Answers: \$1,691/month; \$258,760 of interest



One or more interactive elements has been excluded from this version of the text. You can view them online here:

<https://pressbooks.nsc.ca/mathofmoney/?p=48#oembed-5>

**Problem 1.6.6:** You take out a \$20,000 home improvement loan that will be paid in six equal annual payments. The interest rate on the loan is 8%. Create an amortization schedule for this loan.

Answers: \$4,326.31/month; see video for amortization schedule



One or more interactive elements has been excluded from this version of the text. You can view them online here:

<https://pressbooks.nsc.ca/mathofmoney/?p=48#oembed-6>

**Problem 1.6.7:** Charles gets a \$30,000 car loan at a 4% interest rate. He has arranged with the bank to make four

equal annual payments. Create an amortization schedule for this loan.

Answer: \$8,264.70/month; see video for amortization schedule



One or more interactive elements has been excluded from this version of the text. You can view them online here:

<https://pressbooks.nsc.ca/mathofmoney/?p=48#oembed-7>



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## MODULE 2: DEBT

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## 2.1 INTRODUCTION

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In this section we will explore the different types of debt. We will be using the present value annuity calculations a lot so if you are unsure of how those work, be sure to work through the practice problems or come talk to me!

While we will explore different types of debt soon, let us first consider two concepts first: your credit score and debt repayment.

### CREDIT SCORES<sup>1</sup>

A **credit score** is a numerical expression based on a level analysis of a person's credit files, to represent the creditworthiness of an individual. A credit score is primarily based on a **credit report**, information typically sourced from **credit bureaus**.

Lenders, such as **banks** and credit card companies, use credit scores to evaluate the potential risk posed by lending money to consumers and to mitigate losses due to **bad debt**. Lenders use credit scores to determine who qualifies for a loan, at what **interest rate**, and what credit limits. Lenders also use credit scores to determine which customers are likely to bring in the most revenue. The use of credit or **identity scoring** prior to authorizing access or granting credit is an implementation of a **trusted system**.

1. Adapted from Credit score (2022, September 29), *In Wikipedia*.  
[https://en.wikipedia.org/wiki/Credit\\_score](https://en.wikipedia.org/wiki/Credit_score). CC BY-SA 3.0

Credit scoring is not limited to banks. Other organizations, such as mobile phone companies, insurance companies, landlords, and government departments employ the same techniques. Digital finance companies such as online lenders also use alternative data sources to calculate the creditworthiness of borrowers.

In the United States, a credit score is a number based on a statistical analysis of a person's credit files, that in theory represents the **creditworthiness** of that person, which is the likelihood that people will pay their bills. A credit score is primarily based on **credit report** information, typically from one of the three major **credit bureaus: Experian, TransUnion, and Equifax**. Income and employment history (or lack thereof) are not considered by the major credit bureaus when calculating credit scores.

There are different methods of calculating credit scores. **FICO scores**, the most widely used type of credit score, is a credit score developed by **FICO**, previously known as **Fair Isaac Corporation**. As of 2018, there were 29 different versions of FICO scores in use in the United States. Some of these versions are "industry specific" scores, that is, scores produced for particular market segments, including automotive lending and bankcard (credit card) lending. Industry-specific FICO scores produced for automotive lending are formulated differently than FICO scores produced for bankcard lending. Nearly every consumer will have different FICO scores depending upon which type of FICO score is ordered by a lender; for example, a consumer with several paid-in-full car loans but no reported credit card payment history will generally score better on a FICO automotive-enhanced score than on a FICO bankcard-



enhanced score. FICO also produces several “general purpose” scores which are not tailored to any particular industry. Industry-specific FICO scores range from 250 to 900, whereas general purpose scores range from 300 to 850.

FICO scores are used by many mortgage lenders that use a risk-based system to determine the possibility that the borrower may default on financial obligations to the mortgage lender.

Usage of credit histories in employment screenings increased from 19% in 1996 to 42% in 2006. However, credit reports for employment screening purposes do not include credit scores.<sup>2</sup>

Americans are entitled to one free credit report in every 12-month period from each of the three credit bureaus, but are not entitled to receive a free credit score. The three credit bureaus run **Annualcreditreport.com**, where users can get their free credit reports. Credit scores are available as an add-on feature of the report for a fee. If the consumer disputes an item on a credit report obtained using the free system, under the Fair Credit Reporting Act (FCRA), the credit bureaus have 45 days to investigate, rather than 30 days for reports obtained otherwise.

Under the **Fair Credit Reporting Act**, a consumer is entitled to a free credit report (but not a free credit score) within 60 days of any adverse action (e.g., being denied credit, or receiving substandard credit terms from a lender) taken as a result of their credit score. Under the **Wall Street reform** bill passed on 22 July 2010, a

2. Wernau, J. (2010, April 28). TransUnion battling attempts to ban employment credit checks. *Chicago Tribune*.

consumer is entitled to receive a free credit score if they are denied a loan or insurance due to their credit score.<sup>3</sup>

The generic or classic or general-purpose FICO credit score ranges between 300 and 850. The VantageScore 3.0 score and VantageScore 4.0 score range from 300–850.

In the United States, the **median** generic FICO score was 723 in 2006<sup>4</sup> and 711 in 2011.<sup>5</sup> The performance definition of the FICO risk score (its stated design objective) is to predict the likelihood that a consumer will go 90 days past due or worse in the subsequent 24 months after the score has been calculated. The higher the consumer's score, the less likely he or she will go 90 days past due in the subsequent 24 months after the score has been calculated. Because different lending uses (mortgage, automobile, credit card) have different parameters, FICO algorithms are adjusted according to the predictability of that use. For this reason, a person might have a higher credit score for a revolving credit card debt when compared to a mortgage credit score taken at the same point in time.

The interpretation of a credit score will vary by lender, industry, and the economy as a whole. While 640 has been a divider between “prime” and “subprime”, all considerations about score revolve around the strength

3. Board of Governors of the Federal Reserve System. (August 2007). *Report to the Congress on Credit Scoring and Its Effects on the Availability and Affordability of Credit*. <https://www.federalreserve.gov/boarddocs/RptCongress/creditscore/creditscore.pdf>
4. Harney, K. R. (2006, June 10). New Mortgages Worry Regulators. *Washington Post*. <https://www.washingtonpost.com/wp-dyn/content/article/2006/06/09/AR2006060900027.html>
5. Huynh, F. (2014, February 3). *Scoring Solutions*. <https://www.fico.com/blogs/us-credit-quality-continues-inch-forward>

of the economy in general and investors' appetites for risk in providing the funding for borrowers in particular when the score is evaluated. In 2010, the Federal Housing Administration (FHA) tightened its guidelines regarding credit scores to a small degree, but lenders who have to service and sell the securities packaged for sale into the secondary market largely raised their minimum score to 640 in the absence of strong compensating factors in the borrower's loan profile. In another housing example, Fannie Mae and Freddie Mac began charging extra for loans over 75% of the value that have scores below 740. Furthermore, private mortgage insurance companies will not even provide mortgage insurance for borrowers with scores below 660. Therefore, "prime" is a product of the lender's appetite for the risk profile of the borrower at the time that the borrower is asking for the loan.

Several factors affect individual's credit scores. One factor is the amount an individual borrowed as compared to the amount of credit available to the individual. As an individual borrows, or leverages, more money, the individual's credit score decreases.

# CREDIT SCORE FACTORS

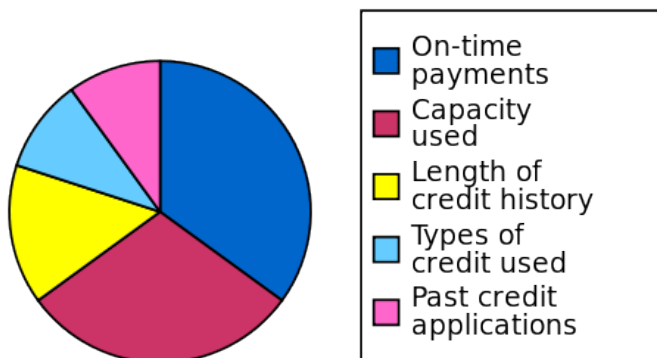


Figure 2.1.1: Factors that Determine a Credit Score

*Money Coach: Credit Scores and Reports 101 (all rights reserved)*



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<https://pressbooks.nsc.ca/mathofmoney/?p=52#oembed-1>

## DEBT REPAYMENT

As a quick note before we dig-in. You will see a variety of interest rate terms like effective interest rate, APY, and APR. These are all essentially the same thing and for the purpose of our class we will treat as the same. Each of these three terms essentially looks at the true interest

rate over the year when you include the concept of compounding periods. The simplest interest to calculate is when interest compounds annually, but there are very few situations where that occurs. Typically financial instruments compound monthly or daily. So as we should remember, the effective interest rate gives you the interest rate if the interest were to compound annually. So, if you have a credit card with a 30% interest rate that compounds daily, your effective interest rate is actually closer to 35% since interest charges are added daily.

### Debt Snowball Method<sup>6</sup>

The **debt-snowball method** is a **debt**-reduction strategy, whereby one who owes on more than one **account** pays off the accounts starting with the smallest **balances** first, while paying the minimum payment on larger debts. Once the smallest debt is paid off, one proceeds to the next larger debt, and so forth, proceeding to the largest ones last.<sup>7</sup> This method is sometimes contrasted with the debt stacking method, also called the “debt avalanche method”, where one pays off accounts on the highest **interest rate** first.<sup>8</sup>

The debt-snowball method is most often applied to repaying **revolving credit** – such as **credit cards**. Under

6. Adapted from Debt snowball method. (2022, November 21), In Wikipedia. [https://en.wikipedia.org/wiki/Debt-snowball\\_method](https://en.wikipedia.org/wiki/Debt-snowball_method) CC BY-SA 3.0
7. Ramsey Solutions. (2022, July 18). *How to Get Out of Debt With the Debt Snowball Plan*. <https://www.federalreserve.gov/boarddocs/RptCongress/creditscore/creditscore.pdf>
8. Ramsey Solutions. (2023, January 4). *Debt Snowball vs. Avalanche: What's the Difference?* <https://www.thebalancemoney.com/debt-snowball-vs-debt-stacking-453633>

the method, extra cash is dedicated to paying debts with the smallest amount owed.<sup>9</sup>

The basic steps in the debt snowball method are as follows:

1. List all debts in ascending order from smallest balance to largest. This is the method's most distinctive feature, in that the order is determined by amount owed, not the rate of interest charged. However, if two debts are very close in amount owed, then the debt with the higher interest rate would be moved above in the list.
2. Commit to pay the minimum payment on every debt.
3. Determine how much extra can be applied towards the smallest debt.
4. Pay the minimum payment plus the extra amount towards that smallest debt until it is paid off. Note that some lenders (mortgage lenders, car companies) will apply extra amounts towards the next payment; in order for the method to work the lenders need to be contacted and told that extra payments are to go directly toward principal reduction. Credit cards usually apply the whole payment during the current cycle.
5. Once a debt is paid in full, add the old minimum payment (plus any extra amount available) from the first debt to the minimum payment on the

9. All About Money. (2009, July 2). *Debt snowball*.  
[https://web.archive.org/web/20140222055100/  
http://www.allaboutmoney.com/debt-advice/debt-  
snowball-0-2646.htm](https://web.archive.org/web/20140222055100/http://www.allaboutmoney.com/debt-advice/debt-snowball-0-2646.htm)

second smallest debt, and apply the new sum to repaying the second smallest debt.

6. Repeat until all debts are paid in full.

In theory, by the time the final debts are reached, the extra amount paid toward the larger debts will grow quickly, similar to a snowball rolling downhill gathering more snow, hence the name.

The theory appeals to human psychology: by paying the smaller debts first, the individual, couple, or family sees fewer bills as more individual debts are paid off, thus giving ongoing positive feedback on their progress towards eliminating their debt.

In situations where a debt has both a higher interest rate and higher balance than another debt, the debt-snowball method will prioritize the smaller debt even though paying the larger debt would be more cost-effective. Several writers and researchers have considered this contradiction between the method and a strictly mathematical approach. Writing in *Forbes*, Rob Berger noted that “humans aren’t really rational creatures” and stresses that research tends to support the debt snowball method in real-world scenarios.<sup>[5]</sup> The primary benefit of the smallest-balance plan is the psychological benefit of seeing results sooner, in that the debtor sees reductions in both the number of creditors owed (and, thus, the number of bills received) and the amounts owed to each creditor. In a 2012 study by Northwestern’s Kellogg School of Management, researchers found that “consumers who tackle small balances first are likelier to eliminate their overall debt” than trying to pay off high interest rate balances first. A 2016 study in *Harvard Business Review* came to a similar conclusion:

We tested a variety of hypotheses and ultimately determined that it is not the size of the repayment or how little is left on a card after a payment that has the biggest impact on people's perception of progress; rather it's what *portion* of the balance they succeed in paying off. Thus focusing on paying down the account with the smallest balance tends to have the most powerful effect on people's sense of progress – and therefore their motivation to continue paying down their debts.<sup>[7]</sup>

Author and radio host [Dave Ramsey](#), a proponent of the debt-snowball method, concedes that an analysis of math and interest leans toward paying the highest interest debt first. However, based on his experience, Ramsey states that personal finance is “20 percent head knowledge and 80 percent behavior” and he argues that people trying to reduce debt need “quick wins” (i.e., paying off the smallest debt) in order to remain motivated toward debt reduction.<sup>[8]</sup>

### **Debt stacking method<sup>10</sup>**

Note: Debt stacking is also referred to as the debt avalanche method and the high rate method.

The debt stacking method (also known as the debt avalanche method) recommends that you make a list of all your debts, ranked by interest rate, from highest to lowest.

For example, you might owe:

- Mastercard, \$2,500—19%, highest interest rate

10. James, M. (2023, January 4). Debt Snowball vs. Avalanche: What's the Difference? *The Balance*. <https://www.thebalance.com/debt-snowball-vs-debt-stacking-453633> (all rights reserved, used under fair use)



- Visa, \$7,500—13%, second-highest interest rate
- Car loan, \$4,000—8%, third-highest interest rate
- [Student loan](#), \$1,900—5%, lowest interest rate

The debt stacking method advises that you make the minimum payment on all your loans. Then, you should throw all of your extra money toward paying off your Mastercard, which has the highest interest rate, at 19%.

Once you've wiped away your Mastercard debt, tackle the Visa balance, which has the second-highest interest rate, at 13%.

It'll take you a long time to repay the Visa, since it has the highest balance, at \$7,500. Stick with it. Whenever you're done, you can start paying off the debts with lower interest rates.

You may feel frustrated after investing so much time and energy toward paying down a loan without feeling the mental victory of crossing it off your list.

*Next Level Life: Debt Snowball versus Debt Avalanche (all rights reserved)*



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<https://pressbooks.nscc.ca/mathhofmoney/?p=52#h5p-6>



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<https://pressbooks.nscc.ca/mathhofmoney/?p=52#h5p-7>

## 2.2 CREDIT CARDS

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### CREDIT CARD BASICS<sup>1</sup>

A credit card is a payment card issued to users (cardholders) to enable the cardholder to pay a merchant for goods and services based on the cardholder's promise to the card issuer to pay them for the amounts plus the other agreed charges.<sup>2</sup> The card issuer (usually a bank) creates a revolving account and grants a line of credit to the cardholder, from which the cardholder can borrow money for payment to a merchant or as a cash advance.

*Money Coach: Credit Cards 101 (all rights reserved)*



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<https://pressbooks.nsc.ca/mathofmoney/?p=54#oembed-1>

### Minimum Monthly Payment

The cardholder must pay a defined minimum portion of

1. Adopted from Credit Card. (2023, January 20). In *Wikipedia*.  
[https://en.wikipedia.org/wiki/Credit\\_card](https://en.wikipedia.org/wiki/Credit_card) CC BY-SA 3.0
2. O'Sullivan, A. & Sheffrin, S. M. (2003). *Economics: Principles in action*. Pearson Prentice Hall.

the amount owed by a due date, or may choose to pay a higher amount. The credit issuer charges interest on the unpaid balance if the billed amount is not paid in full (typically at a much higher rate than most other forms of debt). In addition, if the cardholder fails to make at least the minimum payment by the due date, the issuer may impose a late fee or other penalties. To help mitigate this, some financial institutions can arrange for automatic payments to be deducted from the cardholder's bank account, thus avoiding such penalties altogether, as long as the cardholder has sufficient funds.

### **Grace Period**

A credit card's grace period is the time the cardholder has to pay the balance before interest is assessed on the outstanding balance. Grace periods may vary, but usually range from 20 to 55 days depending on the type of credit card and the issuing bank. Some policies allow for reinstatement after certain conditions are met.

Usually, if a cardholder is late paying the balance, finance charges will be calculated and the grace period does not apply. Finance charges incurred depend on the grace period and balance; with most credit cards there is no grace period if there is any outstanding balance from the previous billing cycle or statement (i.e. interest is applied on both the previous balance and new transactions). However, there are some credit cards that will only apply finance charge on the previous or old balance, excluding new transactions.

### **Interest Payments**

Credit card issuers usually waive interest charges if the

balance is paid in full each month, but typically will charge full interest on the entire outstanding balance from the date of each purchase if the total balance is not paid.

For example, if a user had a \$1,000 transaction and repaid it in full within this grace period, there would be no interest charged. If, however, even \$1.00 of the total amount remained unpaid, interest would be charged on the \$1,000 from the date of purchase until the payment is received. The precise manner in which interest is charged is usually detailed in a cardholder agreement which may be summarized on the back of the monthly statement.

Financial institutions refer to interest charged back to the original time of the transaction and up to the time a payment was made, if not in full, as a residual retail finance charge (RRFC). Thus after an amount has revolved and a payment has been made, the user of the card will still receive interest charges on their statement after paying the next statement in full (in fact the statement may only have a charge for interest that collected up until the date the full balance was paid, i.e. when the balance stopped revolving).

The credit card may simply serve as a form of revolving credit, or it may become a complicated financial instrument with multiple balance segments each at a different interest rate, possibly with a single umbrella credit limit, or with separate credit limits applicable to the various balance segments. Usually this compartmentalization is the result of special incentive offers from the issuing bank, to encourage balance transfers from cards of other issuers. In the event that several interest rates apply to various balance segments, payment allocation is generally at the discretion of the

issuing bank, and payments will therefore usually be allocated towards the lowest rate balances until paid in full before any money is paid towards higher rate balances. Interest rates can vary considerably from card to card, and the interest rate on a particular card may jump dramatically if the card user is late with a payment on that card *or any other credit instrument*, or even if the issuing bank decides to raise its revenue.

### Fraud Protection

Some countries, such as the United States, the United Kingdom, and France, limit the amount for which a consumer can be held liable in the event of fraudulent transactions with a lost or stolen credit card.

*Money Coach: Credit Cards – Mistakes and Best Practices*  
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## CREDIT CARD CALCULATIONS

### Average Daily Balance<sup>3</sup>

Credit card companies do not charge you interest on your ending balance. Instead, they charge you interest on your

“average daily balance” or ADB. Essentially, this is the average debt you had during the month on a daily basis.

*Khan Academy: APR and Effective APR*



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To help understand the process, let us consider an example.

Suppose that your credit card statement dates run from the 1st of the month until the last day of the month. In reality, statement periods can run between any dates, but things will be less complicated if we think of one complete month.

Below is a table that shows your activity for the month:

Date	Activity	Balance
Jan 1	Starting Balance	\$600
Jan 9	Purchase of \$100	\$700
Jan 15	Purchase of \$300	\$1,000
Jan 24	Payment of \$400	\$600
Jan 31	Purchase of \$1,500	\$2,100

To calculate the ADB, we need the following information: Balance, Days at Balance, and Balance x Days at Balance. So let us work through the example above. Remember:

our statement period runs from January 1 – January 31.  
(There are 31 days in January).

- From Jan 1 – Jan 8 (8 days), you had a balance of \$600.
- From Jan 9 – Jan 14 (6 days), you had a balance of \$700.
- From Jan 15 – Jan 23 (9 days), you had a balance of \$1,000.
- From Jan 24 – Jan 30 (7 days), you had a balance of \$600.
- From Jan 31 – Jan 31 ( 1 day), you had a balance of \$2,100.

Quick tip: Never be worried about counting days on your fingers...that’s what I do! And count the number of days to make sure it adds up to the number of days in the month (to be sure you did not double count.)

We can now create the following table:

Balance	Days at Balance	Balance x Days
\$600	8	4,800
\$700	6	4,200
\$1,000	9	9,000
\$600	7	4,200
\$2,100	1	2,100
<b>Total</b>	31	24,300

Then, the ADB is calculated as (Balance x Days)/Days. For this problem, we get  $\$24,300/31=\$783.87$ .

Before we take a look at the interest charged, it should be noted that a large purchase was made at the end of the



month. This purchase will have a minimal impact on the ADB because it was only part of the balance for one day. However, this will have a large impact on the next month assuming that the card is not paid.

Next, we will calculate interest. This is relatively straight forward (at least compared to the compounding interest problems.) This will be done in four steps.

1. Calculate the ADB.
2. Take the interest rate (APR) and divide it by 365.
3. Multiply the result by the ADB.
4. Multiply the result by the number of days in the month.

So for our problem, we saw an ADB of \$783.87. Further, let us suppose that the interest rate (APR) on the card is 21.99%. So following the steps laid out above:

1.  $ADB = \$783.87$ .
2.  $.2199/365=0.000602$
3.  $(\$783.87)(0.000602) = \$0.472$
4.  $(\$0.472)(31) = \$14.63$

*Khan Academy: How Credit Card Interests are Calculated (CC BY)*



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<https://pressbooks.nscc.ca/mathofmoney/?p=54#oembed-4>

The final calculation we need to look at is the monthly payment. There is no standard rule for this as it varies from one credit card to the next. So let us use the following rule for this class:

1. If the balance is \$35 or less, the minimum payment is the entire balance.
2. The minimum payment is the larger of a) \$35 or b) 2% of the balance + interest.

So for our example, 2% of the balance is  $(0.02)(2,100)=42$  plus the interest of \$14.63 gives a minimum monthly payment of \$56.63. Note that while the large purchase at the end of the month did not affect the ADB much, it does affect the minimum payment since we take 2% of the ending balance and not the ADB.

Note: The calculation we just did is technically incorrect. Generally, you have one statement to pay off your credit card before interest is charged. So if you were to pay off her credit card during the next month (February), there would be no interest on your January purchases. We will not add this complication to our problems!

Let's try one together...

Shahad has a credit card that charges 23.99% interest. The activity for June is below.

Date	Activity
June 01	Starting balance of \$1,000
June 05	Purchase of \$300
June 12	Payment of \$800
June 17	Purchase of \$200
June 20	Purchase of \$400

- Calculate the ADB.
- Calculate the interest for the month.
- Calculate the minimum monthly payment using the rule from above (2% + interest or \$35, whichever is more.)

Answers: \$993.33; \$19.86; \$41.86



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## CREDIT CARD USAGE TIPS<sup>4</sup>

Disclaimer: These are things I believe and things I feel

4. New content added to Math of Money edition by J. Z. Klingensmith[

work. This may not work for anyone, so use any tips from me with caution.

1. Never use a debit card...always use a credit card...if you can do so responsibly!
2. Pay your balance each month.
3. It is okay to have multiple credit cards...especially if they are reward-specific.
4. Be sure to use cards once every six months.
5. If you are considering a large purchase, shop for credit cards. They will typically offer 6-18 months of no interest. Again, responsibility is key!
6. Watch for annual fees. Many cards are free, but others do charge an annual fee. This is sometimes worth it, especially for travelers, but not generally worth it for college students/younger people.

### **The Danger of the Monthly Minimum**

Note: I will not show any math here. Rather, I will just illustrate an example.

Suppose you want a new PS5. But you also need a 4k television. So you decide to use a credit card to buy everything and figure you will just make monthly payments. Let us assume that the cost of everything is \$2,000. Also, let us assume you put it on a credit card where your interest rate is 23.99%.

The first issue is that credit card utilization is part of your credit score and college students generally have lower credit limit. So you may only have a total of \$5,000 of credit available (and that is probably high), so you are using 40% or more of your credit limit. This could drop

you credit score by 50+ points which could have a cascading effect on your ability to buy a car, get a job, rent an apartment, etc.

But the second issue is the time it takes to pay down the debt. Using this calculator from [bankrate.com](https://www.bankrate.com), I find that if you just pay the minimum on your card each month assuming that is your only purchase and you make no other purchases, then it will take you 193 months (more than 16 years) to pay off your credit card and you will pay a total of \$5,328.11. This means that you will pay \$3,328.11 in interest! Now realistically, as your income increases, you would be able to pay this off faster, but other bills will likely start as you get older as well. It is best to avoid having baggage like this!

## PAYDAY LOANS

This will be quick, but important. Avoid payday loans. I will illustrate why below, but avoid them. They can create an unbreakable cycle of debt. But in the event you need a payday loan, you need to understand how they work.

A payday loan<sup>5</sup> (also called a payday advance, salary loan, payroll loan, small dollar loan, short term, or cash advance loan) is a short-term unsecured loan, often characterized by high interest rates.

The term “payday” in payday loan refers to when a borrower writes a postdated check to the lender for the payday salary, but receives part of that payday sum in immediate cash from the lender.<sup>[1]</sup> However, in common parlance, the concept also applies regardless of whether repayment of loans is linked to a borrower’s

5. Adapted from Payday Loan. (2022, December 30). In *Wikipedia*.  
[https://en.wikipedia.org/wiki/Payday\\_loan](https://en.wikipedia.org/wiki/Payday_loan) CC BY-SA 3.0

payday.<sup>[2][3][4]</sup> The loans are also sometimes referred to as “cash advances,” though that term can also refer to cash provided against a prearranged line of credit such as a credit card. Legislation regarding payday loans varies widely between different countries, and in federal systems, between different states or provinces.

To prevent usury (unreasonable and excessive rates of interest), some jurisdictions limit the annual percentage rate (APR) that any lender, including payday lenders, can charge. Some jurisdictions outlaw payday lending entirely, and some have very few restrictions on payday lenders.

Payday loans have been linked to higher default rates.<sup>[5][6][7][8]</sup>

The basic loan process involves a lender providing a short-term unsecured loan to be repaid at the borrower's next payday. Typically, some verification of employment or income is involved (via pay stubs and bank statements), although according to one source, some payday lenders do not verify income or run credit checks.<sup>[9]</sup> Individual companies and franchises have their own underwriting criteria.

In the traditional retail model, borrowers visit a payday lending store and secure a small cash loan, with payment due in full at the borrower's next paycheck. The borrower writes a postdated check to the lender in the full amount of the loan plus fees. On the maturity date, the borrower is expected to return to the store to repay the loan in person. If the borrower does not repay the loan in person, the lender may redeem the check. If the account is short on funds to cover the check, the borrower may now face a bounced check fee from their bank in addition to the costs of the loan, and the loan

may incur additional fees or an increased interest rate (or both) as a result of the failure to pay.

In the more recent innovation of online payday loans, consumers complete the loan application online (or in some instances via fax, especially where documentation is required). The funds are then transferred by direct deposit to the borrower's account, and the loan repayment and/or the finance charge is electronically withdrawn on the borrower's next payday.

I will not go over it here, but Wikipedia has an interesting argument in favour of payday lending which can be read about halfway down the page [here](#) (Proponents' stance and counterarguments).

*PBS: Fighting the Debt Trap of Triple-Digit Payday Loans*



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<https://pressbooks.nsc.ca/mathofmoney/?p=54#oembed-6>

## Calculations

Payday loans, as mentioned, are very short-term. So let us imagine that we need a \$500 loan until our next paycheque which is in 8 days. Using CashNetUSA for Alabama (the site does not offer loans in Pennsylvania). They charge \$17.50 per \$100 borrowed. If you absolutely need the \$500, then paying \$87.50 for the money may be worth it. But it is also extremely expensive. Let us use a simple interest calculation to determine the interest rate.

(Note: We did not do simple interest in this class and I will not ask you to do it, but I do want to illustrate the interest rate). When we calculate interest, we find it using the formula  $I = PRT$  where  $I$  is the interest charged,  $P$  is the amount borrowed,  $R$  is the interest rate, and  $T$  is number of years. So in our case,  $I$  is the money we are paying, which is \$87.50,  $P$  is the money borrowed, which is \$500, and  $T$  is the number of years. But since we only have a 10 day loan, then the number of years is  $10/365$ .

We want to solve for  $I$ , so dividing both sides by  $PT$ , we get  $R = I/(PT) = 87.50/((500)(10/365)) = 6.39$ . Now you may look at that and say 6.89% isn't bad at all. But remember that is a decimal. In other problems we would have gotten a value like 0.06 for 6%. So 6.39 is NOT 6.39%...it is 639%!

The issue with payday loans is that people are often unable to pay off the loan, so they have to renew the loan...and in most cases for more as the interest will be added to the loan. This is why maintaining good credit is so important. If you need a short-term loan and have credit cards with ample credit available, you can always use a cash advance. While these are still expensive, they are far superior to payday advances. A typical credit card will charge you a 5% or so fee and around a 25% interest rate. They can also be paid over time just as any other credit card debt would be. Again, I do not advocate using cash advances, but having available credit can save you tons of money.



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## 2.3 AUTOMOBILES

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### BUYING A CAR<sup>1</sup>

Many adults will buy a car several times during their lifetimes. A car is a major purchase. Its price can be as much as or more than one year's disposable income. Its annual operating costs can be substantial, including the cost of fuel, legally mandated insurance premiums, and registration fees, as well as maintenance and perhaps repairs and storage (parking). A car is not only a significant purchase, but also an ongoing commitment.

In the United States, people spend a considerable amount of time in their cars, commuting to work, driving their children to school and various activities, driving to entertainment and recreational activities, and so on. Most people want their car to provide not only transportation, but also comforts and conveniences. You can apply the purchasing model, described in this chapter, to the car purchase.

First, you identify the need: What is your goal in owning a car? What needs will it fulfill? Here are some further questions to consider:

- What kind of driving will you use the car for? Will you depend on it to get you to work, or will you

1. Adapted from [8.2 A MAJOR PURCHASE: BUYING A CAR](#) in [PERSONAL FINANCE](#) by Lumen Learning shared under a [CC BY-NC-SA](#) license.

use it primarily for weekend getaways?

- Do you need carrying capacity (for passengers or “stuff”) or hauling capacity?
- Do you live in a metropolitan area where you will be driving shorter distances at lower speeds and often idling in traffic?
- Do you live in a more rural area where you will be driving longer distances at faster speeds?
- Do you live in a climate where winter or a rainy season would make traction and storage an issue?
- How much time will you spend in the car every day?
- How many miles will you drive each year?
- How long do you expect to keep the car?
- Do you expect to resell or trade in the car?

Your answers to these questions will help you identify the product you want.

All attributes affect price. Product attribution scoring can help you identify the models that most closely fit your goals.

Mary lives on a dirt road in a rural area; she drives about 18,000 miles per year, commuting to her job as an accountant at the corporate headquarters of an auto parts chain and taking her kids to school. She is also a pretty good car mechanic and does basic maintenance herself.

John lives in the city; he walks or takes a bus to his job as a market researcher for an ad agency, but keeps a car to visit his parents in the suburbs. He drives about 5,000 miles per year, often crawling in traffic. All John knows

about a car is that the key goes in the ignition and the fuel goes in the tank.

John and Mary would rate these attributes very differently, and their scoring of the same models would have very different results.

Mary may value fuel efficiency more, as she drives more (and so purchases more fuel). Driving often and with her children, she may rank size, safety, and entertainment features higher than John would, who is in his car less frequently and alone. Mary relies on the car to get to work, so reliability would be more important for her than for John, who drives only for recreational visits. But Mary also knows that she can maintain and repair some things herself, which makes that less of a factor.

Car attributes are widely publicized by car dealers and manufacturers, who are among the top advertisers globally year after year. You can visit dealerships in your area or manufacturers' Web sites. Using the Internet is a more efficient way of narrowing your search. Specialized print and online magazines, such as *Car and Driver*, *Road and Track*, and [Edmunds.com](http://Edmunds.com), offer detailed discussions of model attributes and their actual performance. *Consumer Reports* also offers ratings and reviews and also provides data on frequency of repairs and annual maintenance costs.

You want to be sure to consider not only the price of buying the car, but also the costs of operating it. Fuel, maintenance, repair, insurance, property taxes, and registration may all be affected by the car's attributes, so you should consider operating costs when choosing the product. For example, routine repairs and maintenance are more expensive for some cars. A more fuel-efficient car can significantly lower your fuel costs. A more

valuable car will cost more to insure and will mean higher property (or excise) taxes. Moreover, the costs of fuel, maintenance, insurance, registration, and perhaps property tax on the car will be ongoing expenses—you want to buy a car you can afford *and* afford to drive.

If you are buying a new car, you know its condition, and so you can predict annual maintenance and repair costs and the car's longevity by the history for that model. Depending on how long you expect to own the car, you may also be concerned with its predicted resale value.

Used cars are generally less expensive than new. A used car has fewer miles left in it. Its condition is less certain: you may not know how it has been driven or its repair and maintenance history. This makes it harder to predict annual maintenance and repair costs. Typically, since it is already used when you buy it, you expect little or no resale value. You can gain a significant price savings in the used car market, and there are good used cars for sale. You may just have to look a bit harder to find one.

*The Chevy Car Guy – Questions to NEVER Answer on a Car Lot (all rights reserved)*



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The condition of exterior and interior features can indicate past accidents, repairs, or lack of maintenance that may increase future operating expenses, or just

driving habits that have left a less attractive or less comfortable vehicle.

Services like [Carfax](#) provide research on a vehicle's history based on its VIN (vehicle identification number), including any incidence of accidents, flooding, frame damage, or airbag deployment, the number and type of owners (was it a rental or commercial vehicle?), and the mileage. All these events affect your expectations of the vehicle's longevity, maintenance and repair costs, resale value, and operating costs, which can help you calculate its value and usefulness.

Unless you are an expert yourself, you should always have a trained mechanic inspect a used vehicle before you buy it. With cars, as with any item, the better informed you are, the better you can do as a consumer. Given the cost of a car and its annual expense, there is enough at stake with this purchase to make you cautious.

## IDENTIFY THE MARKET

New cars are sold through car dealerships. The dealer has a contract with the manufacturer to sell its cars in the retail market. Dealers may also offer repair and maintenance services as well as parts and accessories made especially for the models it sells.

New car dealers may also resell cars that they get as

trade-ins, especially of the same models they sell new. Used car dealers typically buy cars through auctions of corporate, rental, or government cars.

Individuals selling a used car can also do so through networking—in an online auction such as eBay, a virtual bulletin board such as Craig’s List, or the bulletin board in the local college snack bar. Dealers will have more information about the market, especially about the supply of cars and price levels for them.

Some people prefer a new car, with its more advanced features and more certain quality, but a used car may be a viable substitute for many purchasers. Many people buy used cars while their incomes are lower, especially in the earlier stages of their adult (working) life. As income rises and concern for convenience, reliability, and safety increases with age and family size, consumers may move into the new car market.

While they are two very different markets, the markets for new and used cars are related. Supply of and demand for new cars affect price levels in the new car market, but also in the used car market. For example, when new car prices are high, more buyers seek out used cars and when low, used car buyers may turn to the new car market.

Demand for cars is affected by macroeconomic factors such as business cycles and inflation. If there is a recession and a rise in unemployment, incomes drop. Demand for new cars will fall. Many people will decide to keep driving their current vehicle until things pick up, unwilling to purchase a long-term asset when they are uncertain about their job and paycheck. That slowing of demand may lower car prices, but will also lower the resale or trade-in value of the current vehicle. For first-time car buyers, that may be a good time to buy.

If there is inflation, it will push up interest rates because the price of borrowing money rises with other prices. Since many people borrow when purchasing a car, that will make the borrowing, and so the purchase, more costly, which will discourage demand.

When the economy is expanding, on the other hand, and inflation and interest rates are low, demand for new cars rises, pushing up prices. In turn, prices are kept in check by competition. As demand for new cars rises, demand for used cars may fall, causing the supply of used cars to rise as more people trade in their cars to buy a new one. They trade them in earlier in the car's life, so the quality of the used cars on the market rises. This may be a good time to buy a used car.

## IDENTIFY THE FINANCING: LOANS AND LEASES

The cost of a car is significant. Car purchases usually require financing through a loan or a lease. Each may require a down payment, which you would take out of your savings. That creates an opportunity cost of losing the return you could have earned on your savings. You also lose liquidity: you are taking cash, a liquid asset, and trading it for a car, a not-so-liquid asset.

Your opportunity cost and the cost of decreasing your



liquidity are costs of buying the car. You can reduce those costs by borrowing more (and putting less money down), but the more you borrow, the higher your costs of borrowing. If you trade in a vehicle, dealers will often use the trade-in value as the down payment and will sell the car to you with “no money down.”

Car loans are available from banks, credit unions, consumer finance companies, and the manufacturers themselves. Be sure to shop around for the best deal, as rates, maturity, and terms can vary. If you shop for the loan before shopping for the car, then the loan negotiation is separate from the car purchase negotiation. Both may be complex deals, and there are many trade-offs to be made. The more separate—and simplified—each negotiation is, the more likely you will be happy with the outcome.

Loans differ by interest rate or annual percentage rate (APR) and by the time to maturity. Both will affect your monthly payments. A loan with a higher APR is costing you more and, all things being equal, will have a higher monthly payment. A loan with a longer maturity will reduce your monthly payment, but if the APR is higher, it is actually costing you more. Loan maturities may range from one to five years; the longer the loan, the more you risk ending up with a loan that’s worth more than your car.

Rebecca buys a used Saturn for \$6,000, with \$1,000 cash down from savings and a GMAC-financed loan at 7.2 APR, on which she pays \$115 a month for forty-eight months. She could have gotten a twenty-four-month loan, but wanted to have smaller monthly payments. After only twenty-five months, she totals her car in a chain collision but luckily escapes injury. Now she needs

another car. The Saturn has no trade-in value, her insurance benefit won't be enough to cover the cost of another car, and she still has to pay off her loan regardless. Rebecca is out of luck, because her debt outlived her asset. If your debt outlives your asset, your ability to get financing when you go to replace that vehicle will be limited, because you still have the old debt to pay off and now are looking to add a new debt—and its payments—to your budget. Rebecca will have to use more savings and may have to pay more for a second loan, if she can get one, increasing her monthly payments or extending her debt over a longer period of time.

### Let's try one together...

Raul purchases a car for \$30,000 and puts \$3,000 down. The fees and other charges amounted to \$1,500. Raul was offered a loan at a rate of 5.2%. He is offered terms of 36-, 48-, 60-, and 72-months. Calculate the monthly payment for each option in addition to the total interest paid.

Answers: \$811.64 (\$2,219 interest); \$624.24 (\$2,963 interest); \$512.00 (\$3,720 interest); \$437.34 (\$4,488 interest)



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<https://pressbooks.nsc.ca/mathofmoney/?p=56#oembed-2>

An alternative to getting a car loan is leasing a car. Leases are a common way of financing a car purchase. A **lease** is a long-term rental agreement with a **buyout option** at maturity. Typically, at the end of the lease, usually three or four years, you can buy the car outright for a certain amount, or you can give it back (and buy or lease another car), which removes the risk of having an asset that outlives its financing. Leases specify an annual mileage limit, that is, the number of miles that you can drive the car in a year before incurring additional costs. Leases also specify the monthly payment and requirements for routine maintenance that will preserve the car's value.

So, lease or borrow? The price of the car should be the same regardless of how it is financed—the car should be worth what it's worth, no matter how it is paid for. The cost of borrowing, in percentage terms, is the interest rate or APR of the loan. The costs of leasing, in dollars, are the down payment, the lease payments, and the buyout. Since the price of the car itself is the same in either case, the present value of all the lease costs should be the same as the price of the car. You can use what you know about the time value of money to calculate the discount rate that produces that price; that is the equivalent annual cost of the lease, in percentage terms.

*The Dave Ramsay Show: Leasing versus Buying a Car (all rights reserved)*



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<https://pressbooks.nscc.ca/mathofmoney/?p=56#oembed-3>

For example, you want to buy a car with a price of \$19,000. You can get a car loan with an APR of 6.5 percent from your bank. You are offered a lease requiring a down payment of \$2,999, monthly payments of \$359 for three years, and a final buyout of \$5,000. The APR of the lease is actually 5.93 percent, which would make it the cheaper financing alternative.

In general, the longer you intend to keep the car, the less sense it makes to lease. If you typically drive a car “into the ground,” until it costs more to repair than replace it, then you are better off borrowing and spreading the costs of financing over a longer period. On the other hand, if you intend to keep the car only for the term of the lease and not to exercise the buyout option, then it is usually more cost effective to lease. You also need to consider whether or not you are likely to stay within the mileage limits of the lease, as the mileage penalties can add significantly to your costs.

Some people will say that they like to borrow and then “own” in order to have an asset that can store value or “build equity.” Given the unpredictable nature of the used car market, however, a car is really not an asset that can be counted on to store value. Thinking of a car as something that you will use up (although over several years) rather than as an asset you can preserve or save will help you make better financial decisions.

When you are buying a car, you want to minimize the

cost of both the car and the financing. If you are purchasing both the car and the financing from the same dealer, you should be careful to discuss them separately. Car dealers, who offer loans and leases as well as cars, often combine the three discussions, offering a break on the financing to make the car more affordable, or offering a break on the car to make the financing more affordable. To complicate matters further, they may also offer a rebate on a certain model or with a certain lease. The more clearly you can separate which costs belongs to which—the car or the financing—the more clearly you can understand and minimize your costs.

## PURCHASE AND POSTPURCHASE

A car purchase requires significant prepurchase activities. Once you have identified and compared appropriate car attributes, a seller, and financing options, all you have to do is drive away, right? Not quite.

Car purchases are one instance where the buyer is expected to haggle over price. The sticker price is the manufacturer's suggested retail price (MSRP) for that vehicle model with those features. Dealers negotiate many of the factors that ultimately determine the value of the purchase: the optional features of the car, the

warranty terms, service discounts on routine maintenance, financing terms, rebates, trade-in value for your old car, and so on.

As more of these factors are discussed at once, the negotiation becomes more and more complex. You can help yourself by keeping the negotiations as simple as possible: negotiate one thing at a time, settle on that, and then negotiate the next factor. Keep track of what has been agreed to as you go along. When each factor has been negotiated, you will have the package deal.

Your ability to get a satisfying deal rests on your abilities as a negotiator. For this reason, many people who find that process distasteful or suspect that their skills are lacking find the car purchasing process distasteful. Dealers know this, and some will try to attract customers by being more transparent about their own costs and about prices. Some even promise the “no-dicker sticker” sale with no haggling over price at all.

As with any product in any market, the more information you have, the better you can negotiate. The more thorough your prepurchase activities, the more satisfying your purchase will be.

While you own the car, you will maximize the benefits enjoyed by operating the vehicle safely and by keeping it in good condition. Routine maintenance (e.g., replacing fluids, rotating tires) can ensure the quality and longevity of your vehicle. New cars come with owner’s manuals that detail a schedule of service requirements and good driving practices for your vehicle. You will be required to keep the car legally insured and registered with the state where you reside, and you must maintain a valid license to drive.

New cars, and some used cars, are sold with a

**warranty**, which is a promise about the quality of the product, made for a certain period of time. The terms and covered repair costs may vary. You should understand the terms of the warranty, especially if something covered should need servicing, so that you know what repairs you may be charged for. The manufacturer, and sometimes the seller, issues the warranty. If you have questions about the warranty after purchasing, it may be best to contact the manufacturer directly.

If you are dissatisfied with your purchase (and the fault seems to be with the car), your first step should be a conversation with your dealer. If the problem is not addressed, you can contact the automobile company directly; its Web site will provide you with a customer service contact. If the dealer and the manufacturer refuse to make good, you should contact your state's consumer affairs division in the attorney general's office. In some states, there are entire state agencies or departments devoted to auto purchases.

For his first car Ray bought a ten-year-old coupe with only 60,000 miles on it for a price that seemed too good to be true. The seller said the good price was in exchange for getting payment in full in cash. The car broke down right away, however, and within two weeks died of a cracked block. When Ray complained, the seller claimed he didn't know about the cracked block and pointed out that there was no warranty on the car, so Ray was out of luck. Fortunately, Ray had read that a defective car, referred to as a "lemon," is covered under laws that protect consumers who unknowingly purchase a car that proves to be defective. **Lemon laws** regulate sales terms, purchase cancellation conditions, and warranty requirements. These laws are enforced on both the

federal and state level in the United States. Other consumer protection laws apply specifically to motor vehicles and vary by state. Ray learned that laws in his state include used cars as well as new ones, and when he told the seller, he was able to get most of his cash back.



## 2.4 MORTGAGES PART I: TERMINOLOGY

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### RENTING VERSUS OWNING<sup>1</sup>

If you have already decided on a goal of home ownership, you have already compared the costs and benefits of the alternative, which is renting. Renting requires relatively few initial legal or financial commitments. The renter signs a lease that spells out the terms of the rental agreement: term, rent, terms of payments and fees, restrictions such as pets or smoking, and charges for damages. A renter is usually required to give the landlord a security deposit to cover the landlord's costs of repairs or cleaning, as necessary, when the tenant moves out. If the deposit is not used, it is returned to the departing tenant (although without any interest earned).



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<https://pressbooks.nsc.ca/mathofmoney/?p=59#oembed-1>

1. Adapted from [9.1 Identify the Product and the Market](#) in [PERSONAL FINANCE](#) by Lumen Learning shared Under a [CC BY-NC-SA](#) license.

Some general advantages and disadvantages of renting and owning are shown in Figure 2.4.1 below.

	Advantage	Disadvantage
<b>Renting</b>	<ul style="list-style-type: none"><li>• Limited financial obligation</li><li>• Limited maintenance expenses</li><li>• More liquidity</li><li>• More mobility</li></ul>	<ul style="list-style-type: none"><li>• No equity growth or store of value</li><li>• Lifestyle limitations (e.g., pets, smoking)</li><li>• Decorating/renovating limitations</li><li>• Less predictable housing expense</li></ul>
<b>Owning</b>	<ul style="list-style-type: none"><li>• Store of value and possible equity growth</li><li>• Lifestyle choices</li><li>• Decorating/renovating choices</li><li>• Pride of ownership</li><li>• Tax deduction for mortgage interest</li><li>• More predictable housing expenses</li></ul>	<ul style="list-style-type: none"><li>• Substantial financial obligation</li><li>• Significant annual expenses</li><li>• Less liquidity</li><li>• Less mobility</li></ul>

Figure 2.4.1: Renting versus Owning

The choice of whether to rent or to own follows the pattern of life stages. People rent early in their adult lives because they typically have fewer financial resources and put a higher value on mobility, usually to keep more career flexibility. Since incomes are usually low, the tax advantages of ownership don't have much benefit.

As family size grows, the quality of life for dependents typically takes precedence, and a family looks for the added space and comfort of a home and its benefits as an investment. This is the mid-adult stage of accumulating

assets and building wealth. As income rises, the tax benefit becomes more valuable, too.

Often, in retirement, with both incomes and family size smaller, older adults will downsize to an apartment, shedding responsibilities and financial commitments.

Home ownership decisions vary: some people just never want the responsibilities of ownership, while some just always want a place of their own.

Finding an apartment is much like finding a home in terms of assessing its attributes, comparing choices, and making a choice. Landlords, property managers, and agents all rent properties and use various media to advertise an available space. Since the rent for an apartment is a regular expense, financed from current income (not long-term debt), you need to find only the apartment and not the financing, which simplifies the process considerably.

## AFFORDABLE HOUSING

We will do calculations shortly, but before we do let us talk a bit about what affordable housing means.

Before looking for a house that offers what you want, you need to identify a price range that you can afford. Most people use financing to purchase a home, so your ability to access financing or get a loan will determine the price range of the house you can buy. Since your home and your financing are long-term commitments, you need to be careful to try to include future changes in your thinking.

For example, Jill and Jack are both twenty-five years old, newly married, and looking to buy their first home. Both work and earn good incomes. The real estate market

is strong, especially with mortgage rates relatively low. They buy a two-bedroom condo in a new development as a starter home.

Fast-forward five years. Jill is expecting their second child; while the couple is happy about the new baby, neither can imagine how they will all fit in their already cramped space. They would love to sell the condo and purchase a larger home with a yard for the kids, but the real estate market has slowed, mortgage rates have risen, and a plant closing last year has driven up unemployment in their area. Jill hasn't worked outside the home since their first child was born two years ago—they are just getting by on one salary and a new baby will increase their expenses—making it even more difficult to think about financing a larger home.

A lender will look at your income, your current debts, and credit history to assess your ability to assume a mortgage. Your credit score is an important tool for the lender, who may also request verification of employment and income from your employer.



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Lenders do their own calculations of how much debt you can afford, based on a reasonable percentage, usually about 33 percent, of your monthly gross income that should go toward your monthly housing costs, or

principal, interest, taxes, and insurance (PITI). If you have other debts, your PITI plus your other debt repayments should be no more than about 38 percent of your gross income. Those percentages will be adjusted for income level, credit score, and amount of the down payment.

Say the lender assumes that 38 percent of your monthly gross income (annual gross income divided by twelve) should cover your PITI plus any other debt payments. Subtracting your other debt payments and estimated cost of taxes and insurance leaves you with a figure for affordable monthly mortgage payments. Dividing that figure by the mortgage factor for your mortgage's maturity and mortgage rate shows the affordable mortgage overall. Knowing what percentage your mortgage will be of the home's purchase price, you can calculate the maximum purchase price of the home that you can afford. That affordable home purchase price is based on your gross income, other debts, taxes, insurance, mortgage rate, mortgage maturity, and down payment.

These kinds of calculations give both you and your lender a much clearer idea of what you can afford. You may want to sit down with a potential lender and have this discussion before you do any serious house hunting, so that you have a price range in mind before you shop. Mortgage affordability calculators are also available online.

## THE SEARCH PROCESS

*Two Cents: How Do You Actually Buy a House? (all rights reserved)*



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After understanding exactly what you are looking for in a home and what you can afford, you can organize your efforts and begin your search.

Typically, buyers use a realtor and realty listings to identify homes for sale. A real estate broker can add value to your search by providing information about the house and property, the neighborhood and its schools, recreational and cultural opportunities, and costs of living.

Remember, however, that the broker or its agent, while helping you gather information and assess your choices, is working for the sellers and will be compensated by the seller when a sale is made. Consider paying for the services of a buyer's agent, a fee-based real estate broker who works for the buyer to identify choices independently of the purchase. The real estate industry is regulated by state and federal laws as well as by self-regulatory bodies, and real estate agents must be licensed to operate.

Increasingly, sellers are marketing their homes directly to save the cost of using a broker. A real estate broker typically takes a negotiable amount up to 6 percent of the purchase price, from which it pays a commission to the real estate agent. "For sale by owner" sites on the

Internet can make the exchange of housing information easier and more convenient for both buyers and sellers. For example, Web sites such as [Picketfencepreview.com](http://Picketfencepreview.com) serve home sellers and buyers directly. Keep in mind, however, that sellers acting as their own brokers and agents are not licensed or regulated and may not be knowledgeable about federal and state laws governing real estate transactions, potentially increasing your risk.

After you narrow your search and choose a prospective home in your price range, you have the home inspected to assess its condition and project the cost of any repairs or renovations. Many states require a home inspection before signing a purchase agreement or as a condition of the agreement.

As with a car, it is best to hire a professional (a structural engineer, contractor, or licensed home inspector) to do the home inspection. For example, see the [American Association of Home Inspectors](#). A professional will be able to spot not only potential problems but also evidence of past problems that may have been fixed improperly or that may recur—for example, water in the basement or leaks in the roof. If there are problems, you will need an estimate for the cost of fixing them. If there are significant and immediate repair or renovation costs projected by the home's condition, you may try to reduce the purchase price of the property by those costs. You don't want any surprises after you buy the house, especially costly ones.

You will also want to do a title search, as required by your lender, to verify that there are no liens or claims outstanding against the property. For example, the previous owners may have had a dispute with a contractor and never paid his bill, and the contractor may

have filed a lien or a claim against the property that must be resolved before the property can change hands. There are several other kinds of liens; for example, a tax lien is imposed to secure payment of overdue taxes.

**Note:** The next two videos are kind of boring and full of legal concepts, but are very important!

*Khan Academy: Titles and Deeds in Real Estate (CC BY)*



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<https://pressbooks.nsc.ca/mathofmoney/?p=59#oembed-4>

*Khan Academy: Title Insurance (CC BY)*



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<https://pressbooks.nsc.ca/mathofmoney/?p=59#oembed-5>

A lawyer or a title search company can do the search, which involves checking the municipal or town records where a lien would be filed. A title search will also reveal if previous owners have deeded any rights—such as development rights or water rights, for example, or



grants of right-of-way across the property—that would diminish its value.

## THE PURCHASE PROCESS

Now that you've chosen your home and figured out the financing, all that's left to do is sign the papers, right?

Once you have found a house, you will make an offer to the seller, who will then accept or reject your offer. If the offer is rejected, you may try to negotiate with the seller or you may decide to forgo this purchase. If your offer is accepted, you and the seller will sign a formal agreement called a purchase and sale agreement, specifying the terms of the sale. You will be required to pay a nonrefundable deposit, or earnest money, when the purchase and sale agreement is signed. That money will be held in escrow or in a restricted account and then applied toward the closing costs at settlement.

The purchase and sale agreement will include the following terms and conditions:

- A legal description of the property, including boundaries, with a site survey contingency
- The sale price and deposit amount
- A mortgage contingency, stating that the sale is contingent on the final approval of your financing
- The closing date and location, mutually agreed upon by buyer and seller
- Conveyances or any agreements made as part of the offer—for example, an agreement as to whether the kitchen appliances are sold with the house

- A home inspection contingency specifying the consequences of a home inspection and any problems that it may find, if not already completed and included in the price negotiation
- Possession date, usually the closing date
- A description of the property insurance policy that will cover the home until the closing date

Property disclosures of any problems with the property that must be legally disclosed, which vary by state, except that lead-paint disclosure is a federal mandate for any housing built before 1978.

After the purchase and sale agreement is signed, any conditions that it specified must be fulfilled before the closing date. If those conditions are the seller's responsibility, you will want to be sure that they have been fulfilled before closing. Read all the documents before you sign them and get copies of everything you sign. Do not hesitate to ask questions. You will live with your mortgage, and your house, for a long time.

## THE COSTS OF A MORTGAGE<sup>2</sup>

Banks, credit unions, finance companies, and mortgage finance companies sell mortgages. They profit by lending and competing for borrowers. It makes sense to shop around for a mortgage, as rates and terms (i.e., the borrowers' costs and conditions) may vary widely. The Internet has made it easy to compare; a quick search for "mortgage rates" yields many Web sites that provide

2. Adapted from [9.2 IDENTIFY THE FINANCING](#) in [PERSONAL FINANCE](#) by Lumen Learning shared Under a [CC BY-NC-SA](#) license.

national and state averages, lenders in your area, comparable rates and terms, and free mortgage calculators.

You may feel more comfortable getting your mortgage through your local bank, which may process the loan and then sell the mortgage to a larger financial institution. The local bank usually continues to service the loan, to collect the payments, but those cash flows are passed through to the financial institution (usually a much larger bank) that has bought the mortgage. This secondary mortgage market allows your local bank to have more liquidity and less risk, as it gets repaid right away, allowing it to make more loans. As long as you continue to make your payments, your only interaction is with the bank that is servicing the loan. Alternatively, local banks may earmark a percentage of mortgages to keep “in house” rather than sell.

The U.S. government assists some groups to obtain home loans, such as Native Americans, Americans with disabilities, and veterans.

Keep in mind that the costs discussed in this chapter, associated with various kinds of mortgages, may change. The real estate market, government housing policies, and government regulation of the mortgage financing market may change at any time. When it is time for you to shop for a mortgage, therefore, be sure you are informed of current developments.

*Brian Kimball: How Does Escrow Work? (all rights reserved)*



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## Types of Mortgages

### *Traditional Mortgage*

The mortgage we will discuss in this class is just a traditional fixed-rate mortgage. As mentioned, failure to provide a 20% down payment will require the payment of private mortgage insurance. The bank may also set other requirements like the purchase of title insurance and homeowner's insurance. Banks are governed by all applicable anti-discrimination and truth-in-lending laws and regulations.

### *VA Loan*<sup>3</sup>

A **VA loan** is a mortgage loan in the United States guaranteed by the United States Department of Veterans Affairs (VA). The program is for American veterans, military members currently serving in the U.S. military, reservists and select surviving spouses (provided they do not remarry) and can be used to purchase single-family homes, condominiums, multi-unit properties, manufactured homes and new construction. The VA does not originate loans, but sets the rules for who may qualify,

3. Adapted from VA Loan. (2022, December 15). In *Wikipedia*.  
[https://en.wikipedia.org/wiki/VA\\_loan](https://en.wikipedia.org/wiki/VA_loan) CC BY-SA 3.0

issues minimum guidelines and requirements under which mortgages may be offered and financially guarantees loans that qualify under the program.

The basic intention of the VA home loan program is to supply home financing to eligible veterans and to help veterans purchase properties with no down payment. The loan may be issued by qualified lenders.

The VA loan allows veterans 103.3 percent financing without private mortgage insurance (PMI) or a 20 percent second mortgage and up to \$6,000 for energy efficient improvements. A VA funding fee of 0 to 3.3% of the loan amount is paid to the VA; this fee may also be financed and some may qualify for an exemption. In a purchase, veterans may borrow up to 103.3% of the sales price or reasonable value of the home, whichever is less. Since there is no monthly PMI, more of the mortgage payment goes directly towards qualifying for the loan amount, allowing for larger loans with the same payment. In a refinance, where a new VA loan is created, veterans may borrow up to 100% of a property's reasonable value, where allowed by state laws. In a refinance where the loan is a VA loan refinancing to VA loan (IRRRL Refinance), the veteran may borrow up to 100.5% of the total loan amount. The additional .5% is the funding fee for a VA Interest Rate Reduction Refinance.

#### *FHA Loan*<sup>4</sup>

An **FHA insured loan** is a US Federal Housing Administration mortgage insurance backed mortgage loan that is provided by an FHA-approved lender. FHA

4. Adapted from FHA Insured Loan. (2023, January 19). In Wikipedia.  
[https://en.wikipedia.org/wiki/FHA\\_insured\\_loan](https://en.wikipedia.org/wiki/FHA_insured_loan) CC BY-SA 3.0

insured loans are a type of federal assistance. They have historically allowed lower-income Americans to borrow money to purchase a home that they would not otherwise be able to afford. Because this type of loan is more geared towards new house owners than real estate investors, FHA loans are different from conventional loans in the sense that the house must be owner-occupant for at least a year.<sup>[1]</sup> Since loans with lower down-payments usually involve more risk to the lender, the home-buyer must pay a two-part mortgage insurance that involves a one-time bulk payment and a monthly payment to compensate for the increased risk.<sup>[1]:15</sup>

The program originated during the Great Depression of the 1930s when the rates of foreclosures and defaults rose sharply, and the program was intended to provide lenders with sufficient insurance. The government subsidized some FHA programs, but the goal was to make it self-supporting based on borrowers' insurance premiums. Over time, private mortgage insurance (PMI) companies came into play. Now FHA primarily serves people who cannot afford a conventional down payment or do not qualify for PMI. The program has since this time been modified to accommodate the heightened recession.

Down payment assistance and community redevelopment programs offer affordable housing opportunities to first-time homebuyers, low- and moderate-income individuals, and families who wish to achieve homeownership. Grant types include seller funded programs, the Grant America Program and others, as well as programs that are funded by the federal government, such as the American Dream Down Payment Initiative. Many down payment grant programs

are run by state and local governments, often using mortgage revenue bond funds.

The FHA employs a two-tiered mortgage insurance premium (MIP) schedule. To obtain mortgage insurance from the Federal Housing Administration, an upfront *mortgage insurance premium* (UFMIP) equal to 1.75% of the base loan amount at closing is required, and is normally financed into the total loan amount by the lender and paid to FHA on the borrower's behalf. There is also a monthly mortgage insurance premium (MIP) which varies based on the amortization term and loan-to-value ratio.<sup>[27]</sup>

The monthly payment is not permanent, however, as there are several ways to get rid of the MIP. One way to remove the monthly payment is to establish at least a 20% equity on the FHA loan, which will allow the homeowner to apply for a refinance on their loan. Since the MIP is to ensure extra security against defaulting, if there are signs of financial instability, the refinance may be declined. However, the easiest and most guaranteed way to remove the MIP is to establish a 22% equity; after which, the mortgage insurance is automatically removed by the lender and is no longer required to be paid.<sup>[28][29]</sup>

## DOWN PAYMENT

Mortgages require a down payment, or a percentage of the purchase price paid in cash upon purchase. Most buyers use cash from savings, the proceeds of a house they are selling, or a family gift.

The size of the down payment does not affect the price of the house, but it can affect the cost of the financing. For a certain house price, the larger the down payment, the smaller the mortgage and, all things being equal, the lower the monthly payments.

We will explore the impact of different sized down payments once we run calculations.

Usually, if the down payment is less than 20 percent of the property's sale price, the borrower has to pay for private mortgage insurance (PMI), which insures the lender against the costs of default. A larger down payment eliminates this expense for the borrower.

The down payment can offset the annual cost of the financing, but it creates opportunity cost and decreases your liquidity as you take money out of savings. Cash will also be needed for the closing costs or transaction costs of this purchase or for any immediate renovations or repairs. Those needs will have to be weighed against your available cash to determine the amount of your down payment.

**Closing costs can run anywhere from 2-5% of the price of the home.** Let us consider a case where you want to buy a \$200,000 house with a 20% down payment and 4% closing costs. This means that you would need close to \$50,000 in cash to purchase the house! If you are not able to come up with that much, there are other options available but, as mentioned earlier, do come with PMI.



## MONTHLY PAYMENT

The monthly payment is the ongoing cash flow obligation of the loan. If you don't pay this payment, you are in default on the loan and may eventually lose the house with no compensation for the money you have already put into it. Your ability to make the monthly payment determines your ability to keep the house.

The interest rate and the maturity (lifetime of the mortgage) determine the monthly payment amount. With a fixed-rate mortgage, the interest rate remains the same over the entire maturity of the mortgage, and so does the monthly payment. Conventional mortgages are fixed-rate mortgages for thirty, twenty, or fifteen years.

The longer the maturity, the greater the interest rate, because the lender faces more risk the longer it takes for the loan to be repaid.

A fixed-rate mortgage is structured as an annuity: regular periodic payments of equal amounts. Some of the payment is repayment of the principal and some is for the interest expense. As you make a payment, your balance gets smaller, and so the interest portion of your next payment is smaller, and the principal payment is larger. In other words, as you continue making payments, you are paying off the balance of the loan faster and faster and paying less and less interest.

In the early years of the mortgage, your payments are mostly interest, while in the last years they are mostly principal. It is important to distinguish between them because the mortgage interest is tax deductible. That tax benefit is greater in the earlier years of the mortgage, when the interest expense is larger. We will take a look at amortization schedules shortly.

## Points

Points are another kind of financing cost. One point is one percent of the mortgage. Points are paid to the lender as a form of prepaid interest when the mortgage originates and are used to decrease the mortgage rate. In other words, paying points is a way of buying a lower mortgage rate.

In deciding whether or not it is worth it to pay points, you need to think about the difference that the lower mortgage rate will make to your monthly payment and how long you will be paying this mortgage. How long will it take for the points to pay for themselves in reduced monthly payments? For example, suppose you have the following choices for a thirty-year, fixed rate, \$200,000 mortgage: a mortgage rate of 6.5 percent with no points or a rate of 6 percent with 2 points. Paying the two points buys you a lower monthly payment and saves you \$64 dollars per month. The two points cost \$4,000 (2 percent of \$200,000). At the rate of \$64 per month, it will take 62.5 months ( $\$4,000 \div 64$ ) or a little over five years for those points to pay for themselves. If you do not plan on having this mortgage for that long, then paying the points is not worth it. Paying points has liquidity and opportunity costs up front that must be weighed against its benefit. Points are part of the closing costs, but borrowers do not have to pay them if they are willing to pay a higher interest rate instead.

## Closing Costs

Other costs of a house purchase are transaction costs, that is, costs of making the transaction happen that are not direct costs of either the home or the financing. These

are referred to as closing costs, as they are paid at the closing, the meeting between buyer and seller where the ownership and loan documents are signed and the property is actually transferred. The buyer pays these closing costs, including the appraisal fee, title insurance, and filing fee for the deed.

The lender will have required an independent appraisal of the home's value to make sure that the amount of the mortgage is reasonable given the value of the house that secures it. The lender will also require a title search and contract for title insurance. The title company will research any claims or liens on the deed; the purchase cannot go forward if the deed may not be freely transferred. Over the term of the mortgage, the title insurance protects against flaws not found in the title and any claims that may result. The buyer also pays a fee to file the property deed with the township, municipality, or county. Some states may also have a property transfer tax that is the responsibility of the buyer.

Closings may take place in the office of the title company handling the transaction or at the registry of deeds. Closings also may take place in the lender's offices, such as a bank, or an attorney's office and usually are mediated between the buyer and the seller through their attorneys. Lawyers who specialize in real estate ensure that all legal requirements are met and all filings of legal documents are completed. For example, before signing, home buyers have a right to review a U.S. Housing and

Urban Development (HUD) Settlement Statement twenty-four hours prior to the closing. This document, along with a truth-in-lending disclosure statement, sets out and explains all the terms of the transaction, all the costs of buying the house, and all closing costs. Both the buyer and the seller must sign the HUD document and are legally bound by it.

## Property Taxes<sup>5</sup>

A **property tax** or millage rate is an ad valorem tax on the value of a property, usually levied on real estate. The tax is levied by the governing authority of the jurisdiction in which the property is located. This can be a national government, a federated state, a county or geographical region or a municipality. Multiple jurisdictions may tax the same property.

It may be imposed annually or at the time of a real estate transaction, such as in real estate transfer tax. This tax can be contrasted to a rent tax, which is based on rental income or imputed rent, and a land value tax, which is a levy on the value of land, excluding the value of buildings and other improvements.

Under a property-tax system, the government requires or performs an appraisal of the monetary value of each property, and tax is assessed in proportion to that value.

The four broad types of property taxes are land, improvements to land (immovable man-made objects, such as buildings), personal property (movable man-made objects) and intangible property. Real property (also called real estate or realty) is the combination of land and

5. Adapted from Property Tax. (2023, January 2). In *Wikipedia*. [https://en.wikipedia.org/wiki/Property\\_tax](https://en.wikipedia.org/wiki/Property_tax) CC BY-SA 3.0

improvements. We will focus on real property taxes in this class.

The *property tax rate* is typically given as a percentage. It may be expressed as a per mil (amount of tax per thousand currency units of property value), which is also known as a *millage rate* or *mill* (one-thousandth of a currency unit). To calculate the property tax, the authority multiplies the assessed value by the mill rate and then divides by 1,000. For example, a property with an assessed value of \$50,000 located in a municipality with a mill rate of 20 mills would have a property tax bill of \$1,000 per year.<sup>[5]</sup>

## Homeowner's Insurance<sup>6</sup>

**Property insurance** provides protection against most risks to property, such as fire, theft and some weather damage. This includes specialized forms of insurance such as fire insurance, flood insurance, earthquake insurance, home insurance, or boiler insurance. Property is insured in two main ways—open perils and named perils.

Open perils cover all the causes of loss not specifically excluded in the policy. Common exclusions on open peril policies include damage resulting from earthquakes, floods, nuclear incidents, acts of terrorism, and war. Named perils require the actual cause of loss to be listed in the policy for insurance to be provided. The more common named perils include such damage-causing events as fire, lightning, explosion, and theft.

There are three types of insurance coverage.

6. Adapted from Property Insurance. (2023, January 28). In *Wikipedia*.  
[https://en.wikipedia.org/wiki/Property\\_insurance](https://en.wikipedia.org/wiki/Property_insurance) CC BY-SA 3.0

Replacement cost coverage pays the cost of repairing or replacing the property with like kind & quality regardless of depreciation or appreciation. Premiums for this type of coverage are based on replacement cost values, and not based on actual cash value.<sup>[5]</sup> Actual cash value coverage provides for replacement cost minus depreciation. Extended replacement cost will pay over the coverage limit if the costs for construction have increased. This generally will not exceed 25% of the limit. When obtaining an insurance policy, the limit is the maximum amount of benefit the insurance company will pay for a given situation or occurrence. Limits also include the ages below or above what an insurance company will not issue a new policy or continue a policy.<sup>[6]</sup>

This amount will need to fluctuate if the cost to replace homes in a neighbourhood is rising; the amount needs to be in step with the actual reconstruction value of the home. In case of a fire, household content replacement is tabulated as a percentage of the value of the home. In case of high-value items, the insurance company may ask to specifically cover these items separate from the other household contents. One last coverage option is to have alternative living arrangements included in a policy. If property damage caused by a covered loss prevents a person from living in their home, policies can pay the expenses of alternate living arrangements (e.g., hotels and restaurant costs) for a specified period of time to compensate for the “loss of use” of the home until the owners can return. The additional living expenses limit can vary, but is typically set at up to 20% of the dwelling coverage limit. Owners need to talk with their insurance company for advice about appropriate coverage and determine what type of limit may be appropriate.<sup>[7]</sup>

Banks generally require insurance on the structure since they are part owners. This is generally included in your monthly bill which we will talk about later.

### **Adjustable Rate Mortgages**

I will not discuss adjustable rate mortgages in this class other than to give information in this section. If you plan on remaining in a house, these are rarely a good idea. Think of these like the “low-low introductory offer” where you pay only \$1 per month for a gym membership...but then realize you are also locked into \$49.99/month for a year once the trial ends.

So far, the discussion has focused on fixed-rate mortgages, that is, mortgages with fixed or constant interest rates, and therefore payments, until maturity. With an adjustable-rate mortgage (ARM), the interest rate—and the monthly payment—can change. If interest rates rise, the monthly payment will increase, and if they fall, it will decrease. By federal law, increases in ARM interest rates cannot rise more than 2 percent at a time, but even with this rate cap, homeowners with ARMs are at risk of seeing their monthly payment increase. Borrowers can limit this interest rate risk with a payment cap, which, however, introduces another risk.

A payment cap limits the amount by which the payment can increase or decrease. That sounds like it would protect the borrower, but if the payment is capped and the interest rate rises, more of the payment pays for the interest expense and less for the principal payment, so the balance is paid down more slowly. If interest rates are high enough, the payment may be too small to pay all the interest expense, and any interest not paid will add to the principal balance of the mortgage.

In other words, instead of paying off the mortgage, your payments may actually increase your debt, and you could end up owing more money than you borrowed, even though you make all your required payments on time. This is called negative amortization. You should make sure you know if your ARM mortgage is this type of loan. You can voluntarily increase your monthly payment amount to avoid the negative effects of a payment cap.

Adjustable-rate mortgages are risky for borrowers. ARMs are usually offered at lower rates than fixed-rate mortgages, however, and may be more affordable. Borrowers who expect an increase in their disposable incomes, which would offset the risk of a higher payment, or who expect a decrease in interest rates, may prefer an adjustable-rate mortgage, which can have a maturity of up to forty years. Otherwise, a fixed-rate mortgage is better.

There are mortgages that combine fixed and variable rates—for example, offering a fixed rate for a specified period of time, and then an adjustable rate. Another type of mortgage is a balloon mortgage that offers fixed monthly payments for a specified period, usually three, five, or seven years, and then a final, large repayment of the principal. There are option ARMs, where you pay either interest only or principal only for the first few years of the loan, which makes it more affordable. While you are paying interest only, however, you are not accumulating equity in your investment.

We will only concern ourselves with traditional fixed-rate mortgages in this class. *Bank of America: Fixed versus Adjustable Rate Mortgages (all rights reserved)*

<https://youtu.be/rJgNbVTYgwM>





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<https://pressbooks.nscc.ca/mathofmoney/?p=59#h5p-9>

## 2.5 MORTGAGES: PART II - CALCULATIONS

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### P&I Payments

When you borrow money, you must pay back what you borrowed (principal) plus extra (interest) to the lender. While your actual mortgage payment will be higher (discussed shortly), we will begin by reviewing the loan payment problems from last module.

Let's try one together...

Sara took out a 30-year mortgage for \$157,000 at a fixed rate of 7.2%. Calculate her monthly payment.

Answer: \$1,065/month



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<https://pressbooks.nsc.ca/mathofmoney/?p=61#oembed-1>

### PMI

When you are unable to produce a down payment of 20%,

banks generally require private mortgage insurance or PMI. Once a borrower surpasses 20% equity, they can stop paying for PMI. As mentioned earlier, PMI is meant to cover the costs of foreclosure for the bank in the event you are unable to continue to pay for the house.

### Let's try one together...

Jo and Zane bought a house for \$146,000, taking out a \$135,000 mortgage. Its been three years and their mortgage balance has dropped to \$132,833.15 and due to renovations, the market value of the house has increased to \$168,000. Can they apply to have PMI removed?

Answer: Yes (20.9% equity)



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<https://pressbooks.nsc.ca/mathofmoney/?p=61#oembed-2>

## Property Tax

Jurisdictions will levy a tax on the assessed value of a property. It should be noted that the assessed value is not necessarily the market value. Some jurisdictions only assess at a certain percentage of the market value.

Additionally, homeowners generally have to pay multiple property taxes. In Pennsylvania it is typical to have to pay property taxes to your city or township, your

county, and your school district. Some of these bills are combined (generally the city/township and county) or each bill may be separate.

As mentioned, property tax is generally given in mills, which is thousandths of a dollar. So if the tax rate is 20 mills, then for every dollar of appraised value, you owe \$1. Some examples follow...

### Let's try one together...

Hamid has a house with a market value of \$525,000 in Peters Township. His jurisdiction uses an appraisal value of 100% of fair market value. The county rate is 2.43 mills, the township rate is 1.622 mills and the school district rate is 14.16 mills. Calculate his property tax bill.

Answer: \$9,564.30



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<https://pressbooks.nsc.ca/mathofmoney/?p=61#oembed-3>

### Let's try one together...

Anne has a house with a market value of \$180,000. Her jurisdiction uses a appraisal value of 80% of fair market

value. The county rate is 11 mills, the city rate is 8 mills and the school district rate is 30 mills. Calculate her total property tax bill.

Answer: \$7,056



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<https://pressbooks.nsc.ca/mathofmoney/?p=61#oembed-7>

Tax rates can vary greatly between municipalities, school districts, and counties. For instance, the total tax rate for a house in Peters Township is around 18 mills whereas the total tax rate for a house in Jefferson Hills is around 30 mills. That means that a \$400,000 house in Peters Township will cost you \$7,200 per year whereas the same house in Jefferson Hills would cost you \$12,000 per year. That comes out to an extra \$400 per month for the house in Jefferson Hills.

## PITI

As mentioned earlier, your mortgage payment is generally more than just the principal and interest. To prevent non-payment of taxes (which would put the ownership of the property in peril) or non-payment of property insurance (which would be disastrous for the bank if something happened to the property and the insurance wasn't active), banks will generally require you

to pay money into an escrow account. An escrow account is simply an account where your money is held. The bank has access to this account and will use the funds to pay your property taxes, homeowner's insurance, and PMI. Therefore, the actual amount you pay each month is called the PITI payment which stands for **P**rincipal, **I**nterest, **T**axes, and **I**nsurance payment. We will run through a few examples, but before we do it is important to point out that when determining what you can afford, you need to look at the PITI and not the P&I. As we saw in the property tax section, houses with the same price tag could have very different PITI payments based on the tax rates.

### Let's try one together...

Jan purchases a house in Jefferson Hills. The purchase price (and we will assume the assessed value) was \$375,000. Jan makes a 10% down payment on the house. The bank approves Jan for a 30-year mortgage at a rate of 3.75%. The bank also makes Jan pay for property tax, homeowners insurance, and PMI with each payment. In Jefferson Hills, the property tax rates are as follows: county rate of 4.73 mills, township rate of 5.66 mills, and school rate of 20.236 mills. The homeowners insurance is expected to cost \$1,200/year. The monthly PMI is \$120. Calculate the PITI payment.

Answer: \$2,740.08



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Let's try one together...

Repeat the problem above, but let the house be located in East Washington. The Washington County rate is 2.43 mills, the East Washington Borough rate is 2.75 mills, and the Washington School tax is 15.158 mills.

Answer: \$2,418.58



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<https://pressbooks.nsc.ca/mathofmoney/?p=61#oembed-4>

## Amortization Schedule

We talked about amortization schedules in the last

module, but let us take a look at the initial payments for a mortgage.

### Let's try one together...

Jack purchases a house for \$300,000. He puts 20% down. The bank charges him a rate of 3.75%. Create an amortization schedule for the first three payments. Note: You need to calculate the payment!  
Answer: Payment = \$1,111.48, see video for amortization table



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<https://pressbooks.nsc.ca/mathofmoney/?p=61#oembed-5>

### Let's try one together...

Suppose that Jack only puts 10% down. Calculate his new payment. Compare his total expenditures (be sure to include the down payment) between 10% and 20% down payments.  
Answer: See video (pays about \$20,000 less with 20% down)





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<https://pressbooks.nsc.ca/mathofmoney/?p=61#oembed-6>

# MODULE 2 PRACTICE PROBLEMS

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**Problem 2.1:** Hendrik has a credit card that charges him 16.99% annual interest. Below is his statement summary for July.

Date	Activity
July 1	Opening Balance of \$1,500
July 9	Purchase of \$600
July 14	Purchase of \$200
July 19	Payment of \$1,000
July 28	Purchase of \$100

1. Calculate the ADB.
2. Calculate the interest payment.
3. Calculate the minimum monthly payment (using interest + 2% rule).

Answers: ABD=\$1,654.84; \$23.88; \$51.88



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<https://pressbooks.nsc.ca/mathofmoney/?p=63#oembed-1>

**Problem 2.2:** Eduards has a credit card that charges him 26.99% annual interest. Below is his statement summary for November.

Date	Activity
Nov 1	Opening Balance of \$3,500
Nov 3	Purchase of \$500
Nov 11	Purchase of \$800
Nov 18	Purchase of \$600
Nov 24	Payment of \$500
Nov 27	Purchase of \$900

1. Calculate the ADB.
2. Calculate the interest payment.
3. Calculate the minimum monthly payment (using interest + 2% rule).

Answers: ABD=\$4,763.33; \$105.67; \$221.67



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<https://pressbooks.nsc.ca/mathofmoney/?p=63#oembed-2>

**Problem 2.3:** Islam purchases a house in Bethel Park for \$275,000. He made a 20% down payment so there will be no PMI. The bank approves a 30-year mortgage at a rate of 3.5%. The bank also makes Islam pay for property tax, homeowners insurance, and PMI with each payment. In Bethel Park, the property tax rates are as follows: county rate of 4.73 mills, municipality rate of 2.73 mills, and school rate of 21.7654 mills. The homeowners insurance is expected to cost \$800/year. Calculate the PITI payment.

Answer: \$1,724.32/month



One or more interactive elements has been excluded from this version of the text. You can view them online here:

<https://pressbooks.nsc.ca/mathofmoney/?p=63#oembed-3>

**Problem 2.4:** Harriet purchases a house in Houston (Pa) for \$195,000. She made a 5% down payment so there will be PMI. The bank approves a 30-year mortgage at a rate of 4.55%. The bank also makes Harriet pay for property tax, homeowners insurance, and PMI with each payment. In Houston, the property tax rates are as follows: county rate of 2.43 mills, municipality rate of 2.52 mills, and

school rate of 12.576 mills. The homeowners insurance is expected to cost \$600/year and the PMI will cost \$80/month. Calculate the PITI payment.

Answer: \$1,358.95/month



One or more interactive elements has been excluded from this version of the text. You can view them online here:

<https://pressbooks.nsc.ca/mathofmoney/?p=63#oembed-4>

**Problem 2.5:** Jevon is looking to purchase a \$300,000 home. He will have a 20% downpayment. He is offered the following loan options:

- 30 years at 3.95%
- 20 years at 3.66%
- 15 years at 3.41%

Calculate the P&I payment for each. Additionally, find the difference in interest payments between the three options.

Answers: \$1,138.89; \$1,411.72; \$1,705.13 (see video for other information)



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view them online here: <https://pressbooks.nsc.ca/mathofmoney/?p=63#oembed-5>

**Problem 2.6:** Let us again consider Jevon from problem 2.5. For each of the three options, create the first month of the amortization schedule.

Answer: See video for amortization schedules.



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<https://pressbooks.nsc.ca/mathofmoney/?p=63#oembed-6>

**Problem 2.7:** Adrian wants to purchase a car for \$32,000. She has a trade-in with \$7,000 worth of equity but there are also \$1,500 worth of fees and taxes. She plans no down payment (other than the trade-in equity). The bank offers her the following loan options:

- 48-month at 1.99%
- 60-month at 2.99%
- 72-month at 3.99%
- 84-month at 4.99%

Calculate the payment for each. Additionally, calculate the difference in interest charges for the different loans.

Answers: \$574.81; \$476.05; \$414.48; \$374.42 (see video for other information)



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<https://pressbooks.nsc.ca/mathofmoney/?p=63#oembed-7>

**Problem 2.8:** Vladimir is going to purchase a car. He has no down payment nor does he have a trade-in. He has been approved for a 60-month loan at a rate of 5.75%. He wants a payment of no more than \$300 per month. What is the most expensive car he can afford?

Answer: \$15,611.40



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## **MODULE 3: INVESTING AND RETIREMENT**

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## 3.1 INVESTMENT BASICS

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<sup>1</sup>Before looking at investment planning and strategy, it is important to take a closer look at the galaxy of investments and markets where investing takes place. Understanding how markets work, how different investments work, and how different investors can use investments is critical to understanding how to begin to plan your investment goals and strategies.

You have looked at using the money markets to save surplus cash for the short term. Investing is primarily about using the capital markets to invest surplus cash for the longer term. As in the money markets, when you invest in the capital markets, you are selling liquidity.

The capital markets developed as a way for buyers to buy liquidity. In Western Europe, where many of our ideas of modern finance began, those early buyers were usually monarchs or members of the nobility, raising capital to finance armies and navies to conquer or defend territories or resources. Many devices and markets were used to raise capital,<sup>2</sup> but the two primary methods that have evolved into modern times are the bond and stock

1. Adapted from [12.1 Investments and Markets: A Brief Overview](#) in [Personal Finance](#) by Lumen Learning shared under a [CC BY-NC-SA](#) license.
2. For a thorough history of the evolution of finance and financial instruments, see Charles P. Kindleberger, *A Financial History of Western Europe* (London: George Allen & Unwin, Ltd., 1984).

markets. We will discuss each on their own later in this module, but a brief introduction is provided below.

*Smart Investing Trends: Investing 401 (all rights reserved)*



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<https://pressbooks.nsc.ca/mathofmoney/?p=66#oembed-2>

## BONDS AND BOND MARKETS

**Bonds** are debt. The bond issuer borrows by selling a bond, promising the buyer regular interest payments and then repayment of the principal at maturity. If a company wants to borrow, it could just go to one lender and borrow. But if the company wants to borrow a lot, it may be difficult to find any one investor with the capital and the inclination to make large a loan, taking a large risk on only one borrower. In this case the company may need to find a lot of lenders who will each lend a little money, and this is done through selling bonds.

A bond is a formal contract to repay borrowed money with interest (often referred to as the coupon) at fixed intervals. Corporations and governments (e.g., federal, state, municipal, and foreign) borrow by issuing bonds. The interest rate on the bond may be a **fixed interest rate** or a **floating interest rate** that changes as underlying interest rates—rates on debt of comparable companies—change. (Underlying interest rates include the prime rate that banks charge their most trustworthy borrowers and the target rates set by the Federal Reserve Bank.)

There are many features of bonds other than the principal and interest, such as the **issue price** (the price you pay to buy the bond when it is first issued) and the **maturity date** (when the issuer of the bond has to repay you). Bonds may also be “callable”: **redeemable** before **maturity** (paid off early). Bonds may also be issued with various **covenants** or conditions that the borrower must meet to protect the bondholders, the lenders. For example, the borrower, the bond issuer, may be required to keep a certain level of cash on hand, relative to its short-term debts, or may not be allowed to issue more debt until this bond is paid off.

Because of the diversity and flexibility of bond features, the bond markets are not as transparent as the stock markets; that is, the relationship between the bond and its price is harder to determine. The U.S. bond market is now more than twice the size (in dollars of capitalization) of all the U.S. stock exchanges combined, with debt of more than \$27 trillion by the end of 2007.<sup>3</sup>

U.S. Treasury bonds are auctioned regularly to banks and large institutional investors by the Treasury Department, but individuals can buy U.S. Treasury bonds directly from the U.S. government. To trade any other kind of bond, you have to go through a broker. The brokerage firm acts as a principal or dealer, buying from or selling to investors, or as an agent for another buyer or seller.

## STOCKS AND STOCK MARKETS

**Stocks** or equity securities are shares of ownership. When you buy a share of stock, you buy a share of the corporation. The size of your share of the corporation is proportional to the size of your stock holding. Since corporations exist to create profit for the owners, when you buy a share of the corporation, you buy a share of its future profits. You are literally sharing in the fortunes of the company.

Unlike bonds, however, shares do not promise you any returns at all. If the company does create a profit, some of that profit may be paid out to owners as a **dividend**, usually in cash but sometimes in additional shares of

4. Financial Industry Regulatory Authority (FINRA), <http://apps.finra.org/> (accessed May 20, 2009).

stock. The company may pay no dividend at all, however, in which case the value of your shares should rise as the company's profits rise. But even if the company is profitable, the value of its shares may not rise, for a variety of reasons having to do more with the markets or the larger economy than with the company itself. Likewise, when you invest in stocks, you share the company's losses, which may decrease the value of your shares.

Corporations issue shares to raise capital. When shares are issued and traded in a public market such as a **stock exchange**, the corporation is "publicly traded." There are many stock exchanges in the United States and around the world. The two best known in the United States are the New York Stock Exchange (now NYSE Euronext), founded in 1792, and the NASDAQ, a computerized trading system managed by the National Association of Securities Dealers (the "AQ" stands for "Automated Quotations").

Only members of an exchange may trade on the exchange, so to buy or sell stocks you must go through a broker who is a member of the exchange. Brokers also manage your account and offer varying levels of advice and access to research. Most brokers have Web-based trading systems. Some discount brokers offer minimal advice and research along with minimal trading commissions and fees.

# COMMODITIES AND DERIVATIVES

**Commodities** are resources or raw materials, including the following:

- Agricultural products (food and fibers), such as soybeans, pork bellies, and cotton
- Energy resources such as oil, coal, and natural gas
- Precious metals such as gold, silver, and copper
- Currencies, such as the dollar, yen, and euro

Commodity trading was formalized because of the risks inherent in producing commodities—raising and harvesting agricultural products or natural resources—and the resulting volatility of commodity prices. As farming and food production became mechanized and required a larger investment of capital, commodity producers and users wanted a way to reduce volatility by locking in prices over the longer term.

The answer was futures and forward contracts. **Futures** and **forward contracts** or *forwards* are a form of **derivatives**, the term for any financial instrument whose value is derived from the value of another security. For example, suppose it is now July 2010. If you know that you will want to have wheat in May of 2011, you could wait until May 2011 and buy the wheat at the market price, which is unknown in July 2010. Or you could buy it now, paying today's price, and store the wheat until May 2011. Doing so would remove your future price uncertainty, but you would incur the cost of storing the wheat.



Alternatively, you could buy a futures contract for May 2011 wheat in July 2010. You would be buying May 2011 wheat at a price that is now known to you (as stated in the futures contract), but you will not take delivery of the wheat until May 2011. The value of the futures contract to you is that you are removing the future price uncertainty without incurring any storage costs. In July 2010 the value of a contract to buy May 2011 wheat depends on what the price of wheat actually turns out to be in May 2011.

Forward contracts are traded privately, as a direct deal made between the seller and the buyer, while futures contracts are traded publicly on an exchange such as the Chicago Mercantile Exchange (CME) or the New York Mercantile Exchange (NYMEX).

When you buy a forward contract for wheat, for example, you are literally buying future wheat, wheat that doesn't yet exist. Buying it now, you avoid any uncertainty about the price, which may change. Likewise, by writing a contract to sell future wheat, you lock in a price for your crop or a return for your investment in seed and fertilizer.

Futures and forward contracts proved so successful in shielding against some risk that they are now written for many more types of "commodities," such as interest rates and stock market indices. More kinds of derivatives have been created as well, such as options. **Options** are the right but not the obligation to buy or sell at a specific price at a specific time in the future. Options are commonly written on shares of stock as well as on stock indices, interest rates, and commodities.

Derivatives such as forwards, futures, and options are used to hedge or protect against an existing risk or to

speculate on a future price. For a number of reasons, commodities and derivatives are more risky than investing in stocks and bonds and are not the best choice for most individual investors.

## MUTUAL FUNDS, INDEX FUNDS, AND EXCHANGE-TRADED FUNDS

A **mutual fund** is an investment portfolio consisting of securities that an individual investor can invest in all at once without having to buy each investment individually. The fund thus allows you to own the performance of many investments while actually buying—and paying the transaction cost for buying—only one investment.

Mutual funds have become popular because they can provide diverse investments with a minimum of transaction costs. In theory, they also provide good returns through the performance of professional portfolio managers.

An **index fund** is a mutual fund designed to mimic the performance of an index, a particular collection of stocks or bonds whose performance is tracked as an indicator of the performance of an entire class or type of security. For example, the Standard & Poor's (S&P) 500 is an index of

the five hundred largest publicly traded corporations, and the famous Dow Jones Industrial Average is an index of thirty stocks of major industrial corporations. An index fund is a mutual fund invested in the same securities as the index and so requires minimal management and should have minimal management fees or costs.

Mutual funds are created and managed by mutual fund companies or by brokerages or even banks. To trade shares of a mutual fund you must have an account with the company, brokerage, or bank. Mutual funds are a large component of individual retirement accounts and of defined contribution plans.

Mutual fund shares are valued at the close of trading each day and orders placed the next day are executed at that price until it closes. An **exchange-traded fund (ETF)** is a mutual fund that trades like a share of stock in that it is valued continuously throughout the day, and trades are executed at the market price.

The ways that capital can be bought and sold is limited only by the imagination. When corporations or governments need financing, they invent ways to entice investors and promise them a return. The last thirty years has seen an explosion in **financial engineering**, the innovation of new financial instruments through mathematical pricing models. This explosion has coincided with the ever-expanding powers of the computer, allowing professional investors to run the millions of calculations involved in sophisticated pricing models. The Internet also gives amateurs instantaneous access to information and accounts.

Much of the modern portfolio theory that spawned these innovations (i.e., the idea of using the predictability of returns to manage portfolios of investments) is based

on an infinite time horizon, looking at performance over very long periods of time. This has been very valuable for institutional investors (e.g., pension funds, insurance companies, endowments, foundations, and trusts) as it gives them the chance to magnify returns over their infinite horizons.

For most individual investors, however, most portfolio theory may present too much risk or just be impractical. Individual investors don't have an infinite time horizon. You have only a comparatively small amount of time to create wealth and to enjoy it. For individual investors, investing is a process of balancing the demands and desires of returns with the costs of risk, before time runs out.



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## RETURN

Returns are always calculated as annual rates of return, or the percentage of return created for each unit (dollar) of original value. If an investment earns 5 percent, for example, that means that for every \$100 invested, you would earn \$5 per year (because  $\$5 = 5\%$  of \$100).

Returns are created in two ways: the investment creates income or the investment gains (or loses) value. To calculate the annual rate of return for an investment, you need to know the income created, the gain (loss) in value, and the original value at the beginning of the year. The percentage return is calculated as:

$$\text{Return} = 100 \times (\text{Income} + \text{Current Value} - \text{Original Value}) / \text{Original Value}.$$

Note that if the ending value is greater than the original value, then  $\text{Ending value} - \text{Original value} > 0$  (is greater than zero), and you have a gain that adds to your return. If the ending value is less, then  $\text{Ending value} - \text{Original value} < 0$  (is less than zero), and you have a loss that detracts from your return. If there is no gain or loss, if  $\text{Ending value} - \text{Original value} = 0$  (is the same), then your return is simply the income that the investment created.

For example, if you buy a share of stock for \$100, and it pays no dividend, and a year later the market price is \$105, then your return =  $100 \times [0 + (105 - 100)] \div 100 = 100 \times 5 \div 100 = 5\%$ . If the same stock paid a dividend of \$2, then your return =  $100 \times [2 + (105 - 100)] \div 100 = 100 \times 7 \div 100 = 7\%$ .

The above calculation only works for one year changes. Due to the impact of compounding growth, we cannot use the simple percent change to determine annual growth over several years. Instead, we return to our

compounding growth formula to determine the annual growth rate. The benefit is that because we are just looking for annual growth rates, we do not need to worry about compounding periods. Therefore, we can use the equation

$$FV = PV(1+r)^t.$$

We need to solve for  $r$  which means that we will need to use fractional roots like we did in module 1.

Let's try one together...

On January 2, 2020, the Amazon stock (NYSE: X) had a price of 1,898.01. On December 31, 2020 it had a price of 3256.93. They did not pay any dividend (nor have they ever paid a dividend.) Calculate the annual return for 2020.

Answer: 71.6%



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Let's try one together...

On January 2, 2020, the US Steel stock (NASDAQ:

AMZN) had a price of 10.82. On December 31, 2020 it had a price of 16.77. They paid a total of \$0.04 per share in dividends. Calculate the annual return for 2020.

Answer: 55.4%



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## Let's try one together...

At the beginning of 2016, the PPG stock (NYSE: PPG) had a price of \$94.49. At the end of 2020 it had a price of \$144.22. Calculate the average *annual* return over this five-year period. (Ignore any potential dividend payment.)

Answer: 8.8%



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## Let's try one together...

At the beginning of 2016, the Dick's Sporting Goods stock (NYSE: DKS) had a price of \$34.49. At the end of 2020 it had a price of \$56.21. Calculate the average *annual* return over this five-year period. (Ignore any potential dividend payment.)

Answer: 10.3%



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While information about current and past returns is useful, investment professionals are more concerned with the **expected return** for the investment, that is, how much it may be expected to earn in the future. Estimating the expected return is complicated because many factors (i.e., current economic conditions, industry conditions, and market conditions) may affect that estimate.

For investments with a long history, a strong indicator of future performance may be past performance. Economic cycles fluctuate, and industry and firm conditions vary, but over the long run, an investment that has survived has weathered all those storms. So you could look at the average of the returns for each year. There are



several ways to do the math, but if you look at the average return for different investments of the same asset class or type (e.g., stocks of large companies) you could compare what they have returned, on average, over time.

If the time period you are looking at is long enough, you can reasonably assume that an investment's average return over time is the return you can expect in the next year. For example, if a company's stock has returned, on average, 9 percent per year over the last twenty years, then if next year is an average year, that investment should return 9 percent again. Over the eighteen-year span from 1990 to 2008, for example, the average return for the S&P 500 was 9.16 percent. Unless you have some reason to believe that next year will *not* be an average year, the average return can be your expected return. The longer the time period you consider, the less volatility there will be in the returns, and the more accurate your prediction of expected returns will be.

Returns are the value created by an investment, through either income or gains. Returns are also your compensation for investing, for taking on some or all of the risk of the investment, whether it is a corporation, government, parcel of real estate, or work of art. Even if there is no risk, you must be paid for the use of liquidity that you give up to the investment (by investing).

Returns are the benefits from investing, but they must be larger than its costs. There are at least two costs to investing: the opportunity cost of giving up cash and giving up all your other uses of that cash until you get it back in the future and the cost of the risk you take—the risk that you won't get it all back.

# RISK

Investment risk is the idea that an investment will not perform as expected, that its actual return will deviate from the expected return. Risk is measured by the amount of volatility, that is, the difference between actual returns and average (expected) returns. This difference is referred to as the **standard deviation**. Returns with a large standard deviation (showing the greatest variance from the average) have higher volatility and are the riskier investments.

What risks are there? What would cause an investment to unexpectedly over- or underperform? Starting from the top (the big picture) and working down, there are

- economic risks,
- industry risks,
- company risks,
- asset class risks,
- market risks.

Economic risks are risks that something will upset the economy as a whole. The economic cycle may swing from expansion to recession, for example; inflation or deflation may increase, unemployment may increase, or interest rates may fluctuate. These macroeconomic factors affect everyone doing business in the economy. Most businesses are cyclical, growing when the economy grows and contracting when the economy contracts.

Consumers tend to spend more disposable income when they are more confident about economic growth and the stability of their jobs and incomes. They tend

to be more willing and able to finance purchases with debt or with credit, expanding their ability to purchase durable goods. So, demand for most goods and services increases as an economy expands, and businesses expand too. An exception is businesses that are countercyclical. Their growth accelerates when the economy is in a downturn and slows when the economy expands. For example, low-priced fast food chains typically have increased sales in an economic downturn because people substitute fast food for more expensive restaurant meals as they worry more about losing their jobs and incomes.

Industry risks usually involve economic factors that affect an entire industry or developments in technology that affect an industry's markets. An example is the effect of a sudden increase in the price of oil (a macroeconomic event) on the airline industry. Every airline is affected by such an event, as an increase in the price of airplane fuel increases airline costs and reduces profits. An industry such as real estate is vulnerable to changes in interest rates. A rise in interest rates, for example, makes it harder for people to borrow money to finance purchases, which depresses the value of real estate.

Company risk refers to the characteristics of specific businesses or firms that affect their performance, making them more or less vulnerable to economic and industry risks. These characteristics include how much debt financing the company uses, how well it creates economies of scale, how efficient its inventory management is, how flexible its labor relationships are, and so on.

The **asset class** that an investment belongs to can also bear on its performance and risk. Investments (assets) are

categorized in terms of the markets they trade in. Broadly defined, asset classes include

- corporate stock or equities (shares in public corporations, domestic, or foreign);
- bonds or the public debts of corporation or governments;
- commodities or resources (e.g., oil, coffee, or gold);
- derivatives or contracts based on the performance of other underlying assets;
- real estate (both residential and commercial);
- fine art and collectibles (e.g., stamps, coins, baseball cards, or vintage cars).

Within those broad categories, there are finer distinctions. For example, corporate stock is classified as large cap, mid cap, or small cap, depending on the size of the corporation as measured by its market capitalization (the aggregate value of its stock). Bonds are distinguished as corporate or government and as short-term, intermediate-term, or long-term, depending on the maturity date.

Risks can affect entire asset classes. Changes in the inflation rate can make corporate bonds more or less valuable, for example, or more or less able to create valuable returns. In addition, changes in a market can affect an investment's value. When the stock market fell unexpectedly and significantly, as it did in October of 1929, 1987, and 2008, all stocks were affected, regardless of relative exposure to other kinds of risk. After such an event, the market is usually less efficient or less liquid;

that is, there is less trading and less efficient pricing of assets (stocks) because there is less information flowing between buyers and sellers. The loss in market efficiency further affects the value of assets traded.

As you can see, the link between risk and return is reciprocal. The question for investors and their advisors is: How can you get higher returns with less risk?

## DIVERSIFICATION

Every investor wants to maximize return, the earnings or gains from giving up surplus cash. And every investor wants to minimize risk, because it is costly. To invest is to assume risk, and you assume risk expecting to be compensated through return. The more risk assumed, the more the promised return. So, to increase return you must increase risk. To lessen risk, you must expect less return, but another way to lessen risk is to diversify—to spread out your investments among a number of different asset classes. Investing in different asset classes reduces your exposure to economic, asset class, and market risks.

Concentrating investment concentrates risk. Diversifying investments spreads risk by having more than one kind of investment and thus more than one kind of risk. To truly diversify, you need to invest in assets that are not vulnerable to one or more kinds of risk. For example, you may want to diversify

- between cyclical and countercyclical investments, reducing economic risk;
- among different sectors of the economy, reducing industry risks;
- among different kinds of investments, reducing

asset class risk;

- among different kinds of firms, reducing company risks.

To diversify well, you have to look at your collection of investments as a whole—as a portfolio—rather than as a gathering of separate investments. If you choose the investments well, if they are truly different from each other, the whole can actually be more valuable than the sum of its parts.

*Fidelity: What's Diversification? (all rights reserved)*



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## STEPS TO DIVERSIFICATION

In traditional portfolio theory, there are three levels or

steps to diversifying: capital allocation, asset allocation, and security selection.

**Capital allocation** is diversifying your capital between risky and riskless investments. A “riskless” asset is the short-term (less than ninety-day) U.S. Treasury bill. Because it has such a short time to maturity, it won’t be much affected by interest rate changes, and it is probably impossible for the U.S. government to become insolvent—go bankrupt—and have to default on its debt within such a short time.

The capital allocation decision is the first diversification decision. It determines the portfolio’s overall exposure to risk, or the proportion of the portfolio that is invested in risky assets. That, in turn, will determine the portfolio’s level of return.

The second diversification decision is **asset allocation**, deciding which asset classes, and therefore which risks and which markets, to invest in. Asset allocations are specified in terms of the percentage of the portfolio’s total value that will be invested in each asset class. To maintain the desired allocation, the percentages are adjusted periodically as asset values change.

Asset allocation is based on the expected returns and relative risk of each asset class and how it will contribute to the return and risk of the portfolio as a whole. If the asset classes you choose are truly diverse, then the portfolio’s risk can be lower than the sum of the assets’ risks.

One example of an asset allocation strategy is **life cycle investing**—changing your asset allocation as you age. When you retire, for example, and forgo income from working, you become dependent on income from your investments. As you approach retirement age, therefore,

you typically shift your asset allocation to less risky asset classes to protect the value of your investments.

**Security selection** is the third step in diversification, choosing individual investments within each asset class. Here is the chance to achieve industry or sector and company diversification. For example, if you decided to include corporate stock in your portfolio (asset allocation), you decide which corporation's stock to invest in. Choosing corporations in different industries, or companies of different sizes or ages, will diversify your stock holdings. You will have less risk than if you invested in just one corporation's stock. Diversification is not defined by the number of investments but by their different characteristics and performance.



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## 3.2 STOCKS

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### Investing in Stocks video by AmeriTrade

<sup>1</sup>Resources have costs, so a company needs money, or capital, which is also a resource. To get that start-up capital, the company could borrow or it could offer a share of ownership, or equity, to those who chip in capital.

If the costs of debt (interest payments) are affordable, the company may choose to borrow, which limits the company's commitment to its capital contributor. When the loan matures and is paid off, the relationship is over.

If the costs of debt are too high, however, or the company is unable to borrow, it seeks equity investors willing to contribute capital in exchange for an unspecified share of the company's profits at some time in the future. In exchange for taking the risk of no exact

1. Adapted from [15.1 Stocks and Stock Markets](#) in [Personal Finance](#) by Lumen Learning shared under a [CC BY-NC-SA](#) license.

return on their investment, equity investors get a say in how the company is run.

Stock represents those shares in the company's future and the right to a say in how the company is run. The original owners—the inventor(s) and entrepreneur(s)—choose equity investors who share their ideals and vision for the company. Usually, the first equity investors are friends, family, or colleagues, allowing the original owners freedom of management. At that point, the corporation is privately held, and the company's stock may be traded privately between owners. There may be restrictions on selling the stock, often the case for a family business, so that control stays within the family.

If successful, however, eventually the company needs more capital to grow and remain competitive. If debt is not desirable, then the company issues more equity, or stock, to raise capital. The company may seek out an **angel investor**, **venture capital** firm, or **private equity** firm. Such investors finance companies in the early stages in exchange for a large ownership and management stake in the company. Their strategy is to buy a significant stake when the company is still “private” and then realize a large gain, typically when the company goes public. The company also may seek a buyer, perhaps a competitive or complementary business.

Alternatively, the company may choose to **go public**, to sell shares of ownership to investors in the public markets. Theoretically, this means sharing control with random strangers because anyone can purchase shares traded in the stock market. It may even mean losing control of the company. Founders can be fired, as Steve Jobs was from Apple in 1985 (although he returned as CEO in 1996).

Going public requires a profound shift in the corporate structure and management. Once a company is publicly traded, it falls under the regulatory scrutiny of federal and state governments, and must regularly file financial reports and analysis. It must broaden participation on the board of directors and allow more oversight of management. Companies go public to raise large amounts of capital to expand products, operations, markets, or to improve or create competitive advantages. To raise public equity capital, companies need to sell stock, and to sell stock they need a market. That's where the stock markets come in.

## PRIMARY AND SECONDARY MARKETS

The private corporation's board of directors, shareholders elected by the shareholders, must authorize the number of shares that can be issued. Since issuing shares means opening up the company to more owners, or sharing it more, only the existing owners have the authority to do so. Usually, it authorizes more shares than it intends to issue, so it has the option of issuing more as need be.

Those **authorized shares** are then issued through an **initial public offering (IPO)**. At that point the company

goes public. The IPO is a **primary market** transaction, which occurs when the stock is initially sold and the proceeds go to the company issuing the stock. After that, the company is publicly traded; its stock is outstanding, or publicly available. Then, whenever the stock changes hands, it is a **secondary market** transaction. The owner of the stock may sell shares and realize the proceeds. When most people think of “the stock market,” they are thinking of the secondary markets.

The existence of secondary markets makes the stock a liquid or tradable asset, which reduces its risk for both the issuing company and the investor buying it. The investor is giving up capital in exchange for a share of the company’s profit, with the risk that there will be no profit or not enough to compensate for the opportunity cost of sacrificing the capital. The secondary markets reduce that risk to the shareholder because the stock can be resold, allowing the shareholder to recover at least some of the invested capital and to make new choices with it.

Meanwhile, the company issuing the stock must pay the investor for assuming some of its risk. The less that risk is, because of the liquidity provided by the secondary markets, the less the company has to pay. The secondary markets decrease the company’s cost of equity capital.

A company hires an investment bank to manage its initial public offering of stock. For efficiency, the bank usually sells the IPO stock to institutional investors. Usually, the original owners of the corporation keep large amounts of stock as well.<sup>2</sup>

When a company goes public, it may issue a relatively small number of shares. Its **market capitalization**—the

total dollar value of its outstanding shares—may therefore be small. The number of individual shareholders, mostly institutional investors and the original owners, also may be small. As a result, the shares may be “thinly traded,” traded infrequently or in small amounts.

Thinly traded shares may add to the volatility of the share price. One large shareholder deciding to sell could cause a decrease in the stock price, for example, whereas for a company with many shares and shareholders, the actions of any one shareholder would not be significant. As always, diversification—in this case of shareholders—decreases risk. Thinly traded shares are less liquid and more risky than shares that trade more frequently.

## STOCK PRICES

The value of a stock is in its ability to create a return, to create income or a gain in value for the investor. With common stock, the income is in the form of a dividend, which the company is not obligated to pay. The potential gain is determined by estimations of the future value of the stock.

If you knew that the future value would likely be more than the current market price—over your transaction costs, tax consequences, and opportunity cost—then you would buy the stock.

If you thought the future value would be less, you would short the stock (borrow it to sell with the intent of buying it back when its price falls), or you would just look for another investment.

Every investor wants to know what a stock will be

worth, which is why so many stock analysts spend so much time estimating future value. Equity analysis is the process of gathering as much information as possible and making the most educated guesses.

Corporations exist to make profit for the owners. The better a corporation is at doing that, the more valuable it is, and the more valuable are its shares. A company also needs to increase earnings, or grow, because the global economy is competitive. A corporation's future value depends on its ability to create and grow earnings.

That ability depends on many factors. Some factors are company-specific, some are specific to the industry or sector, and some are macroeconomic forces.

A company's size is an indicator of its earnings and growth potential. Size may correlate with age. A large company typically is more mature than a smaller one, for example. A larger company may have achieved economies of scale or may have gotten large by eliminating competitors or dominating its market. Size in itself is not an indicator of success, but similarly sized companies tend to have similar earnings growth.<sup>3</sup>

Companies are usually referred to by the size of their market capitalization or market cap, that is, the current market value of the debt and equity they use to finance their assets. Common market cap categories are the sizes micro, small, mid (medium), and large, or

- micro cap, with a market capitalization of less than \$300 million;
- small cap, with a market capitalization between

3. E. F. Fama and K. R. French, "The Cross-section of Expected Stock Returns," *Journal of Finance* 47 (1992): 427–86.

\$300 million and \$2 billion;

- mid cap, with a market capitalization between \$2 billion and \$10 billion;
- large cap, with a market capitalization of more than \$10 billion.

The market capitalization of a company—along with industry and economic indicators—is a valuable indicator of earnings potential.

The economist John Maynard Keynes (1883–1946) famously compared the securities markets with a newspaper beauty contest. You “won” not because you could pick the prettiest contestant, but because you could pick the contestant that everyone else would pick as the prettiest contestant. In other words, the stock market is a popularity contest, but the “best” stock was not necessarily the most popular.

Keynes described investing in the stock market as follows:

“The smart player recognizes that personal criteria of beauty are irrelevant in determining the contest winner. A better strategy is to select those faces the other players are likely to fancy. This logic tends to snowball. After all, the other participants are likely to play the game with at least as keen a perception. Thus, the optimal strategy is not to pick those faces the player thinks are prettiest, or those the other players are likely to fancy, but rather to predict what the average opinion is likely to be about what the average opinion will be.”<sup>4</sup>

In the stock market, the forces of supply and demand

4. Burton G. Malkiel, *A Random Walk Down Wall Street* (New York: W. W. Norton & Company, Inc., 2007).

determine stock prices. The more demand or popularity there is for a company's stock, the higher its price will go (unless the company issues more shares). A stock is popular, and thus in greater demand, if it is thought to be more valuable—that is, if it has more earnings and growth potential.

Sometimes a company is under- or overpriced relative to the going price for similar companies. If the market recognizes the “error,” the stock price should rise or fall as it “corrects” itself.

A **growth stock** is a stock that promises a higher rate of return because the market has underestimated its growth potential. A **value stock** is a stock that has been underpriced for some other reason. For example, investors may be wary of the outlook for its industry. Because it is underpriced, a value stock is expected to provide a higher-than-average return. A **blue-chip stock** is the stock of a large, established company. These stocks tend to generate steady (but not extraordinary) returns each year. Finally, **penny stocks** are stocks with very low prices. These stocks often fail and become worthless, but could produce large gains due to modest price changes. For example, if you buy a stock for \$0.05/share and the price increases to \$0.50/share, then your investment had grown by 10x.



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### 3.3 BONDS

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*TD Ameritrade: Investing Basics – Bonds (all rights reserved)*



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<https://pressbooks.nsc.ca/mathofmoney/?p=72#oembed-1>

Bonds are a relatively old form of financing. Formalized debt arrangements long preceded corporate structure and the idea of equity (stock) as we know it. Venice issued the first known government bonds of the modern era in 1157,<sup>2</sup> while private bonds are cited in British records going back to the thirteenth century.<sup>3</sup> Venice issued bonds to raise funds to finance a Crusade against Constantinople, which included expansion of a shipyard

1. Adapted from [16.1 Bonds and Bond Markets](#) in [Personal Finance](#) by Saylor Academy. [CC BY-NC-SA](#)
2. Isadore Barmash, *The Self-Made Man* (Washington, DC: Beard Books, 2003), 55.
3. George Burton Adams, *The Constitutional History of England* (London: H. Holt, 1921), 93.

attached to the Venetian Arsenal. (Go to [http://en.wikipedia.org/wiki/Venetian\\_Arsenal](http://en.wikipedia.org/wiki/Venetian_Arsenal) to view images.)

## BONDS

In addition to financing government projects, bonds are used by corporations to capitalize growth. Bonds are also a legal arrangement, couched in conditions, obligations, and consequences. As a result of their legal and financial roles, bonds carry a quaint and particular vocabulary. Bonds come in all shapes and sizes to suit the needs of the borrowers and the demands of lenders. Figure 3.3.1 lists the descriptive terms for basic bond features.

Bond Term	Meaning
Issuer	Borrower
Investor	Lender or Creditor
Principal, Face Value, Par Value	Amount Borrowed
Coupon Rate	Interest Rate
Coupon	Interest Payment
Maturity	Due Date
Term	Time until Maturity
Yield to Maturity	Annualized Return on Bond Investment
Market Value	Current Price

Figure 3.3.1: Bond Terminology

The **coupon** is usually paid to the investor twice yearly. It is calculated as a percentage of the **face value**—amount borrowed—so that the annual coupon = coupon rate  $\times$  face value. By convention, each individual bond has a face value of \$1,000. A corporation issuing a bond to raise \$100 million would have to issue 100,000 individual bonds (100,000,000 divided by 1,000). If those bonds pay a 4 percent coupon, a bondholder who owns one of those bonds would receive a coupon of \$40 per year ( $1,000 \times 4\%$ ), or \$20 every six months.

The **coupon rate** of interest on the bond may be fixed or floating and may change. A floating rate is usually based on another interest benchmark, such as the U.S. **prime rate**, a widely recognized benchmark of prevailing interest rates.

A **zero-coupon bond** has a coupon rate of zero: it pays no interest and repays only the principal at maturity. A

“zero” may be attractive to investors, however, because it can be purchased for much less than its face value. If you have a savings bond, then you have a zero-coupon bond!

The face value, the principal amount borrowed, is paid back at maturity. If the bond is **callable**, it may be redeemed after a specified date but before maturity.

Corporations issue corporate bonds, usually with maturities of ten, twenty, or thirty years. Corporate bonds tend to be the most “customized,” with features such as callability, conversion, and covenants.

The U.S. government issues **Treasury bills** for short-term borrowing, **Treasury notes** for intermediate-term borrowing (longer than one year but less than ten years), and **Treasury bonds** for long-term borrowing for more than ten years.

State and municipal governments issue revenue bonds or general obligation bonds. A **revenue bond** is repaid out of the revenue generated by the project that the debt is financing. For example, toll revenue may secure a debt that finances a highway. A **general obligation bond** is backed by the state or municipal government, just as a corporate debenture is backed by the corporation.

Interest from state and **municipal bonds** (also called “munis”) may not be subject to federal income taxes. Also, if you live in that state or municipality, the interest may not be subject to state and local taxes. The tax exemption differs from bond to bond, so you should be sure to check before you invest. Even if the interest is not taxable, however, any gain (or loss) from the sale of the bond is taxed, so you should not think of munis as “tax-free” bonds.

Foreign corporations and governments issue bonds. You should keep in mind, however, that foreign

government defaults are not uncommon. Mexico in 1994, Russia in 1998, Argentina in 2001, and Greece in 2009 are all recent examples. Foreign corporate or sovereign debt also exposes the bondholder to currency risk, as coupons and principal will be paid in the foreign currency.

To provide guidance, **rating agencies** provide bond ratings; that is, they “grade” individual bond issues based on the likelihood of default and thus the risk to the investor. Rating agencies are independent agents that base their ratings on the financial stability of the company, its business strategy, competitive environment, outlook for the industry and the economy—any factors that may affect the company’s ability to meet coupon obligations and pay back debt at maturity.

Ratings agencies such as Fitch Ratings, A. M. Best, Moody’s, and Standard & Poor’s (S&P) are hired by large borrowers to analyze the company and rate its debt. Moody’s also rates government debt. Ratings agencies use an alphabetical system to grade bonds (shown in Figure 3.3.2) based on the highest-to-lowest rankings of two well-known agencies.

Standard & Poor's	Moody's	Grade	Meaning
AAA	Aaa	Investment	Risk is almost zero
AA	Aa	Investment	Low risk
A	A	Investment	Risk if economy declines
BBB	Baa	Investment	Some risk; more if economy declines
BB	Ba	Speculative	Risky
B	B	Speculative	Risky; expected to get worse
CCC	Caa	Speculative	Probable bankruptcy
CC	Ca	Speculative	Probable bankruptcy
C	C	Speculative	In bankruptcy or default
D		Speculative	In bankruptcy or default

Figure 3.3.2: Bond Ratings

A plus sign (+) following a rating indicates that it is likely to be upgraded, while a minus sign (–) following a rating indicates that it is likely to be downgraded.

Bonds rated BBB or Baa and above are considered **investment grade bonds**, relatively low risk and “safe” for both individual and institutional investors. Bonds rated below BBB or Baa are speculative in that they carry some default risk. These are called **speculative grade bonds**, **junk bonds**, or **high-yield bonds**. Because they are riskier, speculative grade bonds need to offer investors a higher return or yield in order to be “priced to sell.”

Although the term “junk bonds” sounds derogatory, not all speculative grade bonds are “worthless” or are issued by “bad” companies. Bonds may receive a speculative rating if their issuers are young companies, in a highly competitive market, or capital intensive, requiring lots of operating capital. Any of those features would make it

harder for a company to meet its bond obligations and thus may consign its bonds to a speculative rating. In the 1980s, for example, companies such as CNN and MCI Communications Corporation issued high-yield bonds, which became lucrative investments as the companies grew into successful corporations.

Default risk is the risk that a company won't have enough cash to meet its interest payments and principal payment at maturity. That risk depends, in turn, on the company's ability to generate cash, profit, and grow to remain competitive. Bond-rating agencies analyze an issuer's default risk by studying its economic, industry, and firm-specific environments and estimate its current and future ability to satisfy its debts. The default risk analysis is similar to equity analysis, but bondholders are more concerned with cash flows—cash to pay back the bondholders—and profits rather than profits alone.

Bond ratings can determine the coupon rate the issuer must offer investors to compensate them for default risk. The higher the risk, the higher the coupon must be. Ratings agencies have been criticized recently for not being objective enough in their ratings of the corporations that hire them. Nevertheless, over the years bond ratings have proven to be a reliable guide for bond investors.



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## 3.4 MUTUAL FUNDS

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1

*Two Cents: What the Heck is a Mutual Fund? (all rights reserved)*



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A mutual fund is a portfolio of securities, consisting of one type of security or a combination of several different types. A fund serves as a convenient way for an investor to have a diversified portfolio of investments in just about any investable asset. The oldest mutual fund is believed to have been founded by Adriaan van Ketwich in 1774. Ketwich invited investors to contribute to a trust fund to spread the risk of investing in foreign bonds. The idea moved from the Netherlands to Scotland to the United

1. Adapted from [17.1 Mutual Funds](#) in [Personal Finance](#) by Saylor Academy. [CC BY-NC-SA](#)

States, where the Boston Personal Property Trust established the first mutual fund in 1893.<sup>2</sup>

The mutual fund's popularity has grown in periods of economic expansion. At the height of the stock market boom in 1929, there were over seven hundred mutual funds in the United States. After 1934, mutual funds fell under the regulatory eye of the Securities and Exchange Commission (SEC), and it wasn't until the 1950s that there were once again over one hundred mutual funds in the United States.

Mutual funds multiplied in the 1970s, spurred on by the creation of IRAs and 401(k) retirement plans, and again in the 1980s and 1990s, inspired by economic growth and the tech stock boom. By the end of 2008, U.S. mutual funds—which account for just over half of the global market—had \$9.6 trillion in assets under management. Forty-five percent of all U.S. households owned mutual funds, compared to 6 percent in 1980. For 69 percent of those households, mutual funds were more than half of their financial assets.<sup>3</sup> Mutual funds play a significant role in individual investment decisions.

A mutual fund provides an investor with cheaper and simpler diversification and security selection, requiring only one transaction to own a diversified portfolio (the mutual fund). By buying shares in the fund rather than individual securities, you achieve extensive diversification for a much lower transaction cost than by investing in individual securities and making individual

2. FinanceScholar.com, <http://www.financescholar.com/history-mutual-funds.html> (accessed June 15, 2009).

3. The Investment Company Institute, *2009 Investment Company Fact Book*, 49th ed., 2009, [http://www.ici.org/pdf/2009\\_factbook.pdf](http://www.ici.org/pdf/2009_factbook.pdf) (accessed June 15, 2009).

transactions. You also receive the benefit of professional security selection, which theoretically minimizes the opportunity costs of lesser choices. So by using a mutual fund, you get more and better security selection and diversification.

A mutual fund also provides stock and bond issuers with a mass market. Rather than selling shares to investors individually (and incurring the costs of doing so), issuers can more easily find a market for their shares in mutual funds.

## STRUCTURES AND TYPES OF MUTUAL FUNDS

Like stocks and bonds, mutual funds may be actively or passively managed. Actively managed funds provide investors with professional management and the expected research, analysis, and watchfulness that goes with it. Passively managed **index funds**, on the other hand, are designed to mirror the performance of a specific index constructed to be representative of an asset class. Recall, for example, that the Standard & Poor's (S&P) 500 Index is designed to mirror the performance of the five hundred largest large cap stocks in the United States.

Mutual funds are structured in three ways:

1. Closed-end funds
2. Open-end funds
3. Exchange-traded funds

**Closed-end funds** are funds for which a limited number of shares are issued. Once all shares have been issued, the fund is “closed” so a new investor can only buy shares from an existing investor. Since the shares are traded on an exchange, the limited supply of shares and the demand for them in that market directly determines the value of the shares for a closed-end fund.

Most mutual funds are **open-end funds** in which investors buy shares directly from the fund and redeem or sell shares back to the fund. The price of a share is its **net asset value (NAV)**, or the market value of each share as determined by the fund’s assets and liabilities and the number of shares that exist. Here is the basic formula for calculating NAV:

$$\text{NAV} = (\text{market value of fund securities} - \text{fund liabilities}) \div \text{number of shares outstanding.}$$

Demand for shares is reflected in the number of shares outstanding, because the fund can create new shares for new investors. NAV calculations are usually done once per day at the close of trading, when mutual fund transactions are recorded.

The NAV is the price that the fund will pay you when you redeem your shares, so it is a gauge of the shares’ value. It will increase if the market value of the securities in the fund increases faster than the number of new shares.

**Exchange-traded funds (ETFs)** are structured like closed-end funds but are traded like stocks. Shares are

traded and priced continuously throughout the day's trading session, rather than once per day at the end of trading. ETFs trade more like individual securities; that is, if you are trying to time a market, they are a more nimble asset to trade than open-end or closed-end funds.

Originally designed as index funds, exchange-traded funds now target just about every asset, sector, and economic region imaginable. Because of this, ETFs have become quite popular, with over \$4.9 trillion (up from a half billion in 2009) invested in over 5,000 funds (up from 700 in 2009).<sup>4</sup>

Another popular type of fund is called a “fund of funds.” This is a mutual fund that owns shares in other mutual funds rather than in specific securities. If you decide to use mutual funds rather than select securities, a fund of funds will provide expertise in choosing funds. These have become popular with the use of retirement year targeted mutual funds where a fund manager adjusts the types of mutual funds based on your age (becoming safer as you approach retirement.)

## MUTUAL FUND FEES AND RETURNS

All funds must disclose their fees to potential investors:

sales fees, management fees, and expenses. A **load fund** charges a sales commission on each share purchase. That sales charge (also called a **front-end load**) is a percentage of the purchase price. A **no-load fund**, in contrast, does not charge a sales commission, because shares may be purchased directly from the fund or through a discount broker. The front-end load can be as much as 8.5 percent, so if you plan to invest often or in large amounts, that can be a substantial charge. For example, a \$5,000 investment may cost you \$425, reducing the amount you have to invest and earn a return.

A fund may charge a **back-end load**, actually a deferred sales charge, paid when you sell your shares instead of when you buy them. The charge may be phased out if you own the shares for a specified length of time, however, usually five to seven years.

A fund may charge a management fee on an annual basis. The management fee is stated as a fixed percentage of the fund's asset value per share. Management fees can range from 0.1 percent to 2.0 percent annually. Typically, a more actively managed fund can be expected to charge a higher management fee, while a passively managed fund such as an index fund should charge a minimal management fee.

A fund may charge an annual **12b-1 fee** or distribution fee, also calculated as not more than 1.0 percent per year of the fund's asset value. Some mutual funds charge other extra fees as well, passing on fund expenses to shareholders. You should consider fee structure and rate when choosing mutual funds, and this can be done through calculations of the expense ratio.

**Note:** Many ETFs carry a very low or zero fee structure.

*Learn to Invest: Introduction to Mutual Fund Fees (all rights reserved)*



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<https://pressbooks.nsc.ca/mathofmoney/?p=74#oembed-2>

## MUTUAL FUND INFORMATION AND STRATEGIES

All mutual fund companies must offer a **prospectus**, a published statement detailing the fund's assets, liabilities, management personnel, and performance record. You should always take the time to read it and to take a closer look at the fund's investments to make sure that the fund will be compatible and appropriate to your investment goals.

For example, suppose you have an investment in an S&P 500 Index fund and now are looking for a global stock fund to complement and diversify your holdings



in domestic (U.S.) equities. You go to the Web site of a large mutual fund company offering hundreds of funds. You find a stock fund called “Global Stock Fund”—sounds like it’s just what you are looking for. Looking closer, however, you can see that this fund is invested in the stocks of companies in Germany, Japan, and the United Kingdom. While they are not U.S. stocks, those economies are similar to the U.S. economy, perhaps too similar to provide the diversity you are looking for.

Or suppose you are looking for a bond fund to create income and security. You find a fund called the “Investment Grade Fixed Income Fund.” On closer inspection, however, you find that the fund does not invest only in investment grade bonds but that the *average* rating of its bonds is investment grade. This means that the fund invests in many investment grade bonds but also in some speculative grade bonds to achieve higher income. While this fund may suit your need for income, it may not be appropriate for your risk tolerance.

Mutual fund companies make this information readily available on Web sites and in prospectuses. You should always make the extra effort to be sure you know what’s in your fund. In addition, mutual funds are widely followed by many performance analysts. Ratings agencies such as Morningstar and investment publications such as *Barron’s* and *Forbes* track, analyze, and report the performance of mutual funds. That information is available online or in print and provides comparisons of mutual funds that you may find helpful in choosing your fund.

In print and online newspapers, mutual fund performance is reported daily in the form of tables that compare the average returns of funds from week to week.

Reported average returns are based on the net asset value per share (NAVPS). Investors can use this information to choose or compare funds and track the performance of funds they own.

In conclusion, since a mutual fund may be made up of any kind or many kinds of securities (e.g., stocks, bonds, real estate, and commodities), it is not really another kind of investment. Rather, it is a way to invest without specifically selecting securities, a way of achieving a desired asset allocation without choosing individual assets.

The advantages of investing in a mutual fund are the diversification available with minimal transaction costs and the professional management or security selection that you buy when you buy into the fund.

Compared to actively managed funds, passively managed or index funds offer similar diversification but with lower management fees and expense ratios because you aren't paying for market timing or security selection skills. The turnover ratio shows how passive or active the fund management is. About half of all equity mutual funds have a turnover ratio of less than 50 percent.<sup>5</sup>



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## 3.5 INVESTMENT TOPICS

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Economic forces and financial behavior can converge to create extreme markets or financial crises, such as booms, bubbles, panics, crashes, or meltdowns. These atypical events actually happen fairly frequently. Between 1618 and 1998, there were thirty-eight financial crises globally, or one every ten years.<sup>1</sup> As an investor, you can expect to weather as many as six crises in your lifetime.

Patterns of events that seem to precipitate and follow the crises are shown in Figure 3.5.1. First a period of economic expansion is sparked by a new technology, the discovery of a new resource, or a change in political balances. This leads to increased production, markets, wealth, consumption, and investment, as well as increased credit and lower interest rates. People are looking for ways to invest their newfound wealth. This leads to an asset bubble, a rapid increase in the price of some asset: bonds, stocks, real estate, or commodities such as cotton, gold, oil, or tulip bulbs that seems to be positioned to prosper from this particular expansion.

1. Charles P. Kindleberger and Robert Aliber, *Manias, Panics, and Crashes*, 5th ed. (Hoboken, NJ: John Wiley & Sons, Inc., 2005).

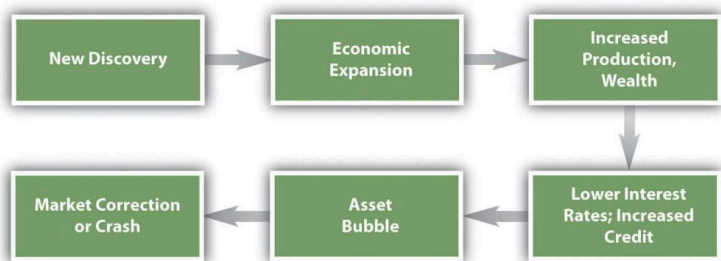


Figure 3.5.1: Asset Bubbles

The bubble continues, reinforced by the behavioral and market consequences that it sparks until some event pricks the bubble. Then asset values quickly deflate, and credit defaults rise, damaging the banking system. Having lost wealth and access to credit, people rein in their demand for consumption and investment, further slowing the economy.

In many cases, the event that started the asset speculation was not a macroeconomic event but nevertheless had consequences to the economy: the end of a war, a change of government, a change in policy, or a new technology. Often the asset that was the object of speculation was a resource for or an application of a new technology or an expansion into new territory that may have been critical to a new emphasis in the economy. In other words, the assets that became the objects of bubbles tended to be the drivers of a “new economy” at the time and thus were rationalized as investments rather than as speculation.

Many irrational financial behaviors—overconfidence, anchoring, availability bias, representativeness—are in play, until finally the market was shocked into reversal by a specific event or simply sank under its own weight.

Economists may argue that this is what you should

expect, that markets expand and contract cyclically as a matter of course. In this view, a crash is nothing more than the correction for a bubble—market efficiency at work.



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## PONZI SCHEMES

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A **Ponzi scheme** ([/ˈpɒnzi/](#), Italian: [\[ˈpɒntsi\]](#); also a **Ponzi game**)<sup>[1]</sup> is a form of [fraud](#) that lures [investors](#) and pays [profits](#) to earlier investors with [funds](#) from more recent investors.<sup>[2]</sup> The scheme leads victims to believe that profits are coming from product sales or other means, and they remain unaware that other investors are the source of funds. A Ponzi scheme can maintain the illusion of a sustainable business as long as new investors contribute new funds, and as long as most of the investors do not demand full repayment and still believe in the non-existent assets they are purported to own.

Among the first recorded incidents to meet the modern definition of Ponzi scheme were carried out from 1869 to 1872 by [Adele Spitzeder](#) in [Germany](#) and by [Sarah Howe](#) in the United States in the 1880s through the “Ladies’

Deposit". Howe offered a solely female clientele an 8% monthly [interest rate](#), and then stole the money that the women had invested. She was eventually discovered and served three years in prison.<sup>[3]</sup> The Ponzi scheme was also previously described in novels; [Charles Dickens'](#) 1844 novel [Martin Chuzzlewit](#) and his 1857 novel [Little Dorrit](#) both feature such a scheme.<sup>[4]</sup>

In the 1920s, [Charles Ponzi](#) carried out this scheme and became well known throughout the United States because of the huge amount of money that he took in.<sup>[5]</sup> His original scheme was based on the legitimate [arbitrage of international reply coupons](#) for postage stamps, but he soon began diverting new investors' money to make payments to earlier investors and to himself.<sup>[6]</sup> Unlike earlier, similar schemes, Ponzi's gained considerable press coverage both within the United States and internationally both while it was being perpetrated and after it collapsed – this notoriety eventually led to the type of scheme being named after him.<sup>[7]</sup>

## Characteristics

Typically, Ponzi schemes require an initial investment and promise above-average returns.<sup>[8]</sup> They use vague verbal guises such as "[hedge futures trading](#)", "[high-yield investment programs](#)", or "[offshore investment](#)" to describe their income strategy. It is common for the operator to take advantage of a lack of investor knowledge or competence, or sometimes claim to use a proprietary, secret investment strategy to avoid giving information about the scheme.

The basic premise of a Ponzi scheme is "[to rob Peter to pay Paul](#)". Initially, the operator pays high returns to attract investors and entice current investors to invest

more money. When other investors begin to participate, a cascade effect begins. The schemer pays a “return” to initial investors from the investments of new participants, rather than from genuine profits.

Often, high returns encourage investors to leave their money in the scheme, so that the operator does not actually have to pay very much to investors. The operator simply sends statements showing how much they have earned, which maintains the deception that the scheme is an investment with high returns. Investors within a Ponzi scheme may even face difficulties when trying to get their money out of the investment.

Operators also try to minimize withdrawals by offering new plans to investors where money cannot be withdrawn for a certain period of time in exchange for higher returns. The operator sees new cash flows as investors cannot transfer money. If a few investors do wish to withdraw their money in accordance with the terms allowed, their requests are usually promptly processed, which gives the illusion to all other investors that the fund is solvent and financially sound.

Ponzi schemes sometimes begin as legitimate investment vehicles, such as [hedge funds](#) that can easily degenerate into a Ponzi-type scheme if they unexpectedly lose money or fail to legitimately earn the returns expected. The operators fabricate false returns or produce fraudulent audit reports instead of admitting their failure to meet expectations, and the operation is then considered a Ponzi scheme.

A wide variety of investment vehicles and strategies, typically legitimate, have become the basis of Ponzi schemes. For instance, [Allen Stanford](#) used bank [certificates of deposit](#) to defraud tens of thousands of

people. Certificates of deposit are usually low-risk and insured instruments, but the Stanford certificates of deposit were fraudulent.<sup>[9]</sup>

According to the [U.S. Securities and Exchange Commission](#) (SEC), many Ponzi schemes share similar characteristics that should be “red flags” for investors.<sup>[10]</sup> The warning signs include:<sup>[10]</sup>

- **High investment returns with little or no risk.** Every investment carries some degree of [risk](#), and investments yielding higher returns typically involve more risk. Any “guaranteed” investment opportunity is often considered suspicious.
- **Overly consistent returns.** Investment values tend to go up and down over time, especially those offering potentially high returns. An investment that continues to generate regular positive returns regardless of overall market conditions is considered suspicious.
- **Unregistered investments.** Ponzi schemes typically involve investments that have not been registered with the SEC or with state regulators. Registration is important because it provides investors with access to key information about the company’s management, products, services, and finances.
- **Unlicensed sellers.** Federal and state securities laws require that investment professionals and their firms be licensed or registered. Most Ponzi schemes involve unlicensed individuals or unregistered firms, the few exceptions usually being the aforementioned investment vehicles



that started out as legitimate operations but failed to earn the expected returns.

- **Secretive or complex strategies.** Investments that cannot be understood or do not give complete information.
- **Issues with paperwork.** Excuses are given regarding why clients cannot review information in writing about an investment. Also, account statement errors and inconsistencies are frequently signs that funds are not being invested as promised.
- **Difficulty receiving payments.** Clients have failures to receive a payment or have difficulty cashing out their investments. Ponzi scheme promoters routinely encourage participants to “roll over” investments and sometimes promise even higher returns on the amount rolled over.

Theoretically it is not impossible at least for certain entities operating as Ponzi scheme to ultimately “succeed” financially, at least so long as a Ponzi scheme was not what the promoters were initially intending to operate. For example, a failing hedge fund reporting fraudulent returns could conceivably “make good” its reported numbers, for example by making a successful high-risk investment. Moreover if the operators of such a scheme are facing the likelihood of imminent collapse accompanied by criminal charges, they may see little additional “risk” to themselves in attempting cover their tracks by engaging in further illegal acts to try and make good the shortfall (for example, by engaging in [insider trading](#)). Especially with lightly-regulated and monitored

investment vehicles like hedge funds, in the absence of a whistleblower or accompanying illegal acts any fraudulent content in reports is often difficult to detect unless and until the investment vehicles ultimately collapse.

Typically, however, if a Ponzi scheme is not stopped by authorities it usually falls apart for one or more of the following reasons:<sup>[6]</sup>

1. The operator vanishes, taking all the remaining investment money. Promoters who intend to abscond often attempt to do so as returns due to be paid are about to exceed new investments, as this is when the investment capital available will be at its maximum.
2. Since the scheme requires a continual stream of investments to fund higher returns, if the number of new investors slows down, the scheme collapses as the operator can no longer pay the promised returns (the higher the returns, the greater the risk of the Ponzi scheme collapsing). Such [liquidity crises](#) often trigger panics, as more people start asking for their money, similar to a [bank run](#).
3. External market forces, such as a sharp decline in the economy, can often hasten the collapse of a Ponzi scheme (for example, the [Madoff investment scandal](#) during the [market downturn of 2008](#)), since they often cause many investors to attempt to withdraw part or all of their funds sooner than they had intended.

Actual losses are extremely difficult to calculate. The

amounts that investors thought they had were never attainable in the first place. The wide gap between “money in” and “fictitious gains” make it virtually impossible to know how much was lost in any Ponzi scheme.<sup>[citation needed]</sup>

In the United States, individuals can halt a Ponzi scheme before its collapse by reporting to the SEC.<sup>[11]</sup> Under the SEC Whistleblower Program,<sup>[12]</sup><sup>[circular reference]</sup> individuals can receive monetary awards for reporting violations of the federal securities laws, including information about Ponzi schemes, if their information leads to a successful SEC enforcement action in which over \$1,000,000 in sanctions is ordered. To report a Ponzi scheme and qualify for an award under the program, the SEC requires that whistleblowers or their attorneys report the tip online through the SEC’s Tip, Complaint or Referral Portal or mail/fax a Form TCR to the SEC Office of the Whistleblower.<sup>[13]</sup>



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## 3.6 RETIREMENT PLANNING

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Retirement planning involves the same steps as any other personal planning: figure out where you'd like to be and then figure out how to get there from where you are. More formally, the first step is to define your goals, even if they are no more specific than "I want to be able to afford a nice life after I stop getting a paycheck." But what is a "nice life," and how will you pay for it?

It may seem impossible or futile to try to project your retirement needs so far from retirement given that there are so many uncertainties in life and retirement may be far away. But that shouldn't keep you from saving. You can try to save as much as possible for now, with the idea that your plans will clarify as you get closer to your retirement, so whatever money you have saved will give you a head start.

Chris and Sam were young urban professionals until their children were born. Tired of pushing strollers through the subways, they bought a home in the suburbs. They are happy to provide a more idyllic lifestyle for their kids but miss the "buzz" and convenience of their urban lifestyle. When their children are on their own and Chris and Sam are ready to retire, they would like to sell their home and move back into the city.

Chris and Sam are planning to use the value of their house to finance a condo in the city, but they also know that real estate prices are often higher in the more desirable urban areas and that living expenses may be higher in the future. Now in their mid-thirties, Chris and Sam are planning to retire in thirty years.

Chris and Sam need to project how much money they will need to have saved by the time they wish to retire. To do that, they need to project both their future capital needs (to buy the condo) and their future living expense in retirement. They also need to project how long they may live after retirement, or how many years' worth of living expenses they will need, so that they won't outlive their savings.

To estimate required savings, in other words, you need to estimate the following:

- Expenses in retirement
- The duration of retirement
- The return on savings in retirement

As difficult as these estimations seem, because it is a long time until retirement and a lot can happen in the meantime, you can start by using what you know about the present.

The amount you need at retirement varies with the expected rate of return on your savings. While you are retired, you will be drawing income from your savings, but your remaining savings will still be earning a return. The more return your savings can earn while you are retired, the less you have to have saved by retirement. The less return your savings can earn in retirement, the more you need to have saved *before* retirement.

## ESTIMATING THE ANNUAL SAVINGS FOR RETIREMENT

In the example above, if you make regular annual deposits into your retirement account for the next thirty years, each deposit would have to be \$23,936, assuming that your account will earn 5 percent for in thirty years. If the rate of return for your savings is less, you would have to save more to have more at retirement. If your retirement savings can earn only 2 percent, for example, you would have to deposit \$60,229 per year to have \$2,443,361 when you retire. Your retirement account grows through your contributions and through its own earnings. The more your account can earn before you retire, the less you will have to contribute to it. On the other hand, the more you can contribute to it, the less it has to earn.

### Let's try one together...

You are currently 20 and plan to retire when you are 60. You want to have \$3 million saved. You are depositing your money in a mutual fund that, over the long-run, earns 5% per year. How much do you need to contribute each month? How does this number change if you start saving at 30? 40? 50?

Answers: \$1,966; \$3,605; \$7,299; \$19,320



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The longer the time you have to save, the sooner you start saving, and the less you need to save each year. Chris and Sam are already in their thirties, so they figure they have thirty years to save for retirement. Had they started in their twenties and had forty years until retirement, they would not have to save so much each year. If they wait until they are around fifty, they will have to save a lot more each year. The more you have to save, the less disposable income you will have to spend on current living expenses, making it harder to save. Clearly, saving early and regularly is the superior strategy.

When you make these calculations, be aware that you are using estimates to figure the money you'll need at retirement. You use the *expected* inflation rate, based on its historic average, to estimate annual expenses, historical statistics on life expectancy to *estimate* the duration of your retirement, and an *estimate* of future savings returns. Estimates must be adjusted because things change. As you progress toward retirement, you'll want to reevaluate these numbers at least annually to be sure you are still saving enough.

*The Motley Fool: How Much do I need to Retire (all rights reserved)*





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## WILLS

Since you won't be here, you will need to leave a written document outlining your instructions regarding your estate. That is your **will**, your legal request for the distribution of your estate, that is, assets that remain after your debts have been satisfied. If you die **intestate**, or without a will, the laws of your state of legal residence will dictate the distribution of your estate.

You can write your own will so long as you are a legal adult and mentally competent. The document has to be witnessed by two or three people who are not inheriting anything under the terms of the will, and it must be dated and signed and, in some states, notarized. A **holographic will** is handwritten; it may be more difficult to validate. A **statutory will** is a preprinted will that you can buy from a store or in a software package. Consider, however, that

a will is a legal document. Having yours drawn up by a lawyer may better insure its completeness and validity in court.

**Probate** is the legal process of validating a will and administering the payment of debts and the distribution of assets by a probate court. Probate courts also distribute property in the absence of a will. Probate is not required in every case, however. Probate is not required if the deceased

- owned assets of little value, allowing for transfer without court supervision;
- owned assets jointly with or “payable on death” to another person;
- owned assets naming another person as beneficiary;
- held all assets in a living trust (a legal entity for managing assets on behalf of beneficiaries).

Besides the details of “who gets what,” a will should name an **executor**, the person or persons who will administer the payment of your debts and the distribution of your remaining assets, according to your wishes as expressed in your will. If you have legal dependents, your will should name a guardian for them. You may also include a “letter of last instruction” stating the location of important documents, safe deposit keys, and bank accounts and specifying your funeral arrangements.

You may change or rewrite your will at any time, but you should definitely do so as your life circumstances change, especially with events such as marriage or divorce, the birth of a child, and the acquisition of

significant assets, such as a house. If the changes in your circumstances are substantial, you should create a new will.

It is possible that you will become mentally or physically disabled before you die and unable to direct management of your assets. To prepare for this possibility, you may create a **living will** with instructions for your care in that event. You may appoint someone—usually a spouse, child, or sibling—who would have **power of attorney**, that is, the right to act on your behalf, especially as regards financial and legal decisions. That power may be limited or unlimited (such as a “durable power of attorney”) and is restricted to certain acts or dependent on certain circumstances.

Along with granting power of attorney, your living will may include a health care proxy, requesting that medical personnel follow the instructions of a designated family member who expresses your wishes concerning your end-of-life treatment. Many people request, for example, that they not be revived or sustained if they cannot experience some quality of life. Be sure to update your living will, however, as over time your views may change and as medical and technological advances change our notions of “quality of life.”

## TRUSTS AND GIFTS

A **trust** is a legal entity created by a trustor, or grantor, who owns assets managed by a trustee or trustees for the benefit of a beneficiary or beneficiaries. A **testamentary trust** may be established by a will so that beneficiaries who are unable to manage assets (minor children or disabled dependents) can benefit from the assets but have them managed for them. A **living trust** is established while the grantor is alive. Unlike a will, it does not become a matter of public record upon your death.

Another way to avoid probate and estate taxes is to gift assets to your beneficiaries while you are alive. Ownership of the assets passes to the beneficiaries at the time of the gift, so the assets are no longer included in your estate. The federal government and many state governments levy a gift tax for gifts exceeding certain limits. In 2009, the annual exclusion from federal tax was \$13,000 per recipient, for example. Also, the federal government does not tax gifts to spouses and to pay others' medical bills or tuitions.

There are limits to this kind of tax-free distribution of funds, however. For example, the federal government considers any "gift" you make within three years prior to your death as part of your taxable estate. Gifting nevertheless is a way to reduce the value of an estate. Some parents also prefer to make funds available or to gift them to their children when the children need them more—for example, earlier in their adult lives when they may not have accrued enough wealth to make a down payment on a house.

Most trusts, whether testamentary or living, revocable or irrevocable, are created to avoid either the probate

process or estate taxes or both. The probate process can be long and costly and therefore a burden for your executor, your beneficiaries (who may have to wait for their distributions), and your estate.

*Hadi Harp: Introduction to Wills and Trusts (all rights reserved)*



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## 3.7 RETIREMENT ACCOUNTS

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Personal Finance. **Provided by:** Saylor Academy. **Located at:** [https://saylordotorg.github.io/text\\_personal-finance](https://saylordotorg.github.io/text_personal-finance). **License:** [CC BY-NC-SA: Attribution-NonCommercial-ShareAlike](#)

While knowing the numbers clarifies the picture of your needs, you must reconcile that picture with the realities that you face now. How will you be able to afford to save what you need for retirement?

There are several savings plans structured to help you save—some offer tax advantages, some don't—but first you need to make a commitment to save.

Saving means not spending a portion of your disposable income. It means delaying gratification or putting off until tomorrow what you could have today. That is often difficult, as you have many demands on your disposable income. You must weigh the benefit of fulfilling those demands with the cost of not saving for retirement, even though benefit in the present is much easier to credit than benefit in the future. Once you resolve to save, however, employer, government, and individual retirement plans are there to help you.

*MoneyCoach: 401(k) and IRA 101 (all rights reserved)*





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## EMPLOYER RETIREMENT ACCOUNTS

Employers may sponsor pension or retirement plans for their employees as part of the employees' total compensation. There are two kinds of employer-sponsored plans: defined benefit plans and defined contribution plans.

A **defined benefit plan** is a retirement plan, sometimes called a **pension plan**, funded by the employer, who promises the employee a specific benefit upon retirement. The employer can be a corporation, labor union, government, or other organization that establishes a retirement plan for its employees. In addition to (or instead of) a defined benefit plan, an employer may also offer a profit-sharing plan, a stock bonus plan, an employee stock ownership plan (ESOP), a thrift plan, or other plan. Each type of plans has advantages and

disadvantages for employers and employees, but all are designed to give employees a way to save for the future and employers a way to attract and keep employees.

The payout for a defined benefit plan is usually an annual or monthly payment for the remainder of the employee's life. In some defined benefit plans, there is also a spousal or survivor's benefit. The amount of the benefit is determined by your wages and length of service with the company.

Many defined benefit plans are structured with a **vesting** option that limits your claim on the retirement fund until you have been with the company for a certain length of time. For example, Paul's employer has a defined benefit plan that provides for Paul to be 50 percent vested after five years and fully vested after seven years. If Paul were to leave the company before he had worked there for five years, none of his retirement fund would be in his account. If he left after six years, half his fund would be kept for him; after ten years, all of it would be.

With a defined benefit plan your income in retirement is constant or "fixed," and it is the employer's responsibility to fund your retirement. This is both an advantage and a disadvantage for the employee. Having your employer fund the plan is an advantage, but having a fixed income in retirement is a drawback during periods of inflation when the purchasing power of each dollar declines. In some plans, that drawback is offset by automatic cost of living increases.

Defined benefit plans also carry some risk. Most companies reserve the right to change or discontinue their pension plans. Furthermore, the pension payout is only as good as the company that pays it. If the company defaults, its pension obligations may be covered by the



**Pension Benefit Guaranty Corporation (PBGC)**, an independent federal government agency. If not, employees are left without the benefit. Even if the company is insured, the PGBC may not cover 100 percent of employees' benefits.

Founded in 1974, the PBGC is funded by insurance premiums paid by employers who sponsor defined benefit plans. If a pension plan ends (e.g., through the employer's bankruptcy) the PBGC assumes pensions payments up to a limit per employee. Currently, the PBGC pays benefits to approximately 640,000 retirees and insures the pensions of about 1,305,000 employees.<sup>1</sup> There is some concern, however, that if too many defined benefit sponsors fail, as could happen in a widespread recession, the PBGC would not be able to fully fund its obligations.

To avoid the responsibility for employee retirement funds, more and more employers sponsor **defined contribution retirement plans**. Under defined contribution plans, each employee has a retirement account, and both the employee and the employer may contribute to the account. The employer may contribute up to a percentage limit or offer to match the employee's contributions, up to a limit. With a matching contribution, if employees choose not to contribute, they lose the opportunity of having the employer's contribution as well as their own. The employee makes untaxed contributions to the account as a payroll deduction, up to a maximum limit specified by the tax code. The maximum for defined contribution plans is 25 percent of the employee's compensation, with a cap in

1. The Pension Benefit Guaranty Corporation, "Mission Statement," <http://www.pbgc.gov/about/about.html> (accessed May 1, 2009).

2009 of \$49,000. Defined contribution plans known as 401(k) plans had a maximum contribution limit in 2009 of \$16,500.

Defined contribution plans have become increasingly popular since section 401(k) was introduced into the tax code in 1978. The **401(k) plans**—or 403b plans for employees of nonprofits and 457 plans for employees of government organizations—offer employees a pretax (or tax-deferred) way to save for retirement to which employers can make a tax-deductible contribution.

The advantages of a 401(k) for the employee are the plan's flexibility and portability and the tax benefit. A defined contribution account belongs to the employee and can go with the employee when he or she leaves that employer. For the employer, there is the lower cost and the opportunity to shift the risk of investing funds onto the employee. There is a ceiling on the employer's costs: either a limited matching contribution or a limit set by the tax code.

The employer offers a selection of investments, but the employee chooses how the funds in his or her account are diversified and invested. Thus, the employee assumes the responsibility—and risk—for investment returns. The employer's contributions are a benefit to the employee. Employers can also make a contribution with company stock, which can create an undiversified account. A portfolio consisting only of your company's stock exposes you to market risk should the company not do well, in which case, you may find yourself losing both your job and your retirement account's value.

## Let's try one together...

Realty Development contributes to its employees 401(k) based on the following design: it matches its employee's contributions dollar-for-dollar up to 6% of their salary. Find the total contributions made to the retirement account for the following employees during a one-year period:

1. Doris earned \$62,000 per year and contributes 0% of her salary to his retirement account.
2. Ebony earned \$49,200 per year and contributes 5% of her salary to his retirement account.
3. Faroud earned \$71,500 per year and contributes 8% of his salary to her retirement account.

Answers: \$0, \$4,920; \$10,010



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# U.S. GOVERNMENT'S RETIREMENT ACCOUNT

*The Retirement Plan Company, LLC: Social Security Basics  
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The federal government offers a mandatory retirement plan for all citizens except federal government employees and railroad workers, known as **Social Security**. Social Security is funded by a mandatory payroll tax shared by employee and employer. That tax, commonly referred to as Federal Insurance Contributions Act (FICA), also funds Medicare. Social Security was signed into law by President Franklin D. Roosevelt in 1935 to provide benefits for old age and survivors and disability insurance for workers (OASDI). The Social Security Administration (SSA) was established to manage these “safety nets.”

Social Security is not an automatic benefit but an entitlement. To qualify for benefits, you must work and contribute FICA taxes for forty quarters (ten years). Retirement benefits may be claimed as early as age sixty-two, but full benefits are not available until age sixty-seven for workers born in 1960 or later. If you continue to earn wage income after you begin collecting Social

Security but before you reach full retirement age, your benefit may be reduced. Once you reach full retirement age, your benefit will not be reduced by additional wage income.

The amount of your benefit is calculated based on the amount of FICA tax paid during your working life and your age at retirement. Up to 85 percent of individual Social Security benefits may be taxable, depending on other sources of income.

Each year, the SSA provides each potential, qualified beneficiary with a projection of the expected monthly benefit amount (in current dollars) for that individual based on the individual's wage history.

Social Security benefits represent a large expenditure by the federal government, and so the program is often the subject of debate. Economists and politicians disagree on whether the system is sustainable. As the population ages, the ratio of beneficiaries to workers increases—that is, there are more retirees collecting benefits relative to the number of workers who are paying into the system.

Many reforms to the system have been suggested, such as extending the eligibility age, increasing the FICA tax to apply to more income (right now it applies only to a limited amount of wages, but not to income from interest, dividends, or investment gains), or having workers manage their Social Security accounts the same way they manage 401(k) plans. Some of these proposals are based on economics, some on politics, and some on social philosophy. Despite its critics, Social Security remains a popular program on which many Americans have come to rely. You should, however, be aware that Social Security can be amended and faces possible underfunding.

Keep in mind that in 1935 when Social Security was created, life expectancy for American males was only sixty-five, the age of Social Security eligibility. Social Security was never meant to be a retirement income, but rather a supplement to retirement income, merely “some measure of protection against...poverty-ridden old age.” As part of the Federal Employees Retirement System (FERS), the U.S. government also offers special retirement plans to its employees, including a Thrift Savings Plan (TSP) for civilians employed by the United States and members of the uniformed services (i.e., Army, Navy, Air Force, Marine Corps, Coast Guard, National Oceanic and Atmospheric Administration, and Public Health Service).

Federal, state, and local government plans; plans for public school teachers and administrators; and church plans are exempt from the rules of the Employee Retirement Income Security Act of 1974 (ERISA) and from some rules that govern retirement plans of private employers under the Internal Revenue Code. In some states, public school teachers pay into a state retirement system and do not pay federal Social Security taxes (or receive Social Security benefits) for the years they are working as teachers.

Nevertheless, many plans for public employees are defined benefit plans providing annuities upon retirement, similar to but separate from plans for employees in the private sector.

# INDIVIDUAL RETIREMENT ACCOUNTS

*TD Ameritrade: Traditional and Roth IRAs (all rights reserved)*



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Any individual can save for retirement without a special “account,” but since the government would like to encourage retirement savings, it has created tax-advantaged accounts to help you do so. Because these accounts provide tax benefits as well as some convenience, it is best to use them first in planning for retirement, although their use may be limited.

Individual retirement accounts (IRAs) were created in 1974 by ERISA. They were initially available only to employees not covered by an employer’s retirement plan. In 1981, participation was amended to include everyone under the age of 70.5.<sup>2</sup> IRAs are personal investment accounts, and as such may be invested in a wide range of financial products: stocks, bonds, certificate of deposits (CDs), mutual funds, and so on. Types of IRAs differ in terms of tax treatment of contributions, withdrawals, and in the limits of contributions.

2. Wikipedia, “Legislative History of IRAs,” [http://en.wikipedia.org/wiki/Individual\\_retirement\\_account](http://en.wikipedia.org/wiki/Individual_retirement_account) (accessed May 23, 2012).

The **Traditional IRA** is an account funded by tax-deductible and/or nondeductible contributions. Deductible contributions are taxed later as funds are withdrawn, but nondeductible contributions are not. In other words, you either pay tax on the money as you put it in, or you pay tax on it as you take it out.

A great advantage of a Traditional IRA is that principal appreciation (interest, dividend income, or capital gain) is not taxed until the funds are withdrawn. Withdrawals may begin without penalty after the age of 59.5. Funds may be withdrawn before age 59.5, but with penalties and taxes applied. Contributions may be made until age 70.5, at which time required minimum distributions (withdrawals) of funds must begin.

Because they create tax advantages, contributions to a Traditional IRA are limited, currently up to \$5,000 (or \$6,000 for someone over the age of fifty). That limit on deductible contributions becomes smaller (the tax benefit is phased out) as income rises. The Internal Revenue Service (IRS) provides a worksheet to calculate how much of your contribution is taxable with your personal income tax return (Form 1040).

For the **Roth IRA**, created in 1997, contributions are not tax deductible, but withdrawals are not taxed. You can continue to contribute at any age, and you do not have to take any minimum required distribution. The great advantage of a Roth IRA is that capital appreciation is not taxed.

As with the Traditional IRA, contributions may be limited depending on your income. If you have both a Traditional and a Roth IRA, you may contribute to both, but your combined contribution is limited.



	Traditional IRA	Roth IRA
Age limit to create the IRA?	Yes, 70½	No
Age limit to contribute?	Yes, 70½	No
Tax-deductible contributions allowed?	Yes	No
Tax-deductible contributions limited?	Yes, by income	N/A
Nondeductible contributions allowed?	Yes	Yes
Nondeductible contributions limited?	Yes	Yes
Withdrawals are taxed?	Yes, of deductible contributions	No
Minimum required distribution?	Yes	No
Age of mandatory distribution?	70½	None
Minimum age for distribution?	Yes, 59½	Yes, 59½

Figure 3.7.1: Differences between IRAs

A **rollover** is a distribution of cash from one retirement fund to another. Funds may be rolled into a Traditional IRA from an employer plan (401(k), 403b, or 457) or from another IRA. You may not deduct a rollover contribution (since you have already deducted it when it was originally contributed), but you are not taxed on the distribution from one fund that you immediately contribute to another. A **transfer** moves a retirement account, a Traditional IRA, from one trustee or asset manager to another. Rollovers and transfers are not taxed if accomplished within sixty days of distribution.



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mathofmoney/?p=82#h5p-16](https://pressbooks.nsc.ca/mathofmoney/?p=82#h5p-16)

## MODULE 3 PRACTICE PROBLEMS

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**Problem 3.1:** At the beginning of 2016, the Westinghouse Air Brake Technologies Corporation (Wabtec) stock (NYSE: WAB) had a price of \$63.16. At the end of 2020 it had a price of \$73.20. Calculate the average annual return over this five-year period. (Ignore any potential dividend payment.)

Answer: 3%



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**Problem 3.2:** You are currently 20 and plan to retire when you are 65. You want to have \$5 million saved. You are depositing your money in a mutual fund that, over the long-run, earns 6% per year. How much do you need to contribute each month? How does this number change if you start saving at 30? 40? 50?

Answer: \$1,814; \$3,509; \$7,215; \$17,193



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**Problem 3.3:** The Washington Group contributes to its employees 401(k) based on the following design: for every 2% deduction for the employee, the company will provide a 1% contribution, up to a 6% company contribution. Find the total contributions made to the retirement account for the following employees during a one-year period:

1. Al earned \$92,000 per year and contributes 0% of his salary to his retirement account.
2. Basir earned \$39,500 per year and contributes 10% of his salary to his retirement account.
3. Cynthia earned \$102,500 per year and contributes 15% of her salary to her retirement account.

Answer: \$0, \$5,925; \$21,525



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## MODULE 4: TAXES

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## 4.1 NON-INCOME TAXES

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<sup>1</sup>Any government that needs to raise revenue and has the legal authority to do so may tax. Tax jurisdictions reflect government authorities. In the United States, federal, state, and municipal governments impose taxes. Similarly, in many countries there are national, provincial or state, county, and municipal taxes. Regional economic alliances, such as the European Union, may also levy taxes.

Jurisdictions may overlap. For example, in the United States, federal, state, and local governments may tax income, which becomes complicated for those earning income in more than one state, or living in one state and working in another. Governments tax income because it is a way to tax broadly based on the ability to pay. Most adults have an income from some source, even if it is a government distribution. Those with higher incomes should be able to pay more taxes, and in theory should be willing to do so, for they have been more successful in or have benefited more from the economy that the government protects. We will talk about income taxes in the next section.

Tax is levied on income from many sources:

- Wages (selling labor)
- Interest, dividends, and gains from investment

1. Adapted from [6.1 Sources of Taxation and Kinds of Taxes](#) in [Personal Finance](#) by Lumen Learning shared under a [CC BY-NC-SA](#) license.

(selling capital)

- Self-employment (operating a business or selling a good or service)
- Property rental
- Royalties (rental of intellectual property)
- “Other” income such as alimony, gambling winnings, or prizes

A **sales tax** or **consumption tax** taxes the consumption financed by income. In the United States, sales taxes are imposed by state or local governments; as yet, there is no national sales tax. Sales taxes are said to be more efficient and fair in that consumption reflects income (income determines ability to consume and therefore level of consumption). Consumption also is hard to hide, making sales tax a good way to collect taxes based on the ability to pay. Consumption taxes typically tax all consumption, including nondiscretionary items such as food, clothing, and housing. Opponents of sales tax argue that it is a **regressive tax**, because those with lower incomes must use a higher percentage of their incomes on nondiscretionary purchases than higher-income people do. In Pennsylvania, for example, food and clothing are not subject to sales tax. This is meant to reduce the burden on lower-income purchasers.

Let's try one together...

You want to buy an Xbox Series X for \$499. You are deciding whether to go to the Century III Best Buy in



Allegheny County or Greensburg Best Buy in Westmoreland County. Calculate the final price at each store. The sales tax in Allegheny County is 7% while the sales tax in Westmoreland County is 6%.

Answers: Allegheny = \$533.93; Westmoreland = \$528.94



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## Let's try one together...

You purchase \$60 worth of clothing, \$100 worth of electronics, \$40 worth of automotive supplies, and \$150 worth of food at the Washington Walmart in Trinity Plaza. The sales tax rate in Washington County is 6%, but 0% for food and clothing. Calculate your grand total for this shopping trip.

Answer: \$148.40



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<https://pressbooks.nsc.ca/mathofmoney/?p=87#oembed-1>

## Let's try one together...

You purchase a variety of crafting supplies at Michael's. Your grand total (after tax) for this trip is \$111.88. Assuming you made this trip in Washington County where the tax rate is 6%, calculate the cost of the crafting supplies you purchased.  
Answer: \$105.55



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The **value-added tax** (VAT) or goods and services tax (GST) is widely used outside the United States. It is a consumption tax, but differs from the sales tax, which is paid only by the consumer as an end user. With a VAT or GST, the value added to the product is taxed at each stage

of production. Governments use a VAT or GST instead of a sales tax to spread the tax burden among producers and consumers, and thus to reduce incentive to evade the tax. A consumption tax, like the sales tax, it is a regressive tax. When traveling abroad, you should be aware that a VAT may add substantially to the cost of a purchase (a meal, accommodations, etc.).

## Exercises

While traveling in Germany, you purchase a variety of souvenirs for a total price of 177,34€. Germany has a VAT of 19%. Calculate the price of the souvenirs. (Remember, the VAT is included in the price you see and not added like the US sales tax.)

Answer: 149,03€



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**Excise taxes** are taxes on specific consumption items such as alcohol, cigarettes, motor vehicles, fuel, or highway use. In some states, excise taxes are justified by the discretionary nature of the purchases and may be criticized as exercises in social engineering (i.e., using the tax code to dictate social behaviors). For example, people

addicted to nicotine or alcohol tend to purchase cigarettes or liquor even if an excise tax increases their cost—and are therefore a reliable source of tax revenue.

Property taxes are used by more local—state, municipal, provincial, and county—governments, and are most commonly imposed on real property (land and buildings) but also on personal assets such as vehicles and boats. Property values theoretically reflect wealth (accrued income) and thus ability to pay taxes. Property values are also a matter of public record (real property is deeded, boats or automobiles are licensed), which allows more efficient tax collection.

**Estate taxes** are taxes on the transfer of wealth from the deceased to the living. Estate taxes are usually imposed on the very wealthiest based on their unusual ability to pay. Because death and the subsequent dispersal of property is legally a matter of public record, estate taxes are generally easy to collect. Estate taxes are controversial because they can be seen as a tax on the very idea of ownership and on incomes that have already been taxed and saved or stored as wealth and properties. Still, estate taxes are a substantial source of revenue for the governments that use them, and so they remain.



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<https://pressbooks.nsc.ca/mathofmoney/?p=87#h5p-17>

## 4.2 INCOME TAX TERMINOLOGY

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<sup>1</sup>The U.S. government relies most on an income tax. The income tax is the most relevant for personal financial planning, as everyone has some sort of income over a lifetime. Most states model their tax systems on the federal model or base their tax rates on federally defined income.

### TAXABLE ENTITIES

There are four taxable entities in the federal system: the individual or family unit, the corporation, the nonprofit corporation, and the trust. Personal financial planning focuses on your decisions as an individual or family unit, but other tax entities can affect individual income. Corporate profit may be distributed to individuals as a **dividend**, for example, which then becomes the individual's taxable income. Likewise, funds established for a specific purpose may distribute money to an

1. Adapted from [6.2 The U.S. Federal Income Tax Process](#) in [Personal Finance](#) by Lumen Learning shared under a [CC BY-NC-SA](#) license.

individual that is taxable as individual income. A **trust**, for example, is a legal arrangement whereby control over property is transferred to a person or organization (the trustee) for the benefit of someone else (the beneficiary). If you were a beneficiary and received a distribution, that money would be taxable as individual income.

The definition of the taxable “individual” is determined by filing status:

- Single, never married, widowed, or divorced
- Married, in which case two adults file as one taxable “individual,” combining all taxable activities and incomes, deductions, exemptions, and credits
- Married filing separately, in which case two married adults file as two separate taxable individuals, individually declaring and defining incomes, deductions, exemptions, and credits
- Head-of-household, for a family of one adult with dependents

Some taxes are levied differently depending on filing status, following the assumption that family structure affects ability to pay taxes.

All taxable entities have to file a declaration of incomes and pay any tax obligations annually. Not everyone who files a return actually pays taxes, however. Individuals with low incomes and tax exempt, nonprofit corporations typically do not. All potential taxpayers nevertheless must declare income and show their obligations to the government. For the individual, that

declaration is filed on Form 1040 (or, if your tax calculations are simple enough, Form 1040EZ).

## INCOME

For individuals, the first step in the process is to calculate total income. Income may come from many sources, and each income must be calculated and declared. Some kinds of income have a separate form or schedule to show their more detailed calculations. The following schedules are the most common for reporting incomes separately by source.

- Schedule B: Interest and Dividend Income
- Schedule C: Business Income
- Schedule SE: Self-Employment Tax
- Schedule D: Capital Gains and Losses
- Schedule E: Rental and Royalty Income; Income from Partnerships, S Corporations, and Trusts
- Schedule F: Farm Income

## OTHER TAXABLE AND NONTAXABLE INCOME

Other taxable income includes alimony, state or local tax refunds, retirement fund distributions from individual retirement accounts (IRAs) and/or pensions, unemployment compensation, and a portion of Social Security benefits.

Your total income is then adjusted for items that the government feels should not be taxed under certain circumstances, such as certain expenses of educators, performing artists, and military reservists; savings in health savings or retirement accounts; moving expenses; a portion of self-employment taxes; student loan interest; tuition and educational fees; and alimony paid. Income that is not taxed by the U.S. government and does not have to be reported as income includes the following:

- Welfare benefits
- Interest from *most* municipal bonds
- *Most* gifts
- *Most* inheritance and bequests
- Workers compensation
- Veteran's benefits
- Federal tax refunds
- *Some* scholarships and fellowships

It's important to read tax filing instructions carefully, however, because not everything you'd think would qualify actually does. The government allows adjustments



to be reported (or not reported) as income only under certain circumstances or up to certain income limits, and some adjustments require special forms.

The result of deducting adjustments from your total income is a calculation of your adjusted gross income (AGI). Your AGI is further adjusted by amounts that may be deducted or exempted from your taxable income and by amounts already credited to your tax obligations.

## DEDUCTIONS AND CREDITS

Deductions reduce taxable income while credits reduce taxes. Deductions are tax breaks for incurring certain expenditures or living in certain circumstances that the government thinks you should not have to include in your taxable income. There are deductions for age and for blindness. For other deductions, there is a standard, lump-sum deduction that you can take, or you may choose to itemize your deductions, that is, detail each one separately and then calculate the total. If your itemized deductions are more than your standard deduction, it makes sense to itemize.

Other deductions involve financial choices that the government encourages by rewarding an extra incentive in the form of a tax break. Home mortgage interest is

a deduction to encourage home ownership, for example; investment interest is a deduction to encourage investment, and charitable donations are deductions to encourage charitable giving.

Deductions are also created for expenditures that may be considered non-discretionary, such as medical and dental expenses, job-related expenses, or state and local income and property taxes. As with income adjustments, you have to read the instructions carefully, however, to know what expenditures qualify as deductions. Some deductions only qualify if they amount to more than a certain percentage of income, while others may be deducted regardless. Some deductions require an additional form to calculate specifics, such as unreimbursed employee or job-related expenses, charitable gifts not given in cash, investment interest, and some mortgage interest.

After deductions are subtracted from adjusted gross income, the remainder is your taxable income. Your tax is based on your taxable income, on a progressive scale. You may have additional taxes, such as self-employment tax, and you may be able to apply credits against your taxes, such as the earned income credit for lower-income taxpayers with children.

Deductions and credits are some of the more disputed areas of the tax code. Because of the depth of dispute about them, they tend to change more frequently than other areas of the tax code. As a taxpayer, you want to stay alert to changes that may be to your advantage or disadvantage. Usually, such changes are phased in and out gradually so you can include them in your financial planning process.

## PAYMENTS AND REFUNDS

Once you have calculated your tax obligation for the year, you can compare that to any taxes you have paid during the year and calculate the amount still owed or the amount to be refunded to you.

You pay taxes during the tax year by having them withheld from your paycheck if you earn income through wages, or by making quarterly estimated tax payments if you have other kinds of income. When you begin employment, you fill out a form (Form W-4) that determines the taxes to be withheld from your regular pay. You may adjust this amount, within limits, at any time. If you have both wages and other incomes, but your wage income is your primary source of income, you may be able to increase the taxes withheld from your wages to cover the taxes on your other income, and thus avoid having to make estimated payments. However, if your nonwage income is substantial, you will have to make estimated payments to avoid a penalty and/or interest.

The government requires that taxes are withheld or paid quarterly during the tax year because it uses tax revenues to finance its expenditures, so it needs a steady and predictable cash flow. Steady payments also greatly decrease the risk of taxes being noncollectable. State and local income taxes must also be paid during the tax year and are similarly withheld from wages or paid quarterly.

Besides income taxes, other taxes are withheld from your wages: payments for Social Security and Medicare. Social Security or the Federal Insurance Contributions Act (FICA) and Medicare are federal government programs. Social Security is insurance against loss of income due to retirement, disability, or loss of a spouse or

parent. Individuals are eligible for benefits based on their own contributions—or their spouse’s or parents’—during their working lives, so technically, the Social Security payment withheld from your current wages is not a tax but a contribution to your own deferred income. Medicare finances health care for the elderly. Both programs were designed to provide minimal benefits to those no longer able to sell their labor in exchange for wage income. In fact, both Social Security and Medicare function as “pay-as-you-go” systems, so your contributions pay for benefits that current beneficiaries receive.

If you have paid more during the tax year than your actual obligation, then you are due a refund of the difference. You may have that amount directly deposited to a bank account, or the government will send you a check.

If you have paid less during the tax year than your actual obligation, then you will have to pay the difference (by check or credit card) and you may have to pay a penalty and/or interest, depending on the size of your payment.

The deadline for filing income tax returns and for paying any necessary amounts is April 15, following the end of the tax year on December 31. You may file to request an extension of that deadline to August 15. Should you miss a deadline without filing for an extension, you will owe penalties and interest, even if your actual tax obligation results in a refund. It really pays to get your return in on time.

**Note:** The following video discusses exemptions. Due to the tax code changes in 2017 there are no longer

exemptions. Just ignore this, but realize they worked much like deductions.

*Money Coach: Taxes 101 (all rights reserved)*



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<https://pressbooks.nsc.ca/mathofmoney/?p=89#oembed-1>

## Calculating Total Taxes

The United States uses a progressive tax system which is one where you pay a larger percentage of your income as you earn more income. The tax brackets for 2021 are shown below in Table 4.2.2.

**Table 4.2.2: US Federal Income Tax Rates, 2021**

Rate	Single	Joint	Head of Household
10%	Up to \$9,950	Up to \$19,900	Up to \$14,200
12%	\$9,951 to \$40,525	\$19,901 to \$81,050	\$14,201 to \$54,200
22%	\$40,526 to \$86,375	\$81,051 to \$172,750	\$54,201 to \$86,350
24%	\$86,376 to \$164,925	\$172,751 to \$329,850	\$86,351 to \$164,900
32%	\$164,926 to \$209,425	\$329,851 to \$418,850	\$164,901 to \$209,400
35%	\$209,426 to \$523,600	\$418,851 to \$628,300	\$209,401 to \$523,600
37%	Over \$523,600	Over \$628,300	Over \$523,600

The standard deduction for 2021 is

- \$12,550 for single filers.
- \$25,100 for married (filing jointly) filers.
- \$18,800 for head of household.

In the next section, we will use this information to see how income taxes are calculated.

### **Tax Payment Terms**

One misunderstanding that people sometimes have is the belief that if they move up to the next tax bracket that their entire income will be subject to the higher tax rate. THIS IS FALSE! For example, for single filers, as we see in Table 4.2.2, income earned in the \$9,951 to \$40,525 bracket is taxed at 12%. So what if it is December 30th and you have earned \$40,525. Your boss asks you to work an additional hour which would increase your annual income to \$40,535, \$10 into the 22% tax bracket. Does this mean that your entire income would now be taxed at 22%? NO! Only the final \$10 is taxed at that rate. To help with this, I introduce a few terms.

From: [https://en.wikipedia.org/wiki/Tax\\_rate](https://en.wikipedia.org/wiki/Tax_rate)

#### *Statutory Tax Rate*

A **statutory tax rate** is the legally imposed rate. An income tax could have multiple statutory rates for different income levels, where a sales tax may have a flat statutory rate.<sup>[1]</sup> The statutory tax rate is expressed as a percentage and will always be higher than the effective tax rate.<sup>[2]</sup>

### *Average/Effective Tax Rate*

An **average tax rate**, also called the effective tax rate, is the ratio of the total amount of taxes paid to the total [tax base](#) (taxable income or spending), expressed as a percentage. If  $t$  is the total tax liability and  $i$  is total income, then the average tax rate is just  $t/i$ .

In a [proportional tax](#), the tax rate is fixed and the average tax rate equals this tax rate. In case of [tax brackets](#), commonly used for [progressive taxes](#), the average tax rate increases as taxable income increases through tax brackets, asymptoting to the top tax rate. For example, consider a system with three tax brackets, 10%, 20%, and 30%, where the 10% rate applies to income from \$1 to \$10,000, the 20% rate applies to income from \$10,001 to \$20,000, and the 30% rate applies to all income above \$20,000. Under this system, someone earning \$25,000 would pay \$1,000 for the first \$10,000 of income (10%); \$2,000 for the second \$10,000 of income (20%); and \$1,500 for the last \$5,000 of income (30%). In total, they would pay \$4,500, or an 18% average tax rate.

### *Marginal Tax Rate*

A **marginal tax rate** is the tax rate on income set at a higher rate for incomes above a designated higher bracket, which in 2016 in the United States was \$415,050. For annual income that was above the cut off point in that higher bracket, the marginal tax rate in 2016 was 39.6%. For income below the \$415,050 cut off, the lower tax rate was 35% or less. [\[3\]](#)[\[4\]](#)

Marginal tax rates are applied to income in countries with progressive taxation schemes, with incremental increases in income taxed in progressively higher [tax](#)

[brackets](#), resulting in the tax burden being distributed amongst those who can most easily afford it.

Marginal taxes are valuable as they allow governments to generate revenue to fund social services in a way that only affects those who will be the least negatively affected.

With a [flat tax](#), by comparison, all income is taxed at the same percentage, regardless of amount. An example is a sales tax where all purchases are taxed equally. A [poll tax](#) is a flat tax of a set dollar amount per person. The marginal tax in these scenarios would be zero, however, these are both forms of regressive taxation and place a higher tax burden on those who are least able to cope with it, and often results in an underfunded government leading to increased deficits.

When we do tax calculations in the next section, we will explore all of these concepts in more depth.

*Money Coach: Do I Need to Pay Federal Taxes? (all rights reserved)*



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mathofmoney/?p=89#h5p-18](https://pressbooks.nsc.ca/mathofmoney/?p=89#h5p-18)

## 4.3 INCOME TAX FORMS

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### W4<sup>1</sup>

The **Form W-4** is used by employers to determine the amount of [tax withholding](#) to deduct from employees' wages. The form is not mailed to the IRS but retained by the employer. Tax withholdings depend on employee's personal situation and ideally should be equal to the annual tax due on the Form 1040. When filling out a Form W-4 an employee calculates the number of Form W-4 allowances to claim based on his or her expected tax filing situation for the year. The amount of money withheld as federal income tax is reduced for each Form W-4 allowance taken. No interest is paid on over-withholding, but penalties might be imposed for under-withholding. Alternatively, or in addition, the employee can send quarterly estimated tax payments directly to the IRS (Form 1040-ES). Quarterly estimates may be required if the employee has additional income (e.g. investments or self-employment income) not subject to withholding or insufficiently withheld. There are specialized versions of this form for other types of payment (W-4P for pensions as an example).

1. From: [https://en.wikipedia.org/wiki/Form\\_W-4](https://en.wikipedia.org/wiki/Form_W-4)

<b>Enter Personal Information</b>	Address _____	▶ Does your name match the name on your social security card? If not, to ensure you get credit for your earnings, contact SSA at 800-772-1213 or go to <a href="http://www.ssa.gov">www.ssa.gov</a> .
	City or town, state, and ZIP code _____	
	(c) <input type="checkbox"/> Single or Married filing separately <input type="checkbox"/> Married filing jointly (or Qualifying widow(er)) <input type="checkbox"/> Head of household (Check only if you're unmarried and pay more than half the costs of keeping up a home for yourself and a qualifying individual.)	

**Complete Steps 2-4 ONLY if they apply to you; otherwise, skip to Step 5.** See page 2 for more information on each step, who can claim exemption from withholding, when to use the online estimator, and privacy.

<b>Step 2: Multiple Jobs or Spouse Works</b>	<p>Complete this step if you (1) hold more than one job at a time, or (2) are married filing jointly and your spouse also works. The correct amount of withholding depends on income earned from all of these jobs.</p> <p>Do <b>only one</b> of the following.</p> <p>(a) Use the estimator at <a href="http://www.irs.gov/W4App">www.irs.gov/W4App</a> for most accurate withholding for this step (and Steps 3-4); or</p> <p>(b) Use the Multiple Jobs Worksheet on page 3 and enter the result in Step 4(c) below for roughly accurate withholding; or</p> <p>(c) If there are only two jobs total, you may check this box. Do the same on Form W-4 for the other job. This option is accurate for jobs with similar pay; otherwise, more tax than necessary may be withheld . . . . . ▶ <input type="checkbox"/></p> <p><b>TIP:</b> To be accurate, submit a 2020 Form W-4 for all other jobs. If you (or your spouse) have self-employment income, including as an independent contractor, use the estimator.</p>
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**Complete Steps 3-4(b) on Form W-4 for only ONE of these jobs.** Leave those steps blank for the other jobs. (Your withholding will be most accurate if you complete Steps 3-4(b) on the Form W-4 for the highest paying job.)

<b>Step 3: Claim Dependents</b>	<p>If your income will be \$200,000 or less (\$400,000 or less if married filing jointly):</p> <p>Multiply the number of qualifying children under age 17 by \$2,000 ▶ \$ _____</p> <p>Multiply the number of other dependents by \$500 . . . . . ▶ \$ _____</p> <p>Add the amounts above and enter the total here . . . . . 3 \$ _____</p>	
<b>Step 4 (optional): Other Adjustments</b>	<p>(a) <b>Other income (not from jobs).</b> If you want tax withheld for other income you expect this year that won't have withholding, enter the amount of other income here. This may include interest, dividends, and retirement income . . . . . 4(a) \$ _____</p> <p>(b) <b>Deductions.</b> If you expect to claim deductions other than the standard deduction and want to reduce your withholding, use the Deductions Worksheet on page 3 and enter the result here . . . . . 4(b) \$ _____</p> <p>(c) <b>Extra withholding.</b> Enter any additional tax you want withheld each pay period . . . . . 4(c) \$ _____</p>	

<b>Step 5: Sign Here</b>	Under penalties of perjury, I declare that this certificate, to the best of my knowledge and belief, is true, correct, and complete.
	<div style="display: flex; justify-content: space-between;"> <div>▶ <u>Employee's signature</u> (This form is not valid unless you sign it.)</div> <div>▶ <u>Date</u></div> </div>

#### 4.3.1: W4 Tax Form

## W2

The Form **W-2**, Wage and Tax Statement, is used to report wages paid to employees and the [taxes withheld](#) from them.<sup>[43]</sup> Employers must complete a Form W-2 for each employee to whom they pay a salary, wage, or other compensation as part of the employment relationship. An employer must mail out the Form W-2 to employees on or before January 31. This deadline gives these taxpayers about 3 months to prepare their returns before the April 15 income tax due date. The form is also used to report [FICA taxes](#) to the [Social Security Administration](#). The

Form W-2, along with Form W-3, generally must be filed by the employer with the Social Security Administration by the end of February. Relevant amounts on Form W-2 are reported by the Social Security Administration to the [Internal Revenue Service](#). In territories, the W-2 is issued with a two letter code indicating which territory, such as W-2GU for Guam. If corrections are made, it can be done on a W-2c. The British-Irish equivalent form to a W-2 is a [P60](#).

22222		Employee's social security number		OMB No. 1545-0008	
b Employer identification number (EIN)		1 Wages, tips, other compensation		2 Federal income tax withheld	
c Employer's name, address, and ZIP code		3 Social security wages		4 Social security tax withheld	
		5 Medicare wages and tips		6 Medicare tax withheld	
		7 Social security tips		8 Allocated tips	
d Control number		9		10 Dependent care benefits	
e Employee's first name and initial Last name Suff.		11 Nonqualified plans		12a	
		13 Statutory employee Retirement plan Third-party sick pay		12b	
		14 Other		12c	
				12d	
f Employee's address and ZIP code					
15 State Employer's state ID number		16 State wages, tips, etc.		17 State income tax	
				18 Local wages, tips, etc.	
				19 Local income tax	
				20 Locality name	

Form **W-2** Wage and Tax Statement 2020 Department of the Treasury—Internal Revenue Service  
Copy 1—For State, City, or Local Tax Department

Figure 4.3.2: W2 Tax Form

## 1099MISC

In the United States, **Form 1099-MISC** is a variant of [Form 1099](#) used to report miscellaneous income. One notable use of Form 1099-MISC was to report amounts paid by a business (including nonprofits<sup>[1]:1</sup>) to a non-corporate US resident [independent contractor](#) for services (in IRS terminology, such payments are *nonemployee compensation*), but starting tax year 2020, this use was moved to the separate [Form 1099-NEC](#). The ubiquity of the form has also led to use of the phrase

“1099 workers” or “the 1099 economy” to refer to the independent contractors themselves.<sup>[2]</sup> Other uses of Form 1099-MISC include rental income, royalties, and [Native American gaming profits](#).<sup>[3]</sup>

The form is issued by the payer (e.g. business) and is due to the recipient (e.g. contractor) by January 31 and to the [IRS](#) by the last day of February<sup>[4]</sup>:<sup>6</sup> each year for work done during the previous tax year.<sup>[3][5]</sup> If the payer is registered to file electronically with the IRS the deadline for filing with the IRS is March 31.<sup>[6]</sup> In accordance with the recently passed [PATH Act](#), these deadlines will be changing so the mailing and transmittal are both January 31 moving forward starting with Tax Year 2016.<sup>[7]</sup>

<input type="checkbox"/> VOID <input type="checkbox"/> CORRECTED					
PAYER'S name, street address, city or town, state or province, country, ZIP or foreign postal code, and telephone no.		1 Rents	OMB No. 1545-0115	<b>Miscellaneous Income</b>	
		\$	<b>2020</b>		
		2 Royalties			
		\$	Form 1099-MISC		
		3 Other income	4 Federal income tax withheld	<b>Copy 1 For State Tax Department</b>	
		\$	\$		
PAYER'S TIN	RECIPIENT'S TIN	5 Fishing boat proceeds	6 Medical and health care payments		
		\$	\$		
RECIPIENT'S name		7 Payer made direct sales of \$5,000 or more of consumer products to a buyer (recipient) for resale <input type="checkbox"/>	8 Substitute payments in lieu of dividends or interest		
Street address (including apt. no.)		9 Crop insurance proceeds	10 Gross proceeds paid to an attorney		
City or town, state or province, country, and ZIP or foreign postal code		\$	\$		
		11	12 Section 409A deferrals		
		\$	\$		
Account number (see instructions)	FATCA filing requirement <input type="checkbox"/>	13 Excess golden parachute payments	14 Nonqualified deferred compensation		
		\$	\$		
		15 State tax withheld	16 State/Payer's state no.	17 State income	
		\$	\$	\$	
		\$	\$	\$	

Form 1099-MISC      www.irs.gov/Form1099MISC      Department of the Treasury - Internal Revenue Service

Figure 4.3.3: 1099Misc Tax Form

## 1099INT

This form shows the amount of interest paid in accounts such as savings accounts.

<input type="checkbox"/> VOID <input type="checkbox"/> CORRECTED		Payer's RTN (optional)		OMB No. 1545-0112 <b>2020</b> Form <b>1099-INT</b>		<b>Interest Income</b>
PAYER'S name, street address, city or town, state or province, country, ZIP or foreign postal code, and telephone no.		1 Interest income \$		2 Early withdrawal penalty \$		
		3 Interest on U.S. Savings Bonds and Treas. obligations \$				
PAYER'S TIN		RECIPIENT'S TIN		4 Federal income tax withheld \$		<b>Copy 1</b>  <b>For State Tax Department</b>
RECIPIENT'S name  Street address (including apt. no.)  City or town, state or province, country, and ZIP or foreign postal code		5 Investment expenses \$		6 Foreign tax paid \$		
City or town, state or province, country, and ZIP or foreign postal code		7 Foreign country or U.S. possession \$		8 Tax-exempt interest \$		
		9 Specified private activity bond interest \$		10 Market discount \$		
Account number (see instructions)		11 Bond premium \$		12 Bond premium on Treasury obligations \$		
		13 Bond premium on tax-exempt bond \$		14 Tax-exempt and tax credit bond CUSIP no. \$		
FATCA filing requirement <input type="checkbox"/>		15 State \$		16 State identification no. \$		
17 State tax withheld \$		18 State tax credit \$		19 State tax credit \$		
Form <b>1099-INT</b>		www.irs.gov/Form1099INT		Department of the Treasury - Internal Revenue Service		

Figure 4.3.4: The 1099INT Tax Form

1098E

The Form **1098-E**, Student Loan Interest Statement, reports interests the taxpayer paid on [student loans](#) that might qualify as an adjustment to income.

<input type="checkbox"/> CORRECTED (if checked)		OMB No. 1545-1576 <b>2020</b> Form <b>1098-E</b>		<b>Student Loan Interest Statement</b>
RECIPIENT'S/LENDER'S name, street address, city or town, state or province, country, ZIP or foreign postal code, and telephone number		1 Student loan interest received by lender \$		
RECIPIENT'S TIN		BORROWER'S TIN		
BORROWER'S name  Street address (including apt. no.)  City or town, state or province, country, and ZIP or foreign postal code		2 If checked, box 1 does not include loan origination fees and/or capitalized interest for loans made before September 1, 2004		This is important tax information and is being furnished to the IRS. If you are required to file a return, a negligence penalty or other sanction may be imposed on you if the IRS determines that an underpayment of tax results because you overstated a deduction for student loan interest.
Account number (see instructions)		3 If checked, box 1 does not include loan origination fees and/or capitalized interest for loans made before September 1, 2004		
Form <b>1098-E</b>		(keep for your records)		www.irs.gov/Form1098E

Figure 4.3.5: 1098e Tax Form

1098T

**Form 1098-T**, Tuition Statement, is an [American IRS tax form](#) filed by eligible education institutions (or those filing on the institution’s behalf) to report payments received and payments due from the paying student. The institution has to report a form for every student that is currently enrolled and paying qualifying tuition and related expenses.<sup>[1]</sup>

Form 1098-T consists of one page, with a red copy to be filed with the IRS, and a black copy to be kept for records or to be sent to the student. There are ten lines that require the institution’s tax information as well as the student’s, tuition payments received and billed, as well as the scholarships granted to the student. There are lines for adjustments to a prior year’s 1098-T, and a checkbox for whether the student is part-time or a graduate student.

☐ CORRECTED

FILER'S name, street address, city or town, state or province, country, ZIP or foreign postal code, and telephone number		1 Payments received for qualified tuition and related expenses \$ 2	OMB No. 1545-1574 <b>2019</b> Form <b>1098-T</b>	<b>Tuition Statement</b>	
FILER'S employer identification no.	STUDENT'S TIN	3	<b>Copy B For Student</b>  This is important tax information and is being furnished to the IRS. This form must be used to complete Form 8863 to claim education credits. Give it to the tax preparer or use it to prepare the tax return.		
STUDENT'S name		4 Adjustments made for a prior year \$			5 Scholarships or grants \$
Street address (including apt. no.)		6 Adjustments to scholarships or grants for a prior year \$			7 Checked if the amount in box 1 includes amounts for an academic period beginning January—March 2020 <input type="checkbox"/>
City or town, state or province, country, and ZIP or foreign postal code		8 Checked if at least half-time student <input type="checkbox"/>			9 Checked if a graduate student <input type="checkbox"/>
Service Provider/Acct. No. (see instr.)		10 Ins. contract reimb./refund \$			

Form **1098-T** (keep for your records) [www.irs.gov/Form1098T](http://www.irs.gov/Form1098T) Department of the Treasury - Internal Revenue Service

Figure 4.3.6: 1098T Tax Form



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[https://pressbooks.nsc.ca/  
mathofmoney/?p=97#h5p-19](https://pressbooks.nsc.ca/mathofmoney/?p=97#h5p-19)



## 4.4 INCOME TAX PROBLEMS

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### *Money Coach: Doing Your Taxes*



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<https://pressbooks.nsc.ca/mathofmoney/?p=99#oembed-1>

For the following questions, we will be using this less realistic, but much easier-to-use tax bracket. It will produce results similar to the US tax code, but is not the same. We will also assume that everybody is married (to work with only one bracket instead of two).

<b>Rate</b>	<b>Income</b>
10%	Up to \$20,000
12%	\$20,001 to \$80,000
22%	\$80,001 to \$170,000
24%	\$170,001 to \$325,000
32%	\$325,001 to \$415,000
35%	\$415,000 to \$625,000
37%	Over \$625,000

Additionally, we will use the following deductions/credits:

- Standard deduction is \$25,000.
- Child tax credit is \$2,000 per child under 17 years old.
- Student loan interest deduction up to \$2,500 per family.
- Any other deductions/credits will be given explicitly.

So let us begin with a basic example first: The Smith family has an income of \$350,000. Ignore any deductions and credits for this example. How do we calculate their

income tax due? We must calculate the tax owed in each bracket. I perform this in the table below.

Income in Bracket	Rate	Tax Due	Income Left
\$20,000	10%	\$2,000	\$330,000
\$60,000	12%	\$7,200	\$270,000
\$90,000	22%	\$19,800	\$180,000
\$155,000	24%	\$37,200	\$25,000
\$25,000	32%	\$8,000	\$0
\$0	35%	\$0	\$0
\$0	37%	\$0	\$0
Total		\$74,200	

So what we see is that tax is due in each bracket and is not a straight percentage. We can now explore the average and marginal tax rates.

The average tax rate for this family is 74,200 (taxes due) divided by \$350,000 (income) which comes out to 21.2%. The marginal tax rate is the tax rate the family would have to pay if they earned \$1 more in income. For this family, that would be 32% (their current tax bracket.) Notice how their first \$20,000 in income is taxed at 10%!

Now, let us go even deeper since we can run this basic calculation.

The Rodriguez family has the following information:

- Their combined family income is \$170,000.
- Their itemized deductions are \$15,000.
- They paid \$1,500 in student loan interest.

- They have two children under 17.
- They have paid \$19,000 in federal withholding.
- Find the tax refund/tax due.

We begin with the families income and then subtract the deductions. They must decide between itemizing deductions or taking the standard deduction. The standard deduction for a family is \$25,000 which is more than their itemized deductions, so they should take the standard deduction. Additionally, even though they took the standard deduction, they can still deduct the student loan interest (one of the few where this is possible.) Therefore, their taxable income is  $\$170,000 - \$25,000 - \$1,500 = \$143,500$ . Next, we calculate the tax due using the table below.

Income in Bracket	Rate	Tax Due	Income Left
\$20,000	10%	\$2,000	\$123,500
\$60,000	12%	\$7,200	\$63,500
\$63,500	22%	\$13,970	\$0
\$0	24%	\$0	\$0
\$0	32%	\$0	\$0
\$0	35%	\$0	\$0
\$0	37%	\$0	\$0
<b>Total</b>		<b>\$23,170</b>	

After we calculate tax due, we then subtract any tax credits. In this case, the family has two eligible children which means a tax credit of  $\$2,000 \times 2 = \$4,000$ . So their new tax bill is  $\$23,170 - \$4,000 = \$19,170$ .

During the year, the family had taxes withheld from their paychecks, so they have actually paid some/most/

all of their taxes. We now compare what is owed versus what they have paid. We see that while they owe \$19,170, they have already paid \$19,000. Since tax due > tax paid, they owe. In this case, they owe  $\text{OWE } \$19,170 - \$19,000 = \$170$ .

We can also calculate their average tax rate as  $19,170 / 170,000 = 11.28\%$  and their marginal tax rate is their current tax bracket which is 22%.

### Let's try one together...

The Chen family has an income of \$240,000. They have \$22,000 in itemized deductions, 1 child under 17, and have paid \$38,000 in taxes already. Calculate their taxes due/refund due and their average tax rate.

Answer: \$200 refund; ATR=15.75%



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<https://pressbooks.nsc.ca/mathofmoney/?p=99#oembed-2>

### Let's try one together...

The Miller family has an income of \$70,000. They have

\$9,000 in itemized deductions, have paid \$3,000 in student loans, 1 child under 17, and have paid \$8,000 in taxes already. Calculate their taxes due/refund due and their average tax rate.

Answer: \$5,300 refund; 3.86%



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# 4.5 OTHER INCOME TAXES

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## OASDI (SOCIAL SECURITY)

Social Security is funded primarily through payroll taxes called Federal Insurance Contributions Act tax (FICA) or Self Employed Contributions Act Tax (SECA). Tax deposits are collected by the Internal Revenue Service (IRS) and are formally entrusted to the Federal Old-Age and Survivors Insurance Trust Fund and the Federal Disability Insurance Trust Fund, the two Social Security Trust Funds. These two trust funds purchase government securities, the interest income from which is used presently to fund the monthly allocations to qualifying citizens. With a few exceptions, all salaried income, up to an amount specifically determined by law (see tax rate table below), is subject to the Social Security payroll tax. All income over said amount is not taxed.<sup>1</sup>

Employee Rate	Employer Rate	Income
6.2%	6.2%	Up to \$142,800
0.0%	0.0%	Over \$142,800

1. Social Security (United States). (). In Wikipedia.  
[https://en.wikipedia.org/wiki/Social\\_Security\\_\(United\\_States\)](https://en.wikipedia.org/wiki/Social_Security_(United_States))

## Let's try one together...

Find the individual social security tax due for the following people. Neglect the employer share. Additionally, find the average tax rate for each.

- Albert = \$72,500
- Bella = \$195,000
- Collette = \$525,000

Answers: \$4,495 (6.2%); \$8,853.60 (4.54%); \$8,853.60 (1.69%)



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## MEDICARE TAX

There is also a tax that funds the Medicare system. Unlike Social Security, there is no wage limit. Additionally, there is an additional tax at higher incomes for employees only. The brackets are below:

Employee Rate	Employer Rate	Income
1.45%	1.45%	Up to \$200,000
2.35%	1.45%	Over \$200,000



## Let's do one together...

Find the Medicare tax due for the following people. Neglect the employer share. Additionally, find the average tax rate for each.

- Albert = \$72,500
- Bella = \$195,000
- Collette = \$525,000

Answers: \$1,051.25 (1.45%); \$2,827.50 (1.45%);  
\$10,537.50 (2.01%)



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<https://pressbooks.nscc.ca/mathofmoney/?p=101#oembed-2>

## STATE AND LOCAL INCOME TAXES

Just as the federal government collects taxes on income, many state and local governments do as well. This varies from one state to the next. For example, Florida does not charge an income tax. Additionally, the way states charge taxes will vary from one to the next. Some states rely more on property taxes and sales tax. Others on income

taxes. Others on fees (for example: Florida charges a variety of excise taxes on tourist activities.)

When a state charges an income tax, they decide the structure of the tax system. Many states have a system similar to the federal government with deductions and credits and a tiered tax system.

Pennsylvania on the other hand has a flat tax rate of 3.07%. Additionally, there are very few deductions and credits. Therefore, most of the time, you will have paid exactly your tax due for the year and will neither owe money nor be due a refund. Many localities in Pennsylvania also charge a 1% local income tax which is also a flat percentage. Some localities can charge more (Pittsburgh charges 3.0%). Your actual rate is split between the rate of your home address and the rate of your work address.



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<https://pressbooks.nsc.ca/mathofmoney/?p=101#h5p-20>

## MODULE 4 PRACTICE PROBLEMS

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**Problem 4.1:** You purchase a new Microsoft Surface from Staples at the old Washington Mall for \$799.99. Find the total price you will have to pay at checkout. The tax rate for Washington County is 6%.

Answer: \$847.99



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<https://pressbooks.nsc.ca/mathofmoney/?p=103#oembed-1>

**Problem 4.2:** You go to Aldi and buy \$95.00 worth of food. But, as is often the case with Aldi, you also buy \$50 worth of stuff in the random stuff aisle. While food is not taxable in Pennsylvania, the other stuff is and is taxed at a rate of 6%. Calculate your grand total.

Answer: \$148



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view them online here: <https://pressbooks.nsc.ca/mathofmoney/?p=103#oembed-2>

**Problem 4.3:** You buy several electronics from Target. Your grand total is \$375.50. Assuming the sales tax rate is 6%, calculate the subtotal; that is, the total *before* tax.

Answer: \$354.25



One or more interactive elements has been excluded from this version of the text. You can view them online here:

<https://pressbooks.nsc.ca/mathofmoney/?p=103#oembed-3>

**Problem 4.4: CHALLENGE:** One thing we did not discuss was sales tax on automobile purchases. In Pennsylvania, you pay the typical sales tax rate for your county on the sale price of the vehicle MINUS any trade-in value. Suppose you want to purchase a new Subaru Forester for \$28,845. You have visited Budd Baer (Washington County, 6%) and Bowser Subaru (Allegheny County, 7%). Calculate the grand total price if you have no trade-in. Calculate the grand total price if both dealerships offer you \$12,000 for your trade-in.

Answers: No trade – Wash = \$30,575.70; Allg = \$30,864.15; Trade-in – Wash = \$17,855.70; Allg = \$18,024.15



One or more interactive elements has been excluded from this version of the text. You can view them online here:

<https://pressbooks.nsc.ca/mathofmoney/?p=103#oembed-4>

**Problem 4.5:** While visiting a friend in Iceland, you purchase groceries for 325 174, 77 ISK. Iceland has a VAT of 24%. Calculate the price of the groceries.

Answer: 262 237,72 ISK



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<https://pressbooks.nsc.ca/mathofmoney/?p=103#oembed-5>

For the following tax problems, use the tax system we used earlier.

Rate	Income
10%	Up to \$20,000
12%	\$20,001 to \$80,000
22%	\$80,001 to \$170,000
24%	\$170,001 to \$325,000
32%	\$325,001 to \$415,000
35%	\$415,000 to \$625,000
37%	Over \$625,000

Additionally, we will use the following deductions/credits:

- Standard deduction is \$25,000.
- Child tax credit is \$2,000 per child under 17 years old.
- Student loan interest deduction up to \$2,500 per family.
- Any other deductions/credits will be given explicitly.

**Problem 4.6:** The Howard family has an income of \$615,000. They have \$39,000 in itemized deductions, 2 children under 17 (NOTE: Under the US tax system, they would not qualify for the child tax credit, but we will just let every child have the tax credit in our class), and have paid \$129,000 in taxes already. Calculate their taxes due/refund due and their average tax rate.

Answers: Owe \$21,850; ATR = 24.14%



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<https://pressbooks.nsc.ca/mathofmoney/?p=103#oembed-6>

**Problem 4.7:** The Tims family has an income of \$165,000. They have \$7,000 in itemized deductions, 2 children under 17, have paid \$4,000 in student loan interest, are eligible for \$1,000 in other tax credits, and have paid \$22,000 in taxes already. Calculate their taxes due/refund due and their average tax rate.

Answers: Refund of \$5,150; ATR = 10.2%



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<https://pressbooks.nsc.ca/mathofmoney/?p=103#oembed-7>

**Problem 4.8:** The Ellis family has an income of \$75,000. They have \$5,000 in itemized deductions, 0 children under 17, and have paid \$6,000 in taxes already. Calculate their taxes due/refund due and their average tax rate.

Answers: Refund of \$400; ATR = 7.47%



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<https://pressbooks.nsc.ca/mathofmoney/?p=103#oembed-8>

**Problem 4.9:** Treat each family in problems 4.6-4.8 as a single individual. Calculate the Social Security tax payment (and average tax rate) and Medicare tax payment (and average tax rate) for each. Ignore the employer contribution. The tax brackets are given below.

Social Security

Employee Rate	Employer Rate	Income
6.2%	6.2%	Up to \$142,800
0.0%	0.0%	Over \$142,800

Medicare

Employee Rate	Employer Rate	Income
1.45%	1.45%	Up to \$200,000
2.35%	1.45%	Over \$200,000

Answers: Howard [SS = \$8,853.60 (1.44%), Medi = \$12,652.50 (2.06%)]; Tims [SS = \$8,853.60 (5.36%), Medi = \$2,932.50 (1.45%)]; Ellis [SS=\$4,650 (6.2%), Medi = \$1,087.50 (1.45%)]





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## MODULE 5: INSURANCE

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## 5.1 HEALTH INSURANCE

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<sup>1</sup>Melissa is a medical transcriptionist who runs a cleaning service on the side. She usually clears about \$24,000 per year from the cleaning service and has come to rely on that money. One day, Melissa slips on a wet floor. She is taken by ambulance to the local hospital, where she is treated for a badly broken wrist and released the next day. Melissa can't clean for about eight weeks, losing close to \$6,000 in earnings.

Soon, medical bills start to arrive. Melissa is not concerned, because she has health insurance through her job as a medical transcriptionist. She is surprised to find out, however, that some of the costs of this accident are not covered, that she has a significant deductible, and that she'll also have to pay the difference between what the doctors billed and what the insurance will pay. Not only did she lose substantial cleaning earnings, but her out-of-pocket costs are mounting as well. This accident is beginning to be very costly.

Melissa is discovering that health insurance is a complicated business. The time to understand your health coverage is before you need it. When you are recovering from an accident or illness, you should not be concerned with your medical bills, yet you may have to be.

1. Adapted from [10.2 Insuring Your Health](#) in [Personal Finance](#) by Lumen Learning. [CC BY-NC-SA 4.0](#)

Medical bills can be a major source of financial ruin. One study found that nearly 2/3 of all personal bankruptcies are tied to medical bills (Himmelstein, 2009). Other studies have shown that number to be a bit lower, but it is a fact that medical-caused bankruptcies impact millions of Americans each year.

Even if you think those numbers are exaggerated, it's still sobering, because no matter how much you try to take care of yourself and to be careful, no one can evade the pure risk of injury or illness. All you can do is try to shift that risk in a way that makes sense for your financial health.

Because of the increasing costs of health care and the increasing complexities of paying for them, the distribution and financing of health care is much discussed and debated in the United States, especially the roles of the federal government and insurance providers. Regardless of the outcome of this debate, momentum is building for change. You should be aware of changes as they occur so that you can incorporate those changes into your budget and financial plans.

## HEALTH INSURANCE COVERAGE

There are many different kinds of coverage and plans for

health insurance. You may have group health insurance offered as an employee benefit or as a member of a professional association. Group plans have lower costs, because the group has some bargaining power with the insurer and can generally secure lower rates for its members. But group plans are not necessarily comprehensive, so you may want to supplement the group coverage with an individual health insurance policy, available to individuals and families.

Sufficient coverage should include **basic insurance** and major medical insurance. A basic insurance policy will cover physician expense, surgical expense, and hospital expense.

- Physician expenses include nonsurgical treatments and lab tests.
- Surgical expenses include surgeons' fees.
- Hospital expenses include room and board and other hospital charges.

The three basic coverages are usually combined under one policy. In addition, health insurance is completed by **major medical insurance**, which covers the costs of a serious injury or illness. Depending on the extent and the nature of your illness or injury, medical bills can quickly exceed your basic coverage limits, so major medical can act as an extension to those limits, saving you from potential financial distress.

Dental insurance also supplements your basic insurance, usually providing reimbursement for preventative treatments and some partial payment of dental services such as fillings, root canals, crowns, extractions, bridgework, and dentures. Vision insurance

provides for eye care, including exams and treatment for eye diseases, as well as for corrective lenses. Depending on your basic coverage limits, dental and vision care could be important for you.

Another feature of basic coverage is a prescription drug plan. Prescriptions may be covered entirely or with a co-pay, or only if the generic version of the drug is available. Your insurer should provide a **formulary** or a list of drugs that are covered. Depending on your plan, prescription coverage may be available only as a supplement to your basic coverage.

## HEALTH INSURANCE COSTS

As health care costs and insurance premiums rise, insurers add cost offsets to make their policies more affordable. Those offsets may include the following:

- **Deductibles**—an amount payable by the insured before any expenses are assumed by the insurer.
- **Co-pays**—partial payment for certain costs—for example, for physician's visits or prescriptions.
- **Coinsurance**—shared payments of expenses by insured and insurer.



Each of these payment features represents responsibilities of the insured, that is, your out-of-pocket costs. The more costs you shoulder, the less risk to the insurer, and so the less you pay for the insurance policy. Making you responsible for initial costs also discourages you from seeking health care more than is necessary or from submitting frivolous health care claims.

*MoneyCoach: Health Insurance 101*



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<https://pressbooks.nsc.ca/mathofmoney/?p=108#oembed-3>

### Let's try one together...

Suppose you have insurance with a \$50 copay, \$500 deductible and 90/10 coinsurance. Below are your medical bills for the year:

Jan 10 – \$200 bill

May 14 – \$150 bill

August 11 – \$400 bill

December 22 – \$800 bill

For each, calculate how much you will be responsible for and how much insurance will pay.

Extra: December 23 – \$15,000 bill

Answer: You pay \$785 and they pay \$765 (see video for breakdown). After the large \$15,000 bill, you will have

paid a total of \$2,330 and they will have paid a total of \$14,220 (see video for breakdown).



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<https://pressbooks.nsc.ca/mathofmoney/?p=108#oembed-1>

## Let's try one together...

Suppose that you take a job which offers three choices for the health insurance plan.

Gold: Premium=\$225/month, \$100 deductible, no coinsurance

Silver: Premium=\$175/month, \$500 deductible, 90/10 coinsurance

Bronze: Premium=\$125/month, \$1,000 deductible, 80/20 coinsurance

Consider the following three people...what should they choose?

You: Healthy, estimated \$300/year in medical expenses

John Doe: Semi-healthy \$1,500/year in medical expenses

Jane Doe: Not healthy \$10,000/year in medical expenses

Answers: Bronze, Bronze, Gold (see video for breakdown)



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<https://pressbooks.nsc.ca/mathofmoney/?p=108#oembed-2>

Costs vary with coverage, coverage limits, and offsets, and they vary widely between insurers. You should be well informed as to the specifics of your coverage, and you should compare rates before you buy. An insurance broker can help you to do this, and there are Web sites designed to help you explore the available options.

## HEALTH INSURANCE AND HEALTH CARE

Health insurance is sold through private insurers, nonprofit service plans, and managed care organizations. Private insurers sell most of their plans to employers as group plans. Individuals are far more likely to purchase insurance through a service plan or managed care.

Private (for-profit) plans in most states are underwritten based on your age, weight, smoking status, and health history and are generally more expensive than other types of plans. You may have to take a medical exam, and specific preexisting conditions—such as asthma, heart disease, anxiety, or diabetes—could be excluded from coverage or used as grounds for increasing the cost of your premium, based on your higher risk. Nevertheless, federal and state laws protect you from being denied health care coverage because of any preexisting condition.

A service plan such as Blue Cross/Blue Shield, for example, consists of regional and state-based nonprofit agencies that sell both group and individual policies. More than half of the health insurance companies in the United States are nonprofits, including, for example, Health Care Service Corporation and Harvard Pilgrim Health Care as among the largest.<sup>2</sup>

**Managed care organizations** became popular in the last thirty years or so with the idea that providing preventative care would lower health care costs. Managed care takes the following forms:

- Health maintenance organizations
- Preferred provider organizations
- Exclusive provider organizations
- Point-of-service plans
- Traditional indemnity plans

The two most familiar kinds of managed care are health

2. (<http://www.nonprofithealthcare.org/resources/BasicFactsAndFigures-NonprofitHealthPlans9.9.08.pdf>)

maintenance organizations (HMOs) and preferred provider organizations (PPOs). A **health maintenance organization** directly hires physicians to provide preventative, basic, and supplemental care. Preventative care should include routine exams and screening tests and immunizations. Basic care should include inpatient and outpatient treatments, emergency care, maternity care, and mental health and substance abuse services. As with any plan, the details for what defines “basic care” will vary, and you should check the fine print to make sure that services are provided. For example, the plan may cover inpatient hospitalizations for a limited number of days in case of a physical illness, but inpatient hospitalization for a more limited number of days for a mental illness.

Supplemental care typically includes the cost of vision and hearing care, prescriptions, prosthetics devices, or home health care. Some or all of this coverage may be limited, or may be available for an added premium. The premium paid to the HMO is a fixed, monthly fee, and you must seek care only within the HMO’s network of care providers.

The most serious constraint of HMOs is the limited choice of doctors and the need to get a referral from your primary care physician (PCP) to obtain the services of any specialist. Depending on where you live and the availability of medical practitioners, this may or may not be an issue for you, but before joining an HMO, you should consider the accessibility and convenience of the care that you are allowed, as well as the limitations of the coverage. For example, if you are diagnosed with a serious disease or need a specific surgical technique, is there an appropriate specialist in the network that you

can consult? Suppose you want a second opinion? The rules differ among HMOs, but these are the kinds of questions that you should be asking. You should also be familiar with the HMO's appeal procedures for coverage denied.

The **preferred provider organization (PPO)** has a different arrangement with affiliated physicians: it negotiates discounted rates directly with health care providers in exchange for making them the “preferred providers” for members seeking care. Care by physicians outside the network may be covered, but with more limitations, or higher co-pays and deductibles. In exchange for offering the flexibility of more choices of provider, the PPO charges a higher premium. Services covered are similar to those covered by an HMO.

The exclusive provider organization works much like the PPO, except that out-of-network services are not covered at all and become out-of-pocket expenses for the insured.

The **point-of-service (POS)** plan also uses a network of contracted, preferred providers. As in an HMO, you choose a primary care physician who then controls referrals to specialists or care beyond preventative and basic care. As in the PPO, out-of-network services may be used, but their coverage is more limited, and you pay higher out-of-pocket expenses for co-pays and deductibles.

*MoneyCoach: Health Insurance 101 – Types of Plans*



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<https://pressbooks.nsc.ca/mathofmoney/?p=108#oembed-4>

## PRIVATE HEALTH CARE FINANCING

In the United States, if someone is not self-insured or uninsured, health insurance coverage is paid for, at least in part, by the employer. As health care costs have risen, employers in all industries have increasingly complained that this cost makes them less competitive in global markets. As an incentive to have more people paying the costs of health care themselves and to be less dependent on employers, the federal government has created tax deductions for savings earmarked for use in paying for health costs. These savings plans are known as flexible spending accounts (FSAs), health reimbursement accounts (HRAs), and health savings accounts (HSAs).

A **flexible savings account** is used to supplement your basic coverage. It is offered by employers and funded by employees: you may have a tax-exempt deduction made

from your paycheck to your flexible spending account. The money from your FSA may be used for care expenses not normally covered by your plan—for example, orthodonture, elder care, or child care. At the end of the year, any money remaining in your account is forfeited; that is, it does not roll over into the next year. Unless you can foresee expenses within the coming year, flexible spending may not be worth the tax break.

A **health reimbursement account** is an account funded by employers. The amount is used to pay the premiums for basic coverage with a high deductible, and any money left over may be used for other health expenses, or, if unused, may be carried over to the next year. The account is yours until you leave your job, when it reverts back to your employer.

A **health savings account** (HSA) allows a tax-deductible contribution from your paycheck to pay the premiums for catastrophic coverage with a high deductible and whatever out-of-pocket health care costs you may have. It is employee funded, employee managed, and employee owned. Thus, it is yours, and you may take it with you when you change jobs.

Figure 5.1.1 shows the differences between these accounts.

FSA	HRA	HSA
<ul style="list-style-type: none"><li>• Funded by employee</li><li>• Use for 1 year</li><li>• Contributions not taxed</li></ul>	<ul style="list-style-type: none"><li>• Funded by employer</li><li>• Use until leave employment</li></ul>	<ul style="list-style-type: none"><li>• Funded by employee</li><li>• No time limit on use</li><li>• Contributions not taxed</li></ul>

Figure 5.1.1: Private Healthcare Funding



A health savings account shifts the responsibility for health insurance from the employer to the employee, although it still gives the employee access to lower group rates on premiums. If you are relatively young and healthy, and your health care need is usually just an annual physical, this seems like an advantageous plan. However, remember that the idea of insurance is to shift risk away from you, to pay someone to assume the risk for you. With a high-deductible policy, you are still bearing a lot of risk. If that risk has the potential to cause a financial disaster, it's too much.

*RPD: HSA vs FSA*



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If you have employer-sponsored health insurance and you leave your job, you may be entitled to keep your insurance for eighteen months (or more under certain circumstances). Under the 1985 Consolidated Budget Omnibus Reconciliation Act (COBRA), an employee at a company with at least twenty employees who notifies the employer of his or her intention to maintain health care coverage is entitled to do so provided the employee pays the premiums. Some states extend this privilege to companies with less than twenty employees, so you should check with your state's insurance commissioner. You may also be able to convert your group coverage

into an individual policy, although with more costly premiums.

The Health Insurance Portability and Accountability Act (HIPAA) of 1996 addresses issues of transferring coverage, especially as happens with a change of jobs. It credits an insured for previous periods of insurance coverage that can be used to offset any waiting periods for coverage of preexisting conditions. In other words, it makes it easier for someone who is changing jobs to maintain continuous coverage of chronic conditions or illnesses.<sup>34</sup>

## PUBLIC HEALTH CARE FINANCING

The federal government, in concert with state governments, provides two major programs to the general public for funding health care: Medicare and

3.

4.<sup>5</sup> (For more information, research the U.S. Department of Health and Human Services at <http://www.hhs.gov>; see, for example, <http://www.hhs.gov/ocr/privacy/hipaa/administrative/statute/hipaastatutepdf.pdf>.)

5. Centers for Medicare and Medicaid Services, U.S. Department of Health and Human Services (n.d.), <http://www.cms.hhs.gov/hipaaGenInfo/> (accessed November 24, 2009).

Medicaid. The federal government also provides services to veterans of the armed forces, and their spouses and dependents, provided they use veterans' health care facilities and providers.

**Medicare** was established in 1965 to provide minimal health care coverage for the elderly, anyone over the age of sixty-five. Medicare offers hospital (Part A), medical (Part B), combined medical and hospital (Part C), and prescription coverage (Part D), as outlined in Figure 5.1.2.

<b>Part A</b>	Hospital	Compulsory	Choice of doctors
<b>Part B</b>	Medical	Optional	Choice of doctors
<b>Part C</b>	Hospital and medical	Optional	HMO or PPO
<b>Part D</b>	Prescriptions	Optional	Purchased through an approved insurer
<b>Medigap</b>	Supplemental	Optional	Individual policies differ

*Figure 5.1.2: Medicare Coverages*

Medicare is really a combination of privately and publicly funded health care; the optional services all require some premium paid by the insured. You may not need Medicare's supplemental plans if you have access to supplemental insurance provided by your former employer or by membership in a union or professional organization.

Medicare does not cover all services. For example, it does not cover dental and vision care, private nursing care, unapproved nursing home care, care in a foreign country, and optional or discretionary (unnecessary) care.

Medicare also determines the limits on payments for services, but physicians may charge more than that for their services (within limits determined by Medicare). You would be responsible for paying the difference. For

these reasons, it is advisable to have supplemental insurance.

Marley thought she didn't need to know anything about Medicare, being young, single, and healthy, but then her sixty-six-year-old father developed a debilitating illness, requiring not only medical care but also assistance with many of his daily living activities. Suddenly, Marley was shouldering the responsibility of arranging her father's care and devising a strategy for financing it. She quickly learned about the care and limits of coverage offered by various Medicare plans.

**Medicaid** was also established in 1965 to provide health care based on income eligibility. It is administered by each state following broad federal guidelines and is jointly financed by the state and federal government. This means that states differ somewhat in the benefits or coverage they offer. If someone is covered by both Medicaid and Medicare, Medicaid pays for expenses not covered by Medicare, such as co-pays and deductibles. Together, Medicare and Medicaid pay about 60 percent of all nursing home costs.<sup>6</sup>

*Medicareful: What is Medicare?*



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6. <sup>7</sup>

7. The Henry J. Kaiser Family Foundation. (January 2006). [\*Paying for Nursing Home Care: Asset Transfer and Qualifying for Medicaid.\*](#)

<https://pressbooks.nscc.ca/mathofmoney/?p=108#oembed-6>

## LONG-TERM CARE INSURANCE

**Long-term care insurance** is designed to insure your care should you be chronically unable to care for yourself. “Care” refers not to medical care, but to care of “activities of daily living” (ADLs) such as bathing, dressing, toileting, eating, and mobility, which may be impaired due to physical or mental illness or injury.

Long-term care coverage is offered as either indemnity coverage or “expense-incurred” policies. With an indemnity policy, you will be paid a specified benefit amount per day regardless of your costs incurred. With an “expense-incurred” policy, you will be reimbursed for your actual expenses incurred. Both types of policies can have limits, either for dollar amounts per day, week, or month or for number of days or years of coverage. Newer policies are designed as integrated policies, offering

pooled benefits and specifying a total dollar limit of benefits that may be used over an unspecified period.

Need for long-term care is anticipated in older age, although anyone of any age may need it. When you buy the policy, you may be far away from needing the coverage. For that reason, many policies offer benefit limits indexed to inflation, to account for cost increases that happen before you receive benefits.

The cost of a long-term care policy varies with your age, coverage, policy features such as inflation indexing, and current health. As with any insurance purchase, you should be as informed as possible, comparing coverage and costs before buying.



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## CHAPTER CREDIT

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## 5.2 PROPERTY INSURANCES

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<sup>1</sup>Property insurance is ownership insurance: it insures that the rights of ownership conferred upon you when you purchased your property will remain intact. Typically, property insurance covers loss of use from either damage or theft; loss of value, or the cost of replacement; and liability for any use of the property that causes damage to others or others' property. For most people, insurable property risks are covered by insuring two kinds of property: car and home.

Loss of use and value can occur from hazards such as fire or weather disasters and from deliberate destruction such as vandalism or theft. When replacement or repair is needed to restore usefulness and value, that cost is the cost of your risk. For example, if your laptop's hard drive crashes, you not only have the cost of replacing or repairing it, but also the cost of being without your laptop for however long that takes. Insuring your laptop shares that risk (and those costs) with the insurer.

Liability is the risk that your use of your property will injure someone or something else. Ownership implies control of, and therefore responsibility for, property use.

For example, you are liable for your dog's attack on a pedestrian and for your fallen tree's damage to a neighbor's fence. You also are liable for damage a friend

1. Adapted from [10.1 Insuring Your Property](#) in [Personal Finance](#) by Lumen learning. [CC BY-NC-SA](#)

causes while driving your car with your permission and for injury to your invited guests who trip over your lawn ornament, fall off your deck, or leave your party drunk.

Legal responsibility can be from

- **negligence**, or responsibility for intentional or unintentional events;
- **vicarious liability**, or responsibility for someone else's use of your possessions or someone else's activity for which you are responsible.

## HOME INSURANCE

Homeowner's insurance insures both the structure and the personal possessions that make the house your home. Renter's insurance protects your possessions even if you are not the owner of your dwelling. You may not think you need insurance until you are the homeowner, but even when you don't need to insure against possible damage or liability for your dwelling, you can still insure your possessions. Even if your furniture came from your aunt's house or a yard sale, it could cost a lot to replace.

If you have especially valuable possessions such as jewelry or fine musical instruments, you may want to insure them separately to get enough coverage for them. Such items are typically referred to as **listed property**



and are insured as **endorsements** added on to a homeowners' or renter's policy. Items should be appraised by a certified appraiser to determine their replacement or insured value.

*Central Insurance: Insurance 101 – Renter's Insurance*



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A good precaution is to have an up-to-date inventory of your possessions such as furniture, clothing, electronics, and appliances, along with photographs or video showing these items in your home. That inventory should be kept somewhere else, such as a safe deposit box. If the house suffered damage, you would then have the inventory to help you document your losses.

A homeowners' policy covers damage to the structure itself as well as any outbuildings on the property and, in some cases, even the landscaping or infrastructure on the grounds, such as a driveway.

A homeowners' policy does not cover

- animals;
- property of renters, or property kept in an apartment regularly rented;
- business property, even if the business is conducted on the residential premises.

According to information from the [Insurances Services Office](#) an insurance industry data and research company, hazards covered by the homeowner's policy include

- fire or lightning;
- windstorm or hail;
- explosion;
- riot or civil commotion;
- damage caused by aircraft;
- damage caused by vehicles;
- smoke;
- vandalism or malicious mischief;
- theft;
- volcanic eruption;
- falling objects;
- weight of ice, snow, or sleet;
- accidental discharge or overflow of water or steam from within a plumbing, heating, air conditioning, or automatic fire-protective sprinkler system, or from a household appliance;
- sudden and accidental tearing apart, cracking, burning, or bulging of a steam or hot water heating, air conditioning, or automatic fire-protective system;
- freezing of a plumbing, heating, air conditioning, or automatic fire-protective sprinkler system, or of a household appliance;
- sudden and accidental damage from artificially

generated electrical current (does not include loss to a tube, transistor, or similar electronic component).

Note that floods and earthquakes are not covered. A homeowner in a flood- or earthquake-prone area may buy special coverage, either from a private insurer or from a federal or state program.


Homeowners' insurance covers the less direct costs of hazards as well. For example, the costs of removing damaged goods or temporary repairs are covered. The cost of temporary housing and extra living expenses while repairs are made is covered, although usually for a limited time or amount.

Homeowners' policies cover liability for injuries on the property and for injuries that the homeowner may accidentally inflict. You may also want to add an **umbrella policy** that covers personal liabilities such as slander, libel, and defamation of character. An umbrella policy may also extend over other assets, such as vehicles or rentals covered by other insurance carriers. If you participate in activities where you are assuming responsibilities for others—you are taking the Cub Scout pack out for a hike, for example, or volunteering at your local recycling center—you may want such extended liability coverage available through your homeowners' policy (also available separately).

*MoneyCoach: Homeowners Insurance 101*



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## HOME INSURANCE COVERAGE: THE BENEFIT

Home insurance policies automatically cover your possessions for up to 40 percent of the house's insured value. You can buy more coverage if you think they are worth more. The benefits are specified as either **actual cash value** or **replacement cost**. Actual cash value tries to estimate the actual market value of the item at the time of loss, so it accounts for the original cost less any depreciation that has occurred. Replacement cost is the cost of replacing the item. For most items, the actual cash value is less.

For example, say your policy insures items at actual cash value. You are claiming the loss of a ten-year-old washer and dryer that were ruined when a pipe burst and your basement flooded. Your coverage could mean a benefit of \$100 (based on the market price of ten-year-

old appliances). However, to replace your appliances with comparable new ones could cost \$1,000 or more.

The actual cash value is almost always less than the replacement value, because prices generally rise over time and because items generally depreciate (rather than appreciate) in value. A policy that specifies benefits as replacement costs offers more actual coverage. **Guaranteed replacement costs** are the full cost of replacing your items, while **extended replacement costs** are capped at some percentage—for example, 125 percent of actual cash value.

## HOME INSURANCE COVERAGE: THE COST

You buy home insurance by paying a premium to the insurance company. The insurance purchase is arranged through a broker, who may represent more than one insurance company. The broker should be knowledgeable about various policies, coverage, and premiums offered by different insurers.

The amount of the premium is determined by the insurer's risk—the more risk, the higher the premium. Risk is determined by

- the insured (the person buying the policy),

- the property insured,
- the amount of coverage.

Insurers may offer discounts for enhancements that lower risks, such as alarm systems or upgraded electrical systems. (Smoke detectors are required by law in every state.) You also may be offered a discount for being a loyal customer, for example, by insuring both your car and home with the same company. Be sure to ask your insurance broker about available discounts for the following:

- Multiple policies (with the same insurer)
- Fire extinguishers
- Sprinkler systems
- Burglar and fire alarms
- Deadbolt locks and fire-safe window grates
- Longtime policyholder
- Upgrades to plumbing, heating, and electrical systems

## AUTO INSURANCE

If you own and drive a car, you must have car insurance. Your car accident may affect not only you and your car, but also the health and property of others. A car accident often involves a second party, and so legal and financial responsibility must be assigned and covered by both parties. In the United States, financial responsibility laws in each state mandate minimal car insurance, although what's "minimal" varies by state.

Conventionally, a victim or plaintiff in an accident is reimbursed by the driver at fault or by his or her insurer. Fault has to be established, and the amount of the claim agreed to. In practice, this has often been done only through extensive litigation.

Some states in the United States and provinces in Canada have adopted some form of **no-fault insurance**, in which, regardless of fault, an injured's own insurance covers his or her damages and injuries, and a victim's ability to sue the driver at fault is limited. The idea is to lower the incidence of court cases and speed up compensation for victims. The states with compulsory no-fault auto insurance, in which personal injury protection (PIP) is required, include Florida, Hawaii, Kansas, Kentucky, Massachusetts, Michigan, Minnesota, New Jersey, New York, North Dakota, Pennsylvania, Utah, and Puerto Rico. The remaining states in the United States use the conventional tort system (suing for damages in court). Understanding the laws of the state where you drive will help you to make better insurance decisions.

## AUTO INSURANCE COVERAGE

*MoneyCoach: Car Insurance 101*



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Auto insurance policies cover two types of consequences: bodily injury and property damage. Each covers three types of financial losses. Figure 5.2.1 shows these different kinds of coverage.

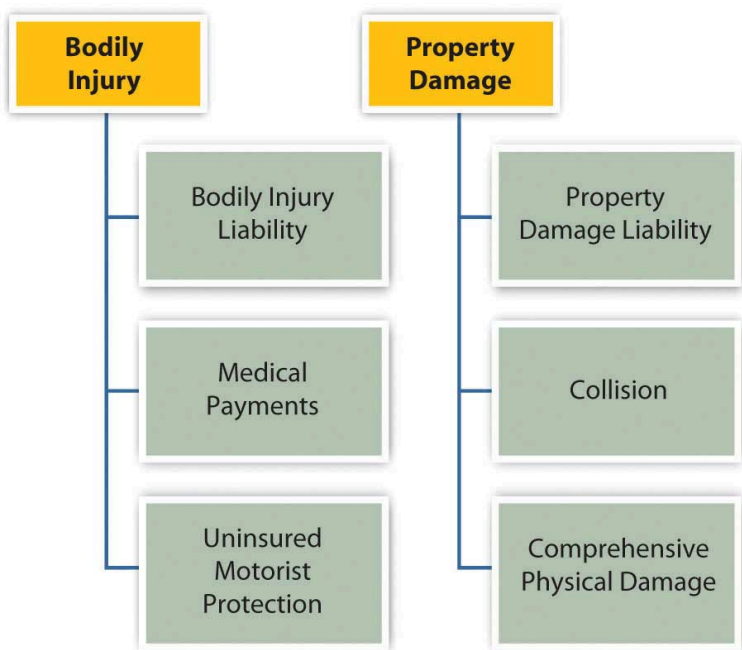


Figure 5.2.1: Types of Auto Insurance

**Bodily injury liability** refers to the financial losses of people in the other car that are injured in an accident you



cause, including their medical expenses, loss of income, and your legal fees. Injuries to people in your car or to yourself are covered by **medical payments coverage**. **Uninsured motorist protection** covers your injuries if the accident is caused by someone with insufficient insurance or by an unidentified driver.

**Property damage liability** covers the costs to other people's property from damage that you cause, while **collision** covers the costs of damage to your own property. Collision coverage is limited to the market value of the car at the time, usually defined by the National Automobile Dealers Association's (NADA) *Official Used Car Guide* or "blue book" (<http://www.nada.org>). To reduce their risk, the lenders financing your car loan will require that you carry adequate collision coverage. **Comprehensive physical damage** covers your losses from anything other than a collision, such as theft, weather damage, acts of nature, or hitting an animal.

Auto insurance coverage is limited, depending on the policy. The limits are typically stated in numbers representing thousands of dollars. For example, 100/300/50 means that \$100,000 is the limit on the payment to one person in an accident; \$300,000 is the limit on the amount paid in total (for all people) per accident; and \$50,000 is the limit on the amount of property damage liability that can be paid out.

Here's an example of how it all works. Kit is driving home one night from a late shift at the convenience store where he works. Sleepy, he drifts into the other lane of the two-lane road and hits an oncoming car driven by Ray. Both Kit and Ray are injured, and both cars are

damaged. Figure 5.2.2 shows how Kit’s insurance will cover the costs.

Type of Insurance	Costs Covered
Bodily Injury Liability	Ray’s medical bills and lost wages
Medical Payments Coverage	Kit’s medical bills
Property Damage Liability	Repairs to Ray’s car
Collision	Repairs to Kit’s car

Figure 5.2.2: An Auto Claim Example

## AUTO INSURANCE COSTS

As with any insurance, the cost of having an insurer assume risk is related to the cost of that risk. The cost of auto insurance is related to three factors that create risk: the car, the driver, and the driving environment—the region or rating territory.

The model, style, and age of the car determine how costly it may be to repair or replace, and therefore the potential cost of damage or collision. The higher that cost is, the higher the cost of insuring the car. For example, a 2020 luxury car will cost more to insure than a 2012 sedan. Also, different models have different safety features that may lower the potential cost of injury to

passengers, and those features may lower the cost of insurance. Different models may come with different security devices or be more or less attractive to thieves, affecting the risk of theft.

The driver is an obvious source of risk as the operator of the car. Insurers use various demographic factors such as age, education level, marital status, gender, and driving habits to determine which kinds of drivers present more risk. Not surprisingly, young drivers (ages sixteen to twenty-four) of both sexes and elderly drivers (over seventy) are the riskiest. Twice as many males as females die in auto accidents, but more females suffer injuries. Nationally, in any year your chances of being injured in a car accident are about one in a thousand.<sup>2</sup>

Your driving history and especially your accident claim history can affect your premiums, as well as your criminal record and credit score. In some states, an accident claim can double your cost of insurance over a number of years. Your driving habits—whether or not you use the car to commute to work, for example—can affect your costs as well. Some states offer credits or points that reduce your premium if you have a safe driving record, are a member of the American Automobile Association (AAA), or have passed a driver education course.

Where you live and drive also matters. Insurers use police statistics to determine rates of traffic accidents, auto theft, and vandalism, for example. If you are in an accident-prone area or higher crime region, you may be

2. <sup>3</sup>

3. The Disaster Center. (n.d.). [The Disaster Center's Motor Vehicle Accident Death and Injury data Index](#).

able to offset those costs by installing safety and security features to your car.

Premium rates vary, so you should always shop around. You can shop through a broker or directly. Online discount auto insurers have become increasingly popular in recent years. Their rates may be lower, but the same cautions apply as for other high-stakes transactions conducted online.

Also, premiums are not the only cost of auto insurance. You should also consider the insurer's reliability in addressing a claim. Chances are you rely on your car to get to school, to work, or for your daily errands or recreational activities. Your car is also a substantial investment, and you may still be paying off debt from financing your car. Losing your car to repairs and perhaps being injured yourself is no small inconvenience and can seriously disrupt your life. You want to be working with an insurer who will cooperate in trying to get you and your car back on the road as soon as possible. You can check your insurer's reputation by the record of complaints against it, filed with your state's agency of banking and insurance, or with your state's attorney general's office.



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## 5.3 INCOME INSURANCES

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<sup>1</sup>As you have learned, assets such as a home or car should be protected from the risk of a loss of value, because assets store wealth, so a loss of value is a loss of wealth.

Your health is also valuable, and the costs of repairing it in the case of accident or illness are significant enough that it also requires insurance coverage. In addition, however, you may have an accident or illness that leaves you permanently impaired or even dead. In either case, your ability to earn income will be restricted or gone. Thus, your income should be insured, especially if you have dependents who would bear the consequences of losing your income. Disability insurance and life insurance are ways of insuring your income against some limitations.

### DISABILITY INSURANCE

**Disability insurance** is designed to insure your income

1. Adapted from [10.3 Insuring Your Income](#) in [Personal Finance](#) by Lumen Learning. [CC BY-NC-SA 4.0](#)

should you survive an injury or illness impaired. The definition of “disability” is a variable feature of most policies. Some define it as being unable to pursue your regular work, while others define it more narrowly as being unable to pursue any work. Some plans pay partial benefits if you return to work part-time, and some do not. As always, you should understand the limits of your plan’s coverage.

The costs of disability insurance are determined by the features and/or conditions of the plan, including the following:

- Waiting period
- Amount of benefits
- Duration of benefits
- Cause of disability
- Payments for loss of vision, hearing, speech, or use of limbs
- Inflation-adjusted benefits
- Guaranteed renewal or noncancelable clause

In general, the greater the number of these features or conditions that apply, the higher your premium.

All plans have a waiting period from the time of disability to the collection of benefits. Most are between 30 and 90 days, but some are as long as 180 days. The longer the waiting period is, generally, the less the premium.

Plans also vary in the amount and duration of benefits. Benefits are usually offered as a percent of your current wages or salary. The more the benefits or the longer the

insurance pays out, the higher the premium. Some plans offer lifetime benefits, while others end benefits at age sixty-five (the age of Medicare eligibility).

In addition, some plans offer benefits in the following cases, all of which carry higher premiums:

- Disability due to accident or illness
- Loss of vision, hearing, speech, or the use of limbs, regardless of disability
- Benefits that automatically increase with the rate of inflation
- Guaranteed renewal, which insures against losing your coverage if your health deteriorates

You may already have some disability insurance through your employer, although in many cases the coverage is minimal. You may also be eligible for Social Security benefits from the federal government or workers' compensation benefit from your state if the disability is due to an on-the-job accident. Other providers of disability benefits include the following:

- The Veterans' Administration (if you are a veteran)
- Automobile insurance (if the disability is due to a car accident)
- Labor unions (if you are a member)
- Civil service provisions (if you are a government employee)

You should know the coverage available to you and if

you find it's not adequate, supplement it with private disability insurance.

## LIFE INSURANCE

**Life insurance** is a way of insuring that your income will continue after your death. If you have a spouse, children, parents, or siblings who are dependent on your income or care, your death would create new financial burdens for them. To avoid that, you can insure your dependents against your loss, at least financially.

There are many kinds of life insurance policies. Before purchasing one, you should determine what it is you want the insurance to accomplish for your survivors. What do you want it to do?

- Pay off the mortgage?
- Put your kids through college?
- Provide income so that your spouse can be home with the kids and not be forced out into the workplace?
- Provide alternative care for your elderly parents or dependent siblings?
- Cover the costs of your medical expenses and funeral?



- Avoid estate taxes?

These are uses of life insurance. Your goals for your life insurance will determine how much benefit you need and what kind of policy you need. Weighed against that are its costs—the amount of premium that you pay and how that fits into your current budget.

Sam and Maggie have two children, ages three and five. Maggie works as a credit analyst in a bank. Sam looks after the household and the children and Maggie's elderly mother, who lives a couple of blocks away. He does her grocery shopping, cleans her apartment, does her laundry, and runs any errands that she may need done. Sam and Maggie live in a condo they bought, financed with a mortgage. They have established college savings accounts for each child, and they try to save regularly.

Sam and Maggie need to insure both their lives, because the loss of either would cause the survivors financial hardship. With Maggie's death, her earnings would be gone, which is how they pay the mortgage and save for their children's education. Insurance on her life should be enough to pay off the mortgage and fund their children's college educations, while providing for the family's living expenses, unless Sam returns to the workforce. With Sam's death, Maggie would have to hire someone to keep house and care for their children, and also someone to keep her mother's house and provide care for her. Insurance on Sam's life should be enough to maintain everyone's quality of living.

## TERM INSURANCE

Maggie's income provides for three expenditures: the mortgage, education savings, and living expenses. While living expenses are an ongoing or permanent need, the mortgage payment and the education savings are not: eventually, the mortgage will be paid off and the children educated. To cover permanent needs, Maggie and Sam should consider permanent insurance, also known as **whole life**, straight life, or cash value insurance. To insure those two temporary goals of paying the mortgage and college tuitions, Maggie and Sam could consider temporary or term insurance.

**Term insurance** is insurance for a limited time period, usually one, five, ten, or twenty years. After that period, the coverage stops. It is used to cover financial needs for a limited time period—for example, to cover the balance due on a mortgage, or education costs. Premiums are lower for term insurance, because the coverage is limited. The premium is based on the amount of coverage and the length of the time period covered.

A term insurance policy may have a renewability option, so that you can renew the policy at the end of its term, or it may have a conversion option, so that you can convert it to a whole life policy and pay a higher premium. If it is multiyear level term or straight term, the premium will remain the same over the term of coverage.

Decreasing term insurance pays a decreasing benefit as the term progresses, which may make sense in covering the balance due on a mortgage, which also decreases with payments over time. On the other hand, you could simply buy a one-year term policy with a smaller benefit each

year and have more flexibility should you decide to make a change.

A return-of-premium (ROP) term policy will return the premiums you have paid if you outlive the term of the policy. On the other hand, the premiums on such policies are higher, and you may do better by simply buying the regular term policy and saving the difference between the premiums.

Term insurance is a more affordable way to insure against a specific risk for a specific time. It is pure insurance, in that it provides risk shifting for a period of time, but unlike whole life, it does not also provide a way to save or invest.

## WHOLE LIFE INSURANCE

Whole life insurance is permanent insurance. That is, you pay a specified premium until you die, at which time your specified benefit is paid to your beneficiary. The amount of the premium is determined by the amount of your benefit and your age and life expectancy when the policy is purchased.

Unlike term insurance, where your premiums simply pay for your coverage or risk shifting, a whole life insurance policy has a **cash surrender value** or cash value that is the value you would receive if you canceled

the policy before you die. You can “cash out” the policy and receive that cash value before you die. In that way, the whole life policy is also an investment vehicle; your premiums are a way of saving and investing, using the insurance company as your investment manager. Whole life premiums are more than term life premiums because you are paying not only to shift risk but also for investment management.

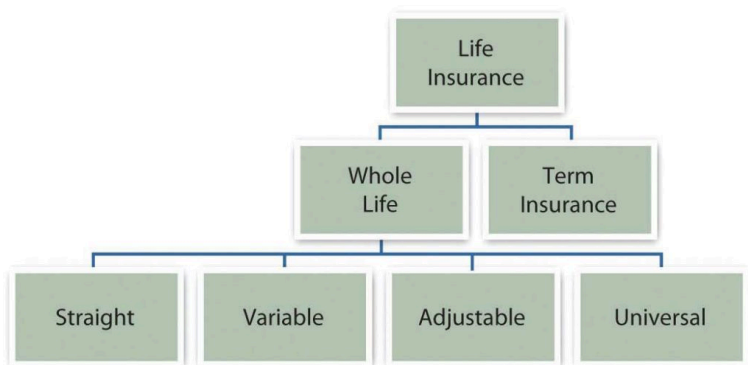
A **variable life** insurance policy has a minimum death benefit guaranteed, but the actual death benefit can be higher depending on the investment returns that the policy has earned. In that case, you are shifting some risk, but also assuming some risk of the investment performance.

An **adjustable life** policy is one where you can adjust the amount of your benefit, and your premium, as your needs change.

A **universal life** policy offers flexible premiums and benefits. The benefit can be increased or decreased without canceling the policy and getting a new one (and thus losing the cash value, as in a basic whole life policy). Premiums are added to the policy’s cash value, as are investment returns, while the insurer deducts the cost of insurance (COI) and any other policy fees.

When purchased, universal life policies may be offered with a single premium payment, a fixed (and regular) premium payment until you die, or a flexible premium where you can determine the amount of each premium, so long as the cash value in the account can cover the insurer’s COI.

Figure 5.3.1 shows the life insurance options.



*Figure 5.3.1: Types of Life Insurance*

So, is it term or whole life? When you purchase a term life policy, you purchase and pay for the insurance only. When you purchase a whole life policy, you purchase insurance plus investment management. You pay more for that additional service, so its value should be greater than its cost (in additional premiums). Whole life policies take some analysis to figure out the real investment returns and fees, and the insurer is valuable to you only if it is a better investment manager than you could have otherwise. There are many choices for investment management. Thus, the additional cost of a whole life policy must be weighed against your choices among investment vehicles. If it's better than your other choices, then you should buy the whole life. If not, then buy term life and save or invest the difference in the premiums.

## CHOOSING A POLICY

All life insurance policies have basic features, which then can be customized with a **rider**—a clause that adds benefits under certain conditions. The standard features include provisions that protect the insured and beneficiaries in cases of missed premium payments, fraud, or suicide. There are also loan provisions granted, so that you can borrow against the cash value of a whole life policy.

Riders are actually extra insurance that you can purchase to cover less common circumstances. Commonly offered riders include

- a waiver of premium payment if the insured becomes completely disabled,
- a double benefit for accidental death,
- guaranteed insurability allowing you to increase your benefit without proof of good health,
- cost of living protection that protects your benefit from inflation,
- accelerated benefits that allow you to spend your benefit before your death if you need to finance long-term care.

Finally, you need to consider the settlement options offered by the policy: the ways that the benefit is paid out to your beneficiaries. The three common options are

- as a lump sum, paid out all at once;
- in installments, paid out over a specified period;

- as interest payments, so that a series of interest payments is made to the beneficiaries until a specified time when the benefit itself is paid out.

You would choose the various options depending on your beneficiaries and their anticipated needs. Understanding these features, riders, and options can help you to identify the appropriate insurance product for your situation. As with any purchase, once you have identified the product, you need to identify the market and the financing.

Many insurers offer many insurance products, usually sold through brokers or agents. Agents are paid on commission, based on the amount of insurance they sell. A captive agent sells the insurance of only one company, while an independent agent sells policies from many insurers. You want a licensed agent that is responsive and will answer questions patiently and professionally. If you die, this may be the person on whom your survivors will have to depend to help them receive their benefits in a troubling time.

You will have to submit an application for a policy and may be required to have a physical exam or release medical records to verify your physical condition. Factors that influence your riskiness are your family medical history, age and weight, and lifestyle choices such as smoking, drinking, and drug use. Your risks will influence the amount of your premiums.

Having analyzed the product and the market, you need to be sure that the premium payments are sustainable for you, that you can add the expense in your operating budget without creating a budget deficit.

*AccuQuote Life Insurance: Types of Life Insurance Explained*



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## LIFE INSURANCE AS A FINANCIAL PLANNING DECISION

Unlike insuring property and health, life insurance can combine two financial planning functions: shifting risk and saving to build wealth. The decision to buy life insurance involves thinking about your choices for both and your opportunity cost in doing so.

Life insurance is about insuring your earnings even after your death. You can create earnings during your lifetime by selling labor or capital. Your death precludes your selling labor or earning income from salary or wages, but if you have assets that can also earn income, they may be able to generate some or even enough income to insure the continued comfort of your dependents, even without your salary or wages.



In other words, the larger your accumulated asset base, the greater its earnings, and the less dependent you are on your own labor for financial support. In that case, you will need less income protection and less life insurance. Besides life insurance, another way to protect your beneficiaries is to accumulate a large enough asset base with a large enough earning potential.

If you can afford the life insurance premiums, then the money that you will pay in premiums is currently part of your budget surplus and is being saved somehow. If it is currently contributing to your children's education savings or to your retirement plan, you will have to weigh the value of protecting current income against insuring your children's education or your future income in retirement. Or that surplus could be used toward generating that larger asset base.

These are tough decisions to weigh because life is risky. If you never have an accident or illness and simply go through life earning plenty and paying off your mortgage and saving for retirement and educating your children, then are all those insurance premiums just wasted? No. Since your financial strategy includes accumulating assets and earning income to satisfy your needs now or in the future, you need to protect those assets and income, at least by shifting the risk of losing them through a chance accident. At the same time, you must make risk-shifting decisions in the context of your other financial goals and decisions.



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## MODULE 5 PRACTICE PROBLEMS

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For problems 5.1-5.4, use the following insurance coverage. Assume a copay must be paid at each visit. For each calculate the total patient responsibility for the year and the total insurance company responsibility for the year.

- \$50 copay
- \$1,000 deductible
- 90/10 coinsurance after deductible

### Problem 5.1

- January 30: \$300
- March 18: \$100
- May 7: \$200
- October 10: \$250
- December 4: \$350

Answer: You pay \$1,200, they pay \$0



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### Problem 5.2

- April 4: \$50
- December 8: \$15,000

Answer: You pay \$2,495, they pay \$12,555



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### Problem 5.3

- January 8: \$500
- February 4: \$700
- April 30: \$600
- June 3: \$900
- August 3: \$1,500
- September 16: \$700
- November 30: \$1,200
- December 15: \$900

Answer: You pay \$1,960, they pay \$5,040.



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### Problem 5.4

- February 6: \$200
- June 12: \$500
- September 9: \$300
- November 1: \$200
- December 10: \$400

Answer: You pay \$1,285, they pay \$315.



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For problems 5.5-5.7, determine which of the following plans is the best (anticipated) option for each person.

### Gold Plan

- \$350/month
- \$100 deductible
- No coinsurance

### **Silver Plan**

- \$200/month
- \$1,000 deductible
- 90/10 coinsurance after deductible

### **Bronze Plan**

- \$50/month
- \$2,500 deductible
- 80/20 coinsurance after deductible

**Problem 5.5:** Antwon expects to have around \$500 in medical bills for the year.

Answers: 4,300; 2,900; 1,100 (pick bronze)



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<https://pressbooks.nsc.ca/mathofmoney/?p=117#oembed-5>

**Problem 5.6:** Bridget expects to have around \$3,500 in medical bills for the year.

Answers: 4,300; 3,650; 3,300 (pick bronze)



One or more interactive elements has been excluded from this version of the text. You can view them online here:

<https://pressbooks.nsc.ca/mathofmoney/?p=117#oembed-6>

**Problem 5.7:** Chad expects to have around \$10,000 in medical bills for the year.

Answers: 4,300; 4,300; 4,600 (pick gold)



One or more interactive elements has been excluded from this version of the text. You can view them online here:

<https://pressbooks.nsc.ca/mathofmoney/?p=117#oembed-7>





## GLOSSARY TERMS

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### **12b-1 fee**

An annual management fee charged to mutual fund shareholders and calculated as a percentage of the assets under management.

### **401(k) plans**

An employer-sponsored defined contribution plan. Contributions may be made by employer, employee, or both. The employee's contributions are tax deferred until distribution after age 59.5 and are limited by the Internal Revenue Code.

### **accounting equation**

Assets = liabilities + equity, or the value of assets must be equal to the value of the debt and equity that financed them. In personal finance, assets = debts + net worth, or net worth = assets – debts.

### **actual cash value**

Personal liability insurance is attached to a homeowner's policy.

### **adjustable life**

Benefits and premium can be adjusted without cancellation of the policy.

**angel investor**

An individual or group providing equity financing; usually a wealthy individual.

**asset allocation**

The strategy of achieving portfolio diversification by investing in different asset classes.

**asset class**

A kind of investment distinguished by its uses and market (e.g., stock, bonds, fine art, real estate, currency).

**assets**

Resources that can be used to create future economic benefit, such as increasing income, decreasing expenses, or storing wealth as an investment.

**authorized shares**

Shares of common or preferred stock that have been authorized for issuance by a corporation's board of directors.

**back-end load**

A deferred sales charge or sales fee charged when shares are redeemed.

**balance sheet**

A list of all assets, liabilities, and equity or net worth, at a given point in time, providing a concise picture of financial condition at that time.

**bankruptcy**

An economic situation when the value of debts is greater than the value of the assets that can be used

to satisfy them. Formal bankruptcy is also a legal process aiming to compensate creditors, governed by the laws of the nation or state in which it occurs.

**basic insurance**

Health insurance that covers the costs of physician expenses, surgical expenses, and hospital expenses.

**Bodily injury liability**

A system of auto insurance where the insured's insurance covers physical and property damage and liability, regardless of "fault" determined.

**Bonds**

Publicly issued and traded long-term debt used by corporations and governments.

**budget deficit**

A shortfall of available funds created when income is less than the expenses.

**budget surplus**

An excess of available funds created when income is greater than the expenses.

**business cycle**

Recurring periods of economy-wide expansion, when the economy is growing, and contraction, when the economy is shrinking. Cycles are often measured by the increase or decrease in the GDP.

**buyout option**

A feature of a lease that offers the option to buy the asset financed by the lease at the end of the lease term.

**callable**

A bond that may be redeemed before maturity.

**Capital allocation**

A strategy of diversifying a portfolio between risky and riskless assets.

**capital gain**

Wealth created when an asset is sold for more than the original investment.

**capital loss**

Wealth lost when an asset is sold for less than the original investment.

**capital market**

A market where long-term liquidity is traded.

**cash surrender value**

The value of a whole life policy—the cash available for the policyholder—if the policy is canceled before the death of the insured.

**Closed-end funds**

A mutual fund that issues a limited number of shares, so that existing shares must be sold to new investors.

**Co-pays**

Partial payment for certain costs, made by the insured.

**Coinsurance**

Shared payments by insured and insurer.

**collision**

Responsibility for damage to property owned by people other than the driver at fault.

**Commodities**

Raw materials—natural resources or agricultural products—used as inputs in processing goods and services.

**Comprehensive physical damage**

Responsibility for damage to the property of the driver at fault.

**consumption tax**

A sales or excise tax that taxes the consumption of discretionary and nondiscretionary goods and services.

**cost of debt**

The cost of borrowing capital because of having to pay interest on the principal.

**cost of equity**

The cost of having to share the benefits—capital gains or income (dividends)—from the investment.

**coupon**

The interest payment on a bond, specified as a feature of the bond at issuance.

**coupon rate**

The interest rate offered on a bond.

**covenants**

A condition placed on bond issuers (borrowers) to protect bondholders (lenders).

**credit market**

A part of the capital market where capital is lent and borrowed through the trading of debt securities such as bonds.

**creditors**

Lenders; anyone to whom debt is owed.

**debt**

Borrowed capital, a liability, a loan that must be repaid.

**Deductibles**

Costs paid by the insured before the insurer provides coverage.

**defined benefit plan**

A pension plan sponsored by an employer in which the employer commits to providing a specific amount of benefit based on wages and tenure to retired employees.

**defined contribution retirement plans**

A pension plan sponsored by an employer in which the employer commits to providing a specific amount of contribution to a retirement account owned by an active employee.

**deflation**

Period characterized by falling prices, increasing purchasing power, and higher currency values (one

unit of currency is worth more because it buys a greater quantity of goods and services).

**depression**

A prolonged and severe recession.

**derivatives**

Financial instruments such as options, futures, forwards, securitized assets, and so on whose value is derived from the value of another asset.

**Disability insurance**

Insurance to protect the insured against the risk of being unable to earn wages or salary as a result of injury or illness.

**diversification**

The strategy of reducing risk by spreading income and investments among a number of different kinds, sources, and locations.

**dividend**

A share of corporate profit distributed to shareholders, usually as cash or corporate stock.

**endorsements**

Valuable property insured separately under a homeowner's policy.

**equity**

An ownership share in an asset, entitling the holder to a share of the future gain (or loss) in asset value and of any future income (or loss) created.

**Estate taxes**

A tax on the intergenerational transfer of wealth after death.

**exchange-traded fund (ETF)**

A fund that tracks an index or a commodity or a basket of assets but is traded like stocks on a stock exchange.

**Exchange-traded funds (ETFs)**

A mutual fund that is structured as a closed-end fund and actively traded on an exchange.

**Excise taxes**

A tax on a specific item produced within a country.

**executor**

The person named in a will who administers the payments of debts and the distribution of assets, as described in the will.

**expected return**

The return expected for an investment based on its average historical performance. Statistically, it is the mean or average of the investment's past performance.

**Expenses**

The costs of consumption or daily living.

**extended replacement costs**

The full cost of replacing insured items at time of loss.



**face value**

For a bond, the amount to be repaid to the bondholder upon redemption.

**financial engineering**

The use of mathematical modeling to create and value new financial instruments and markets.

**fixed interest rate**

A bond interest rate that does not change over time, from issuance to maturity.

**flexible savings account**

An account created with regular payroll deductions by an employee to finance supplemental health care costs. Monies must be expended within a specified time period or forfeited (“use it or lose it”).

**floating interest rate**

A bond interest rate that changes over time, usually related to a benchmark rate such as the U.S. discount rate or prime rate.

**formulary**

A list of drugs covered by an insurer under a prescription drug plan.

**forward contracts**

A private contract to buy or sell an asset at a specified time and price in the future.

**front-end load**

The sales charge for mutual fund shares, quoted as a percentage of the funds invested; it cannot be more than 8.5 percent of investment.

**future value**

The value of a present liquidity or projected series of cash flows in the future, accounting for the effects of time on value.

**Futures**

A publicly traded contract to buy or sell an asset at a specified time and price in the future.

**general obligation bond**

A state or municipal bond secured only by the “full faith and credit” of the issuer.

**go public**

To raise capital by issuing equity shares through a public exchange.

**gross domestic product**

The total value of all final goods and services produced in a year in a nation’s economy. It is used as a fundamental measure of an economy’s growth based on its ability to use resources productively and provide for its members.

**growth stock**

A stock that is expected to offer excessive rates of growth.

**Guaranteed replacement costs**

Cost of replacing insured property at time of loss.

**health maintenance organization**

An organization to provide “managed care” through reliance on primary care physicians and a network of specialists, with an emphasis on preventative care.

**health reimbursement account**

An employer owned and funded account to finance employee health care costs, with the employee choosing the type of coverage.

**health savings account**

Individually owned and financed savings accounts that may be used to finance health care costs with tax-deductible contributions.

**high-yield bonds**

Bonds rated BB or Ba or lower, considered to have significant default risk.

**holographic will**

A handwritten or oral will.

**Income**

Earnings of a given period. In the case of an individual or household, this is generally cash from wages, interest, dividends, or assets (such as rental income from real estate) that can be used for consumption or saved.

**index fund**

A mutual fund designed to track the performance of an index for investors who seek diversification without having to select securities.

**index funds**

A mutual fund designed to track the performance of an index for investors who seek diversification without having to select securities.

**inflation**

Period characterized by rising prices, declining purchasing power, and lower currency values (one unit of currency is worth less because it buys a smaller quantity of goods and services).

**initial public offering (IPO)**

A company's first issuance of stock for trade in the public markets. Companies issue stock publicly to attract more investors and thus more capital for the company. When a company has its IPO is it said to "go public."

**interest**

The cost of debt expressed as an annual percentage of the principal.

**intestate**

To die without a valid will, leaving the disposition of assets and debts to the law.

**investment grade bonds**

Bonds rated BBB or Baa or higher and considered to carry insignificant default risk.

**issue price**

The original market price of a bond at issuance.

**junk bonds**

High yield bonds rated BB or Ba or lower and considered to have significant default risk.

**labor market**

Where labor is traded through hiring or employment

and price is determined by the interaction of employers and employees.

**lease**

A rental agreement used as a form of financing for automobile purchases.

**Lemon laws**

Federal and state laws protecting consumers against products that repeatedly fail to meet standards of performance. The federal Magnuson-Moss Warranty Act was enacted in 1975.

**life cycle investing**

An investment strategy in which asset allocation is based on the investor's age or stage of life.

**Life insurance**

Insurance to compensate beneficiaries against the financial consequences of the death of the insured.

**life stages**

Periods of a person's life based on age and personal circumstances that reflect different needs, goals, and financial capabilities.

**liquidity**

Nearness to cash, or how easily and cheaply—with low transaction costs—an asset can be turned into cash.

**listed property**

Responsibility for another's use of your possessions, or for another's actions, under certain circumstances.

**living trust**

A trust created while the grantor is alive.

**living will**

A document conveying your intentions for your personal care and management of your assets should you become unable to do so before your death.

**load fund**

A mutual fund that charges a sales commission or fee upon investment or purchase of shares; the load is stated as a percentage of invested funds.

**Long-term care insurance**

Insurance to provide for permanent assistance with activities of daily living in the event of disabling injury or illness.

**major medical insurance**

Insurance for the costs of serious injury or illness.

**Managed care organizations**

Organizations or networks of health care providers based on the principle of providing preventative care in order to better health and lower costs of health care. Such organizations also provide for emergency and special treatment services under various systems.

**market capitalization**

The total market value of a corporation's capital.

**maturity**

The date on which payment of a financial obligation is due, such as bond redemption date.

**maturity date**

Date at which a bond matures, or the end of the bond's term, when the bond must be redeemed.

**Medicaid**

A federal program financing health care costs with eligibility based on income.

**medical payments coverage**

Responsibility for financial losses from injuries sustained in an accident for people outside of the car of the driver at fault.

**Medicare**

A federal program financing health care costs with eligibility based on age (for those over age sixty-five).

**MSRP**

The “sticker price” for an item

**municipal bonds**

Bonds issued by a city, town or state to finance public projects. The coupon payments may, under certain circumstances, not be subject to federal income tax for the bondholder.

**mutual fund**

A portfolio of investments created by an investment company such as a brokerage or bank. It is financed as the investment company sells shares of the fund to investors. For investors, a mutual fund provides a way to achieve maximum diversification with minimal transaction costs through economies of scale.

**Negative net worth**

The mathematical result of liabilities being greater than the value of assets, or debts being larger than the value that can be used to meet them.

**negligence**

Failure to take ordinary precautions or the failure to take usual precautions.

**net asset value (NAV)**

When used regarding open-end mutual funds, NAV refers to the redeemable value of each fund share at that time, given the market value of the fund's assets and the number of shares outstanding.

**net worth**

The value of assets owned after creditors' claims (debts) are accounted for, or literally, assets – debts.

**no-fault insurance**

Insured amount capped at a specified percentage of actual cash value.

**no-load fund**

A mutual fund that does not charge a sales commission or fee upon investment or purchase of shares.

**open-end funds**

A mutual fund in which shares are bought from and sold to the fund management; the number of shares is not limited.

**opportunity cost**

The cost of sacrificing the next best choice because



of the choice made; the value of the next best choice, which is forgone once a choice is made.

### **Options**

The right but not the obligation to buy or sell at a specific price at a specific time in the future; commonly written on shares of stock as well as on stock indices, interest rates, and commodities.

### **Pension Benefit Guaranty Corporation (PBGC)**

An agency of the federal government that guarantees defined benefit pensions in the case of employer default.

### **pension plan**

An employer-sponsored, defined benefit plan providing a regular, specified amount of pension, based on wages and years of service.

### **point-of-service (POS)**

A type of managed care in which physicians, hospitals, and other care providers contract with an insurer to provide care at reduced rates upon referral from the insured's primary care physician. Unlike the HMO, out-of-network providers may be used, but on a limited basis.

### **power of attorney**

The legal right to act on your behalf should you become unable to do so before your death.

### **preferred provider organization (PPO)**

A type of managed care in which physicians, hospitals, and other care providers contract with an insurer to provide care at reduced rates upon referral

from the insured's primary care physician. Unlike the HMO, out-of-network providers may be used.

**present value**

The costs of achieving a trade or "doing a deal" that do not contribute to the value of the thing being traded; a cost created by making an economic transaction.

**primary market**

The market in which the initial issuance or initial public offering of a stock occurs.

**prime rate**

A benchmark interest rate understood to be the rate that major banks charge corporate borrowers with the least default risk.

**principal**

The original amount of borrowed capital (a loan).

**private equity**

Equity not traded in a public market or exchange.

**Probate**

The legal process of validating a will and overseeing the orderly payment of debts and the distribution of assets.

**Property damage liability**

Coverage of financial losses from injuries sustained in an accident if the driver at fault has insufficient insurance.

**prospectus**

A written statement of a mutual fund's structure, management, investment objectives, holdings, and historic and current performance; funds are required to make the prospectus available to all potential investors.

**purchasing power**

A currency's usefulness and thus its value as measured by how much it can buy, that is, the quantity of goods and services that can be purchased with one unit of currency.

**rating agencies**

Analysts of bond default risk that assign ratings to bonds.

**recession**

A period of economic contraction lasting at least six consecutive months or two consecutive quarters.

**redeemable**

A bond that is eligible for redemption.

**regressive tax**

A tax rate that decreases as the amount to be taxed increases.

**replacement cost**

Market value of insured property at time of loss.

**revenue bond**

A state or municipal bond that will be repaid from revenues of the specific project it is financing.

**rider**

A clause to a policy that adds specific benefits under specific conditions.

**risk**

In finance, the probability that the value of an asset, income, or investment may decline in the future.

**rollover**

A retirement plan that may accept or distribute funds from another qualified retirement account without tax consequence or penalty.

**Roth IRA**

An individual retirement account for which contributions are not deductible but withdrawals are not taxed.

**secondary market**

A market in which outstanding shares are traded.

**Security selection**

The process of choosing individual securities to be included in the portfolio.

**Social Security**

The mandatory retirement program sponsored by the U.S. government to provide supplemental retirement income. It is funded by a tax (FICA) paid by employers and employees and by self-employed individuals who act as both employer and employee.

**speculative grade bonds**

High yield bonds rated BB or Ba or lower and considered to have significant default risk.

**standard deviation**

In finance, the statistical measure that calculates the frequency and amount by which actual returns differ from the average or expected returns.

**statutory will**

A will written on a preprinted form.

**stock exchange**

An organized market for the trading of corporate shares conducted by members of the exchange.

**Stocks**

Shares issued to account for ownership, as defined by owners' contributions to a corporation.

**sunk costs**

Costs that have been incurred in past transactions and cannot be recovered.

**Term insurance**

Life insurance providing coverage for a specified period of time.

**testamentary trust**

A trust created by a will that becomes effective upon the death of the grantor.

**time value of money**

The impact of the passing of time on the value of money, based on the premise that being separated from liquidity creates opportunity cost.

**Traditional IRA**

An individual retirement account for which

contributions are tax deductible and withdrawals are taxed.

**transfer**

The movement of funds in a tax-advantaged retirement account from one trustee or asset manager to another that is not considered a withdrawal or distribution of funds.

**Treasury bills**

Bonds issued by the U.S. government with a maturity of less than one year.

**Treasury bonds**

Bonds issued by the U.S. government with a maturity of more than ten years.

**Treasury notes**

Bonds issued by the U.S. government with a maturity of between one and ten years.

**trust**

A legal entity created to own and manage assets for the benefit of beneficiaries.

**umbrella policy**

The clause of a homeowner's policy insuring listed property.

**unemployment rate**

A measure of the percentage of people in the labor force who are unemployed, that is, those who would like to be working but cannot find a suitable job.

**Uninsured motorist protection**

Responsibility for financial losses from injuries sustained in an accident for people inside of the car of the driver at fault.

**universal life**

Benefits and premiums are flexible, in terms of both timing and amounts.

**value stock**

A stock whose return is based on its current underpricing by the market.

**value-added tax**

A consumption tax that spreads the tax burden among producers and consumers by taxing the value added to goods at each stage of production and consumption.

**variable life**

Life insurance that provides a guaranteed minimum benefit with potential to be greater depending on investment performance.

**venture capital**

Private equity provided to facilitate excessive growth before the initial public offering of shares.

**vesting**

The process of earning full ownership in an employer-sponsored retirement plan according to length of service.

**vicarious liability**

Strict liability, responsibility for intentional or unintentional events.

**warranty**

A manufacturer's guarantee of product performance for a period of time.

**whole life**

Life insurance providing coverage until the insured's death; it can also be used as an investment instrument.

**will**

A legal document detailing the disposition of assets upon death.

**zero-coupon bond**

A bond that has a coupon rate of zero, and therefore a coupon of zero. Its only cash flow return is the principal repayment at maturity.



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## MATH FOR MONEY

- New practice problem videos created
- H5P exercises added / used to replace Lumen exercises
- Original text added

<b>Chapter Mapping</b>	CC BY-NC-SA	CC BY-NC-SA
Math for Money(3rd)	Personal Finance/ Lumen Edition(2nd)	Personal Finance / Saylor Edition (1st)
1.1	1.1 &1.2	1.1 and 1.2
1.2	2.1,2.2 & 2.4	1.3-1.6
1.3	3.1	1.7
1.4	12.1	1.8
1.5	1.9 and 1.10	1.9 and 1.10
	1.11 removed – concept of the natural exponential e and continuous compounding growth	1.11
1.6	1.12	1.12
1.7 Practice – NEW		
2.1	Wikipedia	
2.2	Wikipedia	
2.3	8.2	
2.4	9.1 & 9.2	
2.5 NEW		
2.6 Practice – NEW		
3.1	12.1	
3.2	15.1	
3.3		
3.4		
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3.8 Practice – NEW		
4.1	6.1	

4.2	6.2	
4.3	Wikipedia	
4.4		
4.5	Wikipedia	
4.6 Practice – NEW		
5.1	10.2	10.2
5.2	10.1	10.1
5.3	10.3	10.3
5.4 Practice – NEW		

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