

# Math for Trades



# MATH FOR TRADES

Volume 1

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Format	Internet required?	Device	Required apps	Accessibility Features	Screen reader compatible
Online webbook	Yes	Computer, tablet, phone	An Internet browser (Chrome, Firefox, Edge, or Safari)	WCAG 2.0 AA compliant, option to enlarge text, and compatible with browser text-to-speech tools	Yes
PDF	No	Computer, print copy	Adobe Reader (for reading on a computer) or a printer	Ability to highlight and annotate the text. If reading on the computer, you can zoom in.	Unsure
EPUB and MOBI	No	Computer, tablet, phone	Kindle app (MOBI) or eReader app (EPUB)	Option to enlarge text, change font style, size, and colour.	Unsure
HTML	No	Computer, tablet, phone	An Internet browser (Chrome, Firefox, Edge, or Safari)	WCAG 2.0 AA compliant and compatible with browser text-to-speech tools.	Yes

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However, the interactive activities and video answers are also provided in alternate formats for people not using the webbook. In the case of the videos answers, the answers can also be found in [Appendix B: Answers to Practice Questions](#). As for the chapter quizzes, those questions have been made available in a static format in [Appendix C: Offline Copies of Chapter Quizzes](#).

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The attached style guide was used in the editing of this book: [Style Sheet for Math for Trades \[Word file\]](#)



# PART I

# INTRODUCTION

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## Math for Trades Introduction: Video transcript

Welcome, everyone, to Math for Trades!

Isn't math kind of like learning a new language? It has its own set of words, sentences, its own structure, and its own rules.

Well, in this *Math for Trades: Volume 1*, we're going to start building all those. We're gonna build the words. We're going to take the words and build sentences. We're going to find the rules, we're going to create structure, and we're going to do that in steps.

So, Chapter 1 you can kind of look at as the first step. It's "Whole Numbers." And then what we do is once we've gone through that, we take a step up to Chapter 2, to "Fractions." Chapter 3, another step, "Decimals." And then finally, Chapter 4, which is "Percent."

Now, when you're going through this book, there's a few things you want to think about. One is, take your time but be efficient. Remember this is about you. What do you want out of this? If you find that you know whole numbers, and you don't need to go through it? Perfect. Don't go through it. But you might need a little extra help with fractions. Check out the fractions part, then. This is about you, and this is for you, so do whatever it is that works for you.

Follow the steps. Examples are laid out in steps. If you follow those, that's the key. That'll help you to get from step one to step four or step five and understand the process in between.

Think about the concepts and relationships. Everything in math is interrelated, so the more you understand concepts and the more you understand relationships, the easier it gets as you go along.

Watch the videos after you've tried the questions. This is the cool part of this book. You'll see these everywhere in the book [*shows a Math For Trades video icon*] and what they are is video answers. So when we have practice questions for you, get your pen or pencil out, get your paper

out, and give it a try. Think about what you're doing or think about the process you're going through in your mind as you try it. And then check out the answers. See how you did.

Also, study in an environment with no distractions. No phone, no TV, nothing like that. That would really help because you'll be all in and that goes along with the next one.

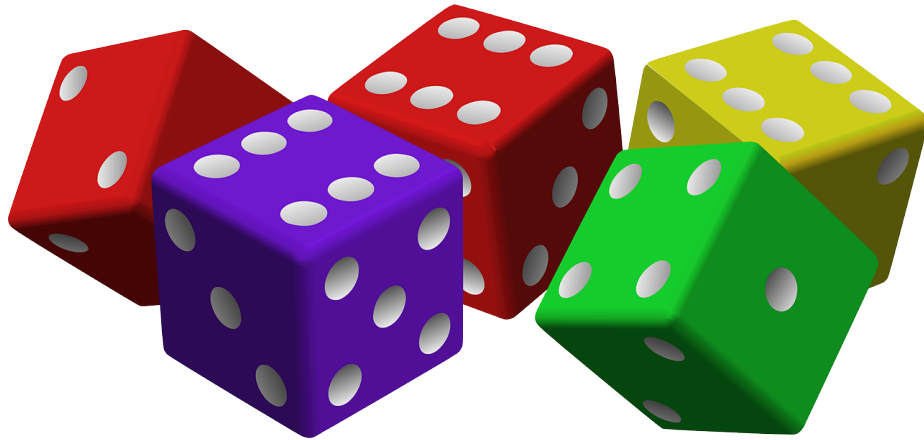
Get all in for 20 to 30 minutes, 110%, and then take a break. This will probably help you retain more information and be more efficient with your study time. Okay? So 20 to 30 minutes.

And then there is one more thing, and this is the most important thing because if you don't do this, it gets tougher and tougher. Have fun with this. Enjoy the process.



## PART II

# WHOLE NUMBERS



### Outcomes

- Identify counting numbers and whole numbers.
- Locate digits in the place value system.
- Add whole numbers.
- Subtract whole numbers.
- Multiply whole numbers.
- Divide whole numbers.



1.

# THE PLACE VALUE SYSTEM

---

Have you ever heard the term “whole number”? It might seem like a weird concept: aren’t all numbers whole? I ate six apples this week, or I charged my electric vehicle for three hours yesterday. Those are whole numbers, right?



Well, not all numbers are whole. You could have eaten  $3\frac{1}{2}$  apples during a particularly slow apple-eating week, and maybe you didn’t drive your electric vehicle all that much yesterday, so you only had to charge it for 2.3 hours. Those are not whole numbers.

The purpose of this chapter is to define what a whole number is and then learn to work with those whole numbers, be it by adding, subtracting, multiplying, or dividing.

All the chapters in this textbook, including this one, will have examples and exercises geared towards the trades. It contains content relevant to all trades, no matter what trade you are in.

## Counting Numbers and Whole Numbers



A number of years ago, a student named Harpreet went through a plumbing apprenticeship program. After completing his schooling and receiving his Red Seal plumbing ticket, he has decided to open up his own plumbing company.

Like most new entrepreneurs, he experienced a steep learning curve, especially when it came to ordering material for jobs and organizing and scheduling when he had a number of jobs on the go.

Although he might not be thinking about counting numbers or whole numbers, he has ended up working with these when ordering materials and during the day-to-day operations of the company.



If we want to start with the very basics, we must start with **counting numbers**, which are sometimes referred to as **natural numbers**. These are the most basic units in algebra and the ones we use to count objects. For instance, Harpreet had to order one toilet, two sets of taps, three fittings, four braided hoses, and five chocolate bars (it seems like Harpreet has a bit of a sweet tooth).

One more thing: in counting numbers, there are no decimal or fraction parts. Also, there are no negatives and no zero.

Okay, so we have no zero. But what if we just decided to add a zero to the set of counting numbers? Would that be so bad? No, it wouldn't, but then we couldn't call them counting numbers anymore. The set of counting numbers plus zero becomes what is known as the set of **whole numbers**.

**Counting numbers:** 1, 2, 3, 4, 5, ...

**Whole numbers:** 0, 1, 2, 3, 4, 5, ...

The discovery of the number zero was a big step in the history of mathematics. Don't ask me why, but apparently it was. As for Harpreet, let's hope that he bids jobs properly and that his profit doesn't amount to zero.

## The Place Value System

Worried that he might estimate jobs improperly, Harpreet decides he needs some help with estimating and bidding jobs, as well as some help with the books. He calls up his friend Jamieson and asks if he wants to help. Jamieson, who has been in the plumbing industry for a number of years, has had a recurring back injury and figures this might be a good opportunity to rest his back for a while and help his friend out at the same time.

The first job Jamieson bids on is for \$931. Take a look at the three numbers in 931 and notice where each one is located. This is very important, because if Jamieson reversed the numbers, he would have bid \$139 and most likely lost money on the job.

Writing the correct number relies on what our number system calls the **place value system**. The 9, the 3, and the 1 are all located in different spots in the larger number, and each of them is called a **digit**. Putting the digits in the wrong locations could result in disaster when dealing with money and in many other situations.



Here's a visual example: say I were to take a wallet, open it up, and take out the money. Suppose that wallet had two \$100 bills, three \$20 bills and one \$5 bill. What would the total amount of money be in the wallet?

I would have:

$$\begin{aligned} 2 \times \$100 &= \$200 \\ 3 \times \$20 &= \$60 \\ 1 \times \$5 &= \$5 \end{aligned}$$

Adding those up, I would have:

$$\$200 + \$60 + \$5 = \$265$$

Notice that each of the original values has a specific place in the total. Positioning the digits in the reverse order would make it appear as if we had a lot more money than we actually have:

**\$562**

This is the importance of the place value system.

Another aspect of this system is known as the **base-10 concept**. Each position in a number is a specific place. If we use the number 265 as an example, we would see that:

5 = ones place

6 = tens place

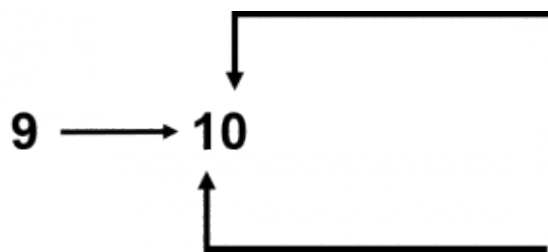
2 = hundreds place

Each of the digits in the place value system can have a value anywhere between zero and nine. When the value of a digit increases past nine, we start again at zero but add one to the value of the digit in the next highest place value.

The simplest way to think about this is to go from 9 to 10:

**9 → 10**

In this case, the ones place goes back to zero and the tens place increases by one.



The ones place has gone from nine to zero.

There was no value in the tens place when we started, but now, the tens place has increased by one.

Now, you might be thinking that this really is far too basic to be reading, but these are the building blocks of the language of algebra. Although the example used here is quite simple, when you get to larger and more complex numbers, the mathematical grammar that you

are learning here is the basis for understanding more complicated mathematical grammar in the chapters to follow.

Now let's look at a bigger number. One way to understand the place value system is to take a number such as:

5, 217, 364, 958

If we were to sound out this number, we would say:

**Five billion, two hundred and seventeen million, three hundred and sixty-four thousand, nine hundred and fifty-eight.**

It's read that way because each digit corresponds to a certain value in the place value system. Take a look at the following place value chart, in which we start with the ones and work our way up:

Ones	8
Tens	5
Hundreds	9
Thousands	4
Ten thousands	6
Hundred thousands	3
Millions	7
Ten millions	1
Hundred millions	2
Billions	5
Ten billions	–
Hundred billions	–

This pattern would continue on into the trillions, quadrillions, quintillions, and so on.

Another common way to express numbers in the place value system is demonstrated in the table below.

Place Value														
Trillions			Billions			Millions			Thousands			Ones		
Hundred Trillions	Ten Trillions	Trillions	Hundred Billions	Ten Billions	Billions	Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
					5	2	1	7	3	6	4	9	5	8

Place value table with values added.

Going from left to right, we get:

- The digit 5 is in the billions place. Its value is 5,000,000,000.
- The digit 2 is in the hundred millions place. Its value is 200,000,000.
- The digit 1 is in the ten millions place. Its value is 10,000,000.
- The digit 7 is in the millions place. Its value is 7,000,000.
- The digit 3 is in the hundred thousands place. Its value is 300,000.
- The digit 6 is in the ten thousands place. Its value is 60,000.
- The digit 4 is in the thousands place. Its value is 4,000.
- The digit 9 is in the hundreds place. Its value is 900.
- The digit 5 is in the tens place. Its value is 50.
- The digit 8 is in the ones place. Its value is 8.

Try an example.

### Example

As you go through this question, think of the actual value of each of the digits. For example, the 4 in the leftmost place of the number below represents 40,000,000. Doing this will help you to visualize the value each number actually stands for.

Find the place value of each of the following digits:

**45, 837, 249**

Use the following layout as a guideline to help you answer the question.

4 = \_\_\_\_\_

5 = \_\_\_\_\_

8 = \_\_\_\_\_

3 = \_\_\_\_\_

7 = \_\_\_\_\_

2 = \_\_\_\_\_

4 = \_\_\_\_\_

9 = \_\_\_\_\_

Inserting the numbers into the table below is also a good place to start:



Place Value														
Trillions			Billions			Millions			Thousands			Ones		
Hundred Trillions	Ten Trillions	Trillions	Hundred Billions	Ten Billions	Billions	Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
							4	5	8	3	7	2	4	9

**Solution:**

Using the table above, you get:

4 = ten millions

5 = millions

8 = hundred thousands

3 = ten thousands

7 = thousands

2 = hundreds

4 = tens

9 = ones

## Practice Questions

Try a couple more examples and check out the video answers.

Find the place value of each digit of the following numbers.

### Question 1

**385, 922, 102**

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### Question 2

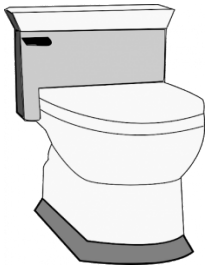
**20, 433, 876, 011**

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2.

## ADDING WHOLE NUMBERS

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Harpreet and Jamieson are currently working a job that requires seven toilets to be installed in one area of the building and six toilets in another. In total, they need to install 13 toilets.

Now, that might not be too hard to do in our heads, but what if we need to add larger numbers? For instance, a job that they are bidding on has 53 toilets and 34 kitchen sinks. That might be harder to add in your head.

An easy way to add is to use a calculator, and if we did, we would get an answer of 87. But learning to add these two whole numbers using math principles can help us not only add the numbers, but also visualize how these numbers go together.



Let's go back to the place value system to help us out:

First, look at 53, which has 5 tens and 3 ones.

Then, look at 34, which has 3 tens and 4 ones.

Add the tens up: 5 tens + 3 tens = 8 tens.

Add the ones up: 3 ones + 4 ones = 7 ones.

When we put the original numbers in the place value system and add, we get a total of 87. Separating this number into each of its digits and positioning them in the place value system can help us visualize the value of each of the digits.

Follow the steps outlined below to see how this process looks mathematically.

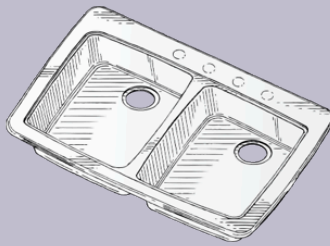
Example

We'll use the same numbers from the previous example:



53

+



34

=



**Step 1:** Put the question into a formula that is easy to work with.

$$\begin{array}{r} 53 \\ + 34 \\ \hline \end{array}$$

**Step 2:** Add up the ones. In this case, we have 3 and 4. Together, they add up to 7.

$$\begin{array}{r} 53 \\ + 34 \\ \hline 7 \end{array}$$

**Step 3:** Add up the tens. In this case, we have 5 and 3. Together, they add up to 8.

$$\begin{array}{r} 53 \\ + 34 \\ \hline 87 \end{array}$$

That example worked out well, as neither the ones nor the tens added up to more than 9. But what if they did? Take a look at the next example to see how this works.



The word “sum” is math language for adding numbers together. When we say, “What is the sum of 27 and 45?” what we are saying is, “What is the total of those numbers if we added them together?”

### Example

Find the sum of 27 and 45.

To get the sum of these numbers, we will go back to our three steps.

**Step 1:** Put the question into a formula that is easy to work with.

$$\begin{array}{r} 27 \\ + 45 \\ \hline \end{array}$$

**Step 2:** Add up the ones. In this case, we have 7 and 5. Together, they add up to 12. This is where we do what is called “carrying the one.” Take a look at the following image to see this in action:

carrying the one

$$\begin{array}{r} 1 \\ 27 \\ + 45 \\ \hline 2 \end{array}$$

**Note:** In this case, we do not put the number 12 at the bottom of the equation. Instead, we “carry the one” into the next spot (the tens) in the place value system.

**Step 3:** Add up the tens. In this case, we have 2 and 4. Together, they add up to 6. We also have to take into account the one that we carried over. We add that in as well to get a total of 7 in the tens place.

$$\begin{array}{r} 1 \\ 27 \\ + 45 \\ \hline 72 \end{array}$$

**Note:** If it turned out that the tens added up to more than 9, we could carry the one again into the hundreds column. Take a look below to see how this works.

$$\begin{array}{r} 72 \\ + 85 \\ \hline 157 \end{array}$$

As there are no other values in the hundreds column in this question, we just go ahead and place the one in the hundreds column.

## Practice Questions

Try a couple practice questions yourself and check the video answers to see how you did. Make sure to follow the steps outlined above and think about the place value of each of the digits.

### Question 1

$$\begin{array}{r} 97 \\ + 74 \\ \hline \end{array}$$

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### Question 2

$$\begin{array}{r} 128 \\ + 456 \\ \hline \end{array}$$

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3.

## SUBTRACTING WHOLE NUMBERS

---



As his company grows, Harpreet asks Jamieson to bid on more jobs. Harpreet has had to hire a few plumbers to help with the work, as things are going well for the business and profits are climbing. It's going so well that they have had to turn down some of the jobs they are bidding on (and getting).

In fact, last month, they bid on 17 jobs and got all of them. They didn't figure they would get all 17 and actually had to turn down 5 of the jobs, leaving them with 12 jobs. This is an example of subtracting whole numbers.



The key to subtracting whole numbers is to find the difference between the two numbers. If we started with 17 jobs, and Harpreet turned down 5 of those jobs, then the number of jobs he would have taken is 12. We could also look at it as 5 being the difference between the number of jobs he got and the number of jobs he took. We could write this formula down as something like the following:

$$17 - 5 = 12$$

OR

$$17 - 2 = 5$$

We'll use the first way it is written down as our example, and then what we'll do is change that into another format that will be easier to work with.

$$\begin{array}{r} 17 \\ - 5 \\ \hline ? \end{array}$$

What you might note is that the way the question is written is similar to how we worked the equation when we were adding whole numbers. Writing it this way gives us a better representation of the ones and the tens columns, which we'll need to use when working through the question.



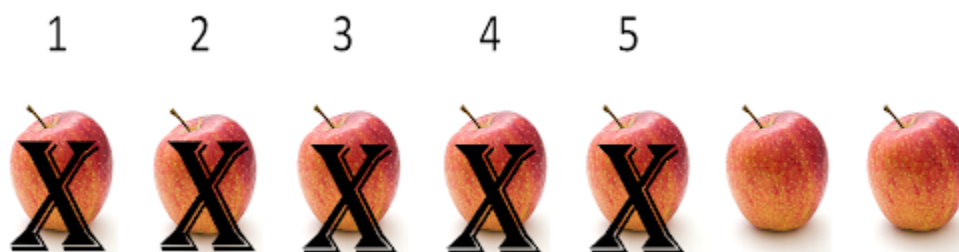
$$\begin{array}{r}
 \text{tens} \quad \quad \text{ones} \\
 \quad \swarrow \quad \searrow \\
 17 \\
 - 5 \\
 \hline
 \end{array}$$

Subtracting, like adding, requires us to work through each of the columns one by one until we reach our final answer. We'll answer this question visually in order to get the picture.

Start with 7 apples in the ones column.



Remove (or subtract) 5 of those apples, and you are left with 2 apples.



That takes care of the ones columns. We started with 7 apples, then we subtracted 5 apples, leaving us with 2 apples in the ones column. We would say that 2 is the difference between 7 and 5.

$$\begin{array}{r}
 \quad \quad \text{ones} \\
 \quad \quad \swarrow \\
 17 \\
 - 5 \\
 \hline
 2
 \end{array}$$

Now off to the tens column.

$$\begin{array}{r} \text{tens} \rightarrow 17 \\ - 5 \\ \hline 2 \end{array}$$

What is interesting to note here is that there is only one number in the tens column, and that happens to be the number 1. This makes things easy, as there is no work for us to do. We just move the 1 down into the tens column of the answer, and we then have our final answer.

$$\begin{array}{r} 17 \\ - 5 \\ \hline 12 \end{array}$$

Okay, that was pretty straightforward. Now we'll try something a little more challenging.

### Example

Imagine things worked out differently for Harpreet and Jamieson. Let's say that, of those 17 jobs they bid on and successfully got, they had to turn down 9 of them. How many of those jobs would they have taken? Before you continue reading and see the answer, try and visualize what it would look like when we put those numbers into the formula. Do you see the problem?



If we were to start with the ones column as we did in the last example, the problem would show up right away. The problem is that, if you try to subtract 9 from 7, you would end up below zero. So we have to come up with some method of subtracting that compensates for that.

$$7 - 9 = \text{less than zero}$$

$$\begin{array}{r} 17 \\ - 9 \\ \hline ? \end{array}$$

What we end up doing is borrowing from the tens column. We would end up with something that looked like this:

$$\begin{array}{r} 1 \\ 07 \\ - 9 \\ \hline 8 \end{array}$$

When we borrow from the tens column, we are borrowing a value of ten and adding it to the ones column to help the ones. We end up with 17 in the ones column, which is now more than enough to deal with the 9 being subtracted. Also, whatever value we had in the tens column (in this case, it was a 1) is reduced by 1 to account for the fact that it has been borrowed.

## Example

We'll go through another example in which we return to our plumbers, Harpreet and Jamieson. The three employees they hired are Dixon, Kavanir, and Arman, and it's a good thing they hired them. Those 17 jobs they originally bid on involved the installation of 246 fixtures, including bathtubs, toilets, and sinks. Harpreet and Jamieson could not do all that work by themselves.

But, due to the fact that they turned down 5 jobs, they won't need to install 75 of those fixtures, so with the help of the three new employees, they should be able to complete all the jobs. The question is, "How many fixtures will they have to install?"



As usual, start with the mathematical formula that allows us to properly answer the question. But this time, we'll go through the process using steps, so that when you look back, you can see how it breaks down.

You may have noticed in the first couple of sections that we often go through examples using steps. This is done to break down a large question into manageable parts. If you follow this idea when working through math problems, it can help keep you on track.

Okay, back to the problem:

**Step 1:** Put the question into a format that is easy to work with.

$$\begin{array}{r} 246 \\ - 75 \\ \hline \end{array}$$

**Step 2:** Subtract the ones. In this case, we have 6 minus 5, which equals 1.

$$\begin{array}{r} 246 \\ - 75 \\ \hline 1 \end{array}$$

**Step 3:** Subtract the tens. Here, we have the issue wherein the number on top (4) is less than the number on the bottom (7), so we have to borrow from the hundreds. The 2 in the hundreds column has to be reduced by 1, and then that 1 is added to the tens. We end up with 14 minus 7, which equals 7.

$$\begin{array}{r} 1 \phantom{0} 1 \\ \times 246 \\ - 75 \\ \hline 71 \end{array}$$

**Step 4:** Subtract the hundreds. In this case, we only have the 1, so 1 minus 0 is 1.

$$\begin{array}{r} 1 \phantom{0} 1 \\ \times 246 \\ - 75 \\ \hline 171 \end{array}$$

## Practice Questions

Try a couple example questions yourself and check the video answers to see how you did.

Question 1

$$\begin{array}{r} 94 \\ - 62 \\ \hline \end{array}$$

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## Question 2

$$\begin{array}{r} 434 \\ - 78 \\ \hline \end{array}$$

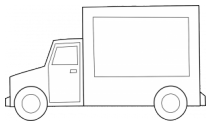
[https://video.bccampus.ca/id/O\\_51r50q8w?width=608&height=402&playerId=23448552](https://video.bccampus.ca/id/O_51r50q8w?width=608&height=402&playerId=23448552)

4.

## MULTIPLYING WHOLE NUMBERS

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### Multiplying Small Whole Numbers

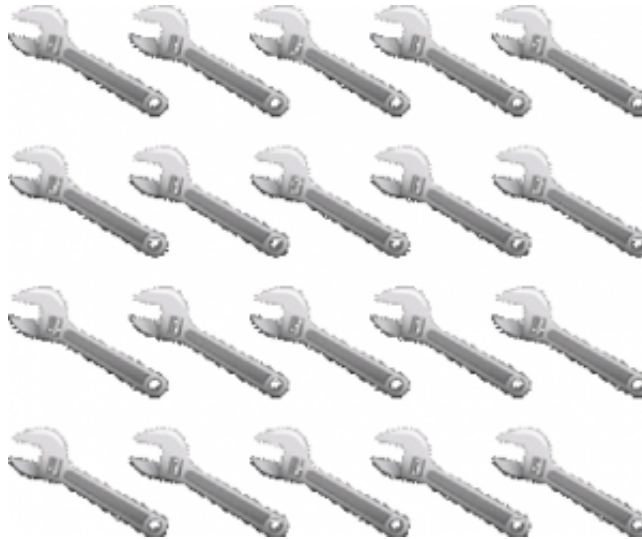


As Harpreet's company grows, he needs to buy work vans for himself and his employees. He's able to get a deal and buys 4 vans. Each of those vans needs to be equipped with tools, so he starts with wrenches. He decides that each van needs 5 wrenches. How many wrenches would that be in total?

This is an example of multiplying whole numbers. The way we could sound it out or write it is as follows:

**Four times five OR  $4 \times 5$**

If we looked at it from a more visual perspective, it would look like this:



In this case we could physically count up all the wrenches, and we would end up with 20. We could also count up each row of 5 and add them as follows:

$$5 + 5 + 5 + 5 = 20$$

Or we could take one row of 5 and multiply it 4 times because there are 4 rows. The equation would look like this:

$$4 \times 5 = 20$$

Each method should get us the same answer, but going 4 times 5 gets us to the answer quicker. If we were to

multiply larger numbers such as 8 time 9 ( $8 \times 9$ ), we could spend a long time counting wrenches or a long time adding numbers. Multiplication simplifies the process.

Before moving on to multiplying larger numbers, why don't we go through another example of multiplying smaller numbers and use visuals to help us with our answer.

### Example

Harpreet decides to buy some screwdrivers for the van and decides that each van needs 7 screwdrivers. He also decides to buy 2 extra sets for the shop. In total he decides to buy 6 sets of screwdrivers.

The first thing we should do is write the question in a form that we can work with.

$$\begin{array}{ccc} & 6 \times 7 = & \\ \nearrow & & \nwarrow \\ \text{Number of sets of} & & \text{Number of screwdrivers} \\ \text{screwdrivers} & & \text{in each set} \end{array}$$

Remember you can also think of it like this:

$$7 + 7 + 7 + 7 + 7 + 7 =$$

And once again we could look at it visually and count up the screwdrivers.





If we were to count them up, we would find that we have 42 screwdrivers. If we took our calculator and plugged in 6 times 7, we would get the same answer. If we added 7 plus 7 plus 7 and so on, we would also end up with 42.

<b>1</b> 1×1 = 1 1×2 = 2 1×3 = 3 1×4 = 4 1×5 = 5 1×6 = 6 1×7 = 7 1×8 = 8 1×9 = 9 1×10 = 10	<b>2</b> 2×1 = 2 2×2 = 4 2×3 = 6 2×4 = 8 2×5 = 10 2×6 = 12 2×7 = 14 2×8 = 16 2×9 = 18 2×10 = 20	<b>3</b> 3×1 = 3 3×2 = 6 3×3 = 9 3×4 = 12 3×5 = 15 3×6 = 18 3×7 = 21 3×8 = 24 3×9 = 27 3×10 = 30	<b>4</b> 4×1 = 4 4×2 = 8 4×3 = 12 4×4 = 16 4×5 = 20 4×6 = 24 4×7 = 28 4×8 = 32 4×9 = 36 4×10 = 40	<b>5</b> 5×1 = 5 5×2 = 10 5×3 = 15 5×4 = 20 5×5 = 25 5×6 = 30 5×7 = 35 5×8 = 40 5×9 = 45 5×10 = 50
<b>6</b> 6×1 = 6 6×2 = 12 6×3 = 18 6×4 = 24 6×5 = 30 6×6 = 36 6×7 = 42 6×8 = 48 6×9 = 54 6×10 = 60	<b>7</b> 7×1 = 7 7×2 = 14 7×3 = 21 7×4 = 28 7×5 = 35 7×6 = 42 7×7 = 49 7×8 = 56 7×9 = 63 7×10 = 70	<b>8</b> 8×1 = 8 8×2 = 16 8×3 = 24 8×4 = 32 8×5 = 40 8×6 = 48 8×7 = 56 8×8 = 64 8×9 = 72 8×10 = 80	<b>9</b> 9×1 = 9 9×2 = 18 9×3 = 27 9×4 = 36 9×5 = 45 9×6 = 54 9×7 = 63 9×8 = 72 9×9 = 81 9×10 = 90	<b>10</b> 10×1 = 10 10×2 = 20 10×3 = 30 10×4 = 40 10×5 = 50 10×6 = 60 10×7 = 70 10×8 = 80 10×9 = 90 10×10 = 100

A list of times tables for the numbers 1 to 10. Click on the image to see it full size or refer to the [Appendix A: Times Tables](#) chapter at the back of the book for a full list.

Now, we don't want to have to go through all that work of counting screwdrivers every time we multiply small numbers together. What students end up doing is using their calculator or just memorizing their "times tables." Times tables are a list of numbers being multiplied, starting at 1 times 1 and going up to 12 times 12. Take a look at the picture to the left to get an idea of what I'm talking about here. The idea is that you memorize these time tables so that when you see small numbers being multiplied, you can just access your memory for the answer.

Although memorizing your times tables is great for small numbers, you still will run up against multiplying larger numbers. The following explanation goes through

how we tackle this issue from a mathematical viewpoint.

## Multiplying Large Whole Numbers

Suppose that we want to multiply large numbers together. How do you think that would be done? The

following method is used to calculate large whole numbers, and we're going to go right to an example complete with the steps for this one as its quite the process, and we really don't want to go through this twice.

What is the product of four hundred and thirty seven times three hundred and ninety two?



The word "product" in math terms means the multiplying of two numbers together.



One thing to keep in mind is that the process we are going to work our way through here takes some time and is a lot of work. If during the process you are asking yourself, *Would it be easier to just use a calculator?*, you would have a good case. But the point of this exercise is once again to help with our understanding of how numbers go together. Remember that once you understand the basics of math, the more difficult concepts will be handled more easily. Taking the time now to fully understand the concept will help you down the road and save you money down that same road.

### Example

$$437 \times 392 = ?$$

**Step 1:** As always, the first step is to put the question in a format that is easy to work with.

$$\begin{array}{r} 437 \\ \times 392 \\ \hline \end{array}$$

The procedure is generally just multiplying a bunch of smaller numbers together to finally get the larger number. This is where knowing your times tables really comes in handy. The number on the bottom of the equation (in this case 392) is going to be worked with and changed much more than the upper number (437).

**Step 2:** We start the process by acting as if we are just multiplying the 437 by 2, the number in the ones column. So think of it as looking like the equation below.

$$\begin{array}{r} 437 \\ \times 2 \\ \hline \end{array}$$

How this works is that you will first take the 2 and multiply it by the 7, then the 2 will be multiplied by the 3, and finally the 2 will be multiplied by the 4. Now this doesn't all occur in one step. Take a look below to see how this is done.

To start with, multiply the 2 by the 7 to get 14 ( **$2 \times 7 = 14$** ).

Enter the 4 into the "ones" column and carry the 1. Remember carrying the one? As the number 14 is too large to be put into the ones column, we have to take the extra and carry it into the next calculation with the tens column.

$$\begin{array}{r} \textcircled{1} \\ 437 \\ \times 2 \\ \hline \textcircled{4} \end{array}$$

**Step 3:** Now multiply the 2 by the 3 to get 6. The 1 that we just carried is then added to the 6 to get 7. Put this answer in the tens column ( **$2 \times 3 + 1 = 7$** ).

$$\begin{array}{r} \textcircled{1} \\ 437 \\ \times 2 \\ \hline \textcircled{74} \end{array}$$

**Step 4:** Take the 2 and multiply it by the 4 to get 8. Put this number in the hundreds column ( **$2 \times 4 = 8$** ).

$$\begin{array}{r}
 \phantom{0}^1 437 \\
 \times \phantom{0} 2 \\
 \hline
 \textcircled{8}74
 \end{array}$$

**Note:** 874 is not the final answer. We still have a few steps to go. Once again you might be asking yourself if doing this with a calculator would be easier, and you'd be correct. It would be way easier but working through the question using this method will help you visualize the process behind multiplication.

Also, before you move on, take a second and go back through the initial steps. Make sure you understand the process we just went through.

**Step 5:** The next step would be to do the same thing with the number in the tens column. In this case, the 9. Think of it as looking at the equation like the one below.

$$\begin{array}{r}
 437 \\
 \times 9 \\
 \hline
 \end{array}$$

Note the empty space beside the 9. When we begin to take the 9 and multiply it by the number 437, this empty space in the ones column beside the 9 must be taken into account. This is done by placing a zero to start the process and placing it in the ones column. It would look something like is shown below.

$$\begin{array}{r}
 437 \\
 \times 9 \\
 \hline
 874 \\
 0
 \end{array}$$

**Step 6:** Now go ahead and multiply the 9 by all three numbers in the top portion of the equation, starting with the 7. When we take the 9 and multiply it by the 7, we get 63. The 3 is placed in the tens column below the 7 and then carry the 6 ( $9 \times 7 = 63$ ).

$$\begin{array}{r} \textcircled{6} \\ 437 \\ \times 9 \\ \hline 874 \\ \textcircled{30} \end{array}$$

**Step 7:** Next multiply the 9 by the 3 to get 27. We then add the 6 that was carried over to get 33. The 3 goes in the hundreds column below the 8 and the other 3 is carried into the hundreds column to be used in the next calculation ( $9 \times 3 + 6 = 33$ ).

$$\begin{array}{r} \textcircled{3}^6 \\ 437 \\ \times 9 \\ \hline 874 \\ \textcircled{330} \end{array}$$

**Step 8:** The 9 is then multiplied by the 4 to get 36. Then add the 3 that was carried over to get 39. As this is the last calculation for this part, put the 3 and the 9 into the answer. ( $9 \times 4 + 3 = 39$ )

$$\begin{array}{r} \textcircled{3}^6 \\ 437 \\ \times 9 \\ \hline 874 \\ \textcircled{39330} \end{array}$$

Now do the same procedure using the 3 from the number 392. As the 3 is from the hundreds column, it can be thought of as looking at the equation below. Once again, in the answer we have to account

for the fact that we are using a number in the hundreds column and add two zeros in the answer to start off the process.

**Step 9:** Take the 3 and multiply it by the 7 to get 21. Repeat the same procedure as before ( $3 \times 7 = 21$ ).

$$\begin{array}{r}
 \textcircled{2} \\
 437 \\
 \times 3 \\
 \hline
 874 \\
 39330 \\
 \textcircled{1}00
 \end{array}$$

**Step 10:** Now take the 3 and multiply it by the 3 to get 9 and add the 2 carried over from the first multiplication. This will give you 11 ( $3 \times 3 + 2 = 11$ ).

$$\begin{array}{r}
 \textcircled{1}^2 \\
 437 \\
 \times 3 \\
 \hline
 874 \\
 39330 \\
 \textcircled{11}00
 \end{array}$$

**Step 11:** Take the 3 and multiply it by the 4 to get 12. Add the 1 to get 13 ( $3 \times 4 + 1 = 13$ ).

$$\begin{array}{r}
 \textcircled{1}^2 \\
 437 \\
 \times 3 \\
 \hline
 874 \\
 39330 \\
 \textcircled{131}00
 \end{array}$$

**Step 12:** Finally, add up the 874, the 39,330 and the 131,100 together to get your final answer.

$$\begin{array}{r}
 874 \\
 39330 \\
 131100 \\
 \hline
 171304
 \end{array}$$

You might want to do a few things at this point. One is take a break and relax. That was a lot of work to go through. Take a minute to visually work through the process in your mind. If you feel you need to, go back through the work again and make sure you understand what is going on.

The second thing you might want to do at this point is check the answer using a calculator. Plug the numbers in and see what you get.

$$437 \times 392 = 171,304$$



## Practice Questions

Try a couple example questions yourself, and check the video answers to see how you did. Remember that this is a long process so take your time and pay attention to detail.

Question 1

$$\begin{array}{r} 522 \\ \times 196 \\ \hline \end{array}$$

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## Question 2

$$\begin{array}{r} 647 \\ \times 381 \\ \hline \end{array}$$

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5.

## DIVIDING WHOLE NUMBERS

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Harpreet has ended up with a total of 12 jobs on the go. He has decided to hire 3 more people to help him with all the work. They are Dixon, Kavanir, and Arman. He now has 4 employees including Jamieson and he splits the jobs evenly between the 4 of them. How many jobs will each of them be doing?

This would be an example of dividing whole numbers. Harpreet has a total of 12 jobs and he has 4 employees splitting the number of jobs evenly. We would write

this as:

$$12 \div 4 = ?$$

How would we go about solving this? Well let's look at it visually to start. We have 12 jobs to work with.

**JOB JOB JOB JOB JOB JOB JOB JOB JOB JOB JOB JOB**  
**1 2 3 4 5 6 7 8 9 10 11 12**

What we do now when dividing is take the number we have (12) and split it into the number of groups we are going to have. In this case we have 4 employees so we will split it into 4 even groups.

<b>JOB</b>	<b>JOB</b>	<b>JOB</b>	<b>JOB</b>	<b>JOB</b>	<b>JOB</b>	<b>JOB</b>	<b>JOB</b>	<b>JOB</b>	<b>JOB</b>	<b>JOB</b>	<b>JOB</b>
1	2	3	4	5	6	7	8	9	10	11	12
<b>1</b>			<b>2</b>			<b>3</b>			<b>4</b>		

You can now count that there are 3 jobs in every group. So what we get in the end is:

$$12 \div 4 = 3$$

Like multiplying, dividing small whole numbers is usually pretty straight forward. But what if we had a case where we have larger numbers. How would you go about doing this calculation without the use of a calculator?

What we use is a system called “LONG DIVISION” and it looks something like the following:

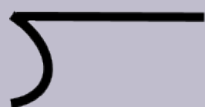
$$\begin{array}{r}
 0.736842 \\
 \hline
 190 \overline{) 140.000000} \\
 \underline{133 \phantom{0}} \phantom{00} \\
 7 \phantom{00} \phantom{00} \\
 \underline{5 \phantom{70}} \phantom{00} \\
 1 \phantom{300} \phantom{00} \\
 \underline{1 \phantom{140}} \phantom{00} \\
 1600 \phantom{00} \\
 \underline{1520} \phantom{00} \\
 800 \phantom{00} \\
 \underline{760} \phantom{00} \\
 400 \phantom{00} \\
 \underline{380}
 \end{array}$$

### Example

We'll use this process of long division to answer the following question. How many times does 5 go into 90?

**Step 1:** Set up the equation in a workable form.

$$5 \overline{)90}$$



Note: the symbol to the left is used to designate a long division question. The placement of the five in the equation indicates that it is the number to be divided into the 90. Essentially, the equation is asking how many times 5 can go into 90.

**Step 2:** Take the 5 and divide it into the first digit in the number to be divided into. In this case this is the 9. We have to figure out how many times 5 goes into 9 without going over.

This is where knowing our times tables really comes in handy. We know that:

$$5 \times 1 = 5$$

and

$$5 \times 2 = 10$$

Therefore we know that 5 goes into 9 one complete time with 4 left over.

$$\begin{array}{r} 1 \\ 5 \overline{)90} \\ \underline{5} \\ 4 \end{array}$$

To complete this step we need to bring down the next number in the equation. In this case, it's the 0. What we have now is 40 and the next step becomes how many times 5 goes into 40.

$$\begin{array}{r} 1 \\ 5 \overline{)90} \\ \underline{5 \downarrow} \\ 40 \end{array}$$

**Step 3:** Figure out how many times 5 goes into 40 without going over. If we go back to our multiplying, we might be able to remember that 5 goes into 40 exactly 8 times.

$$\begin{array}{r} 18 \\ 5 \overline{)90} \\ \underline{5} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

As we end up with zero there are no more numbers to work with through which means we are done.

Try another example.

Example

How many times does 7 go into 167?

**Step 1:** Set up the equation in a workable form.

$$7 \overline{)167}$$

**Step 2:** Take the 7 and divide it into the first digit in the number to be divided into. In this case this is the 1, and we can see that this is not going to work. As such, we just move one step to the right and divide the 7 into 16.

$$\begin{array}{r} 2 \\ 7 \overline{)167} \\ \underline{14} \phantom{0} \\ 2 \phantom{0} \end{array}$$

To complete this step we need to bring down the next number in the equation. In this case it's the 7. We now have 27 and the next step becomes how many times 7 goes into 27.

$$\begin{array}{r} 2 \\ 7 \overline{)167} \\ \underline{14} \downarrow \\ 27 \end{array}$$

**Step 3:** Figure out how many times 7 goes into 27 without going over.

$$\begin{array}{r}
 23 \\
 7 \overline{)167} \\
 \underline{14} \phantom{0} \\
 27 \\
 \underline{21} \\
 6
 \end{array}$$

We end up with a little different scenario here than in the first question. When we get to the end of the question we have 6 left over. What this is telling us is that 7 goes into 167 twenty three times with 6 left over.

## Practice Questions

Try a couple example questions yourself and check the video answers to see how you did. Remember to set the question up in a format that works.

Question 1

$$130 \div 8 =$$

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Question 2

$$684 \div 12 =$$

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6.

## WHOLE NUMBERS QUIZ

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*An interactive or media element has been excluded from this version of the text. You can view it online here:*

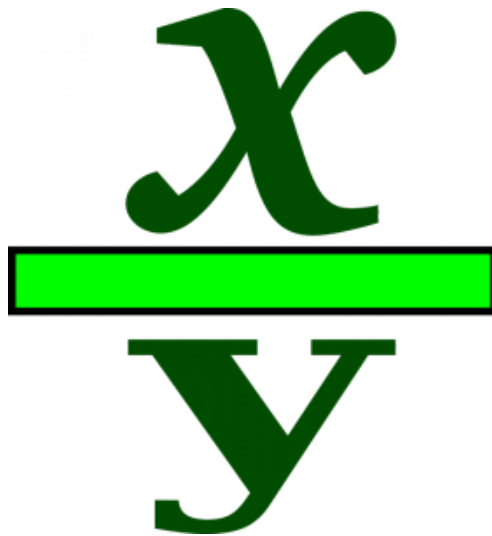
<https://pressbooks.nsc.ca/mathtrades/?p=109>

If using the print, PDF, or eBook copy of this book, navigate to the above link to complete the quiz. However, the quiz questions are also provided in [Appendix C](#) at the end of the book for offline use.



## PART III

# FRACTIONS



### Outcomes

- Identify and describe different types of fractions.
- Add and subtract fractions.
- Multiply fractions.
- Divide fractions.



7.

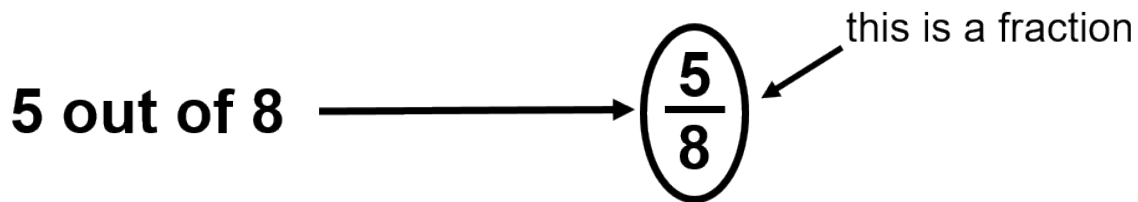
## THE BASICS OF FRACTIONS

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What exactly is a fraction, anyway? Have you ever dealt with fractions in either your schooling or your work? Well, a fraction is a part (or portion) of a whole.

Say you ordered a pizza and there were a total of 8 slices. You were hungry that day and you had 5 of them, therefore eating 5 out of the 8 slices. That can be represented as a fraction.



### Defining a Fraction

Our story of fractions begins with Abigail, Hanna, and Naomi, who are electrical apprentices going through their schooling at the same time who hope to open a company together once they get their Red Seal Electrical Tickets.

We'll start with a couple of definitions. Every fraction has two parts: the numerator and the denominator. Let's take a look at a fraction to define each.

**1**  

---

**3**

**Numerator:** The number above the line in a fraction. It indicates how many parts of the whole are being counted.

**Denominator:** The number below the line in a fraction. It indicates how many total parts are in the whole.

If Abigail, Hanna, and Naomi did end up owning a company, each would own  $\frac{1}{3}$  of that company. Each person represents 1 owner, and together, there are 3 owners in the whole company.

Here are a few more examples of fractions:

$$\frac{1}{2} \quad \frac{3}{8} \quad \frac{5}{16} \quad \frac{4}{9} \quad \frac{7}{15} \quad \frac{77}{145}$$

## Mixed Numbers and Improper Fractions

The examples above are all of typical fractions, but we don't always see fractions in that form. There are two other styles of fractions that we deal with: **mixed numbers** and **improper fractions**.

### Mixed numbers

Let's say the three apprentices get together one night to talk about the future, and they order 2 pizzas, each with 8 pieces.

Ham and Pineapple



Vegetarian



(I know the pizzas look exactly the same, but you'll have to trust me on this one. One thing for sure is that each pizza has 8 slices, and someone has gone ahead and had a taste test of both pizzas.)

We need to break this down: we have a total of 2 pizzas, each with 8 slices. That makes for a total of 16 slices. If the apprentices eat 1 whole pizza, they will have eaten 8 out of 8 slices.

$$\frac{8}{8} = 1$$

Now let's say that one of the three of them has another slice from the second pizza. They will have now eaten 1 whole pizza plus 1 slice.

$$1 + \frac{1}{8} = 1\frac{1}{8} \leftarrow \text{Mixed number}$$

This is what is known as a mixed number. A mixed number can be defined as the following:

**Mixed number:** A combination of a whole number and a fraction.

Next, we cover improper fractions.

## Improper fractions

**Improper fraction:** A fraction in which the numerator is larger than the denominator.

What this means is that the number on the top of the fraction is larger than the number on the bottom. We'll stick to our pizza example. Together, the apprentices have eaten a total of 9 slices. This accounts for 1 whole pizza plus 1 slice from the second pizza. Written as an improper fraction, the number of pizzas eaten would look like this:

$$\frac{9}{8}$$

Now, we want to change a mixed number into an improper fraction, and then do the reverse and take an improper fraction and change it back to a mixed number.

### Example

Change the following mixed number into an improper fraction:

$$1\frac{3}{4}$$

**Step 1:** Change the whole number into a fraction, with the denominator being 4.

$$1 = \frac{4}{4}$$

**Step 2:** Add the two fractions together. Now, we'll have to jump ahead a little here, as we haven't covered adding fractions yet. I'll give you the cheap and easy version here. As long as the denominators are the same, we are all good. When adding fractions, we simply keep the denominators the same and add the numerators. (We'll go through adding fractions thoroughly in the next chapter.)

$$\frac{4}{4} + \frac{3}{4} = \frac{7}{4}$$

So:

$$1\frac{3}{4} = \frac{7}{4}$$

Another way to find your answer would be as follows:

$$1\frac{3}{4} = 1 \overset{+}{\underset{\times}{\times}} \frac{3}{4} = \frac{7}{4}$$

That may look a little confusing, but follow me through it. With the mixed number  $1\frac{3}{4}$ , take the 4 and multiply it by the 1. Then add 3, and you end up with 7. It's the same answer — just a different way of getting there.

Try going from a mixed number to an improper fraction by yourself.

### Practice Question A

Change the following into an improper fraction. Check the video answer to see how you did.

$$3\frac{3}{8}$$

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All right: hopefully, you've got the mixed-number-to-improper-fraction calculation down. But what about the reverse? We should go through an example of that as well, and then give you a chance to work through one yourself.

## Example

Change the following improper fraction into a mixed number:

$$\frac{27}{6}$$

**Step 1:** Find out how many times 6 goes into 27. We can do this using long division. The good news here is that we already went through long division in the first chapter. If you need to review it, take a look back to see how it's done (see [Dividing Whole Numbers](#)).

$$\begin{array}{r} 4 \\ 6 \overline{)27} \\ \underline{24} \\ 3 \end{array}$$

What we end up with is 6 going into 27 four times with 3 left over. So our mixed number becomes:

$$4\frac{3}{6}$$

Try another practice question.

Practice Question B

Change the following improper fraction into a mixed number. Check out the video answer to see how you did.

$$\frac{17}{3}$$

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## BRAIN BREAK

Take a couple moments to review what you just learned before moving on.

## Reducing Fractions

Before we move on to adding and subtracting fractions, we should touch on another concept known as reducing fractions. Reducing is what we do when we want to make a smaller version of a fraction that still has the same mathematical value as the original.

Back to our pizza. Once again, we have 8 slices per pizza. Now, say we eat 4 of those slices. We have eaten:

$$\frac{4}{8} \begin{array}{l} \text{number of slices eaten} \\ \text{number of slices in pizza} \end{array}$$

If someone asked you how much pizza you had, how would you answer? Would you say, “I had 4 out of a possible 8 slices,” or would you say, “I ate half the pizza”? I think we’d all agree that we would just say that we ate half the pizza, as 4 pieces would amount to half the pizza. If we were to write half as a fraction, it would look like this:

$$\frac{1}{2}$$

We could then conclude that the two fractions represent the same thing mathematically, and they are just two different ways to represent the same thing. You could look at it this way: I cut two pieces of wood. One is



12 inches in length, and the other is 1 foot in length. They are the same length — their lengths are just expressed in different ways. So in the end, we end up with this:

$$\frac{4}{8} = \frac{1}{2}$$

What we've done is reduced the fraction from 4 over 8 to 1 over 2 without changing the actual value represented. How this was done mathematically is we took the original numerator of 4 and divided it by 4. What is done to one part of the fraction must also be done to the other, so we also divided the denominator of 8 by 4, resulting in a fraction of 1 over 2.

$$\frac{4}{8} \div 4 = \frac{1}{2}$$

Doing the same thing to both the numerator and the denominator guarantees that the original fraction and the final fraction are equal in value.

We reduce fractions when we can evenly divide the same number into both the numerator and the denominator. In our example, 4 can be divided into both. Note that the number 2 can also be divided into both the numerator and the denominator. If we divided both by 2, we would get:

$$\frac{4}{8} \div 2 = \frac{2}{4}$$

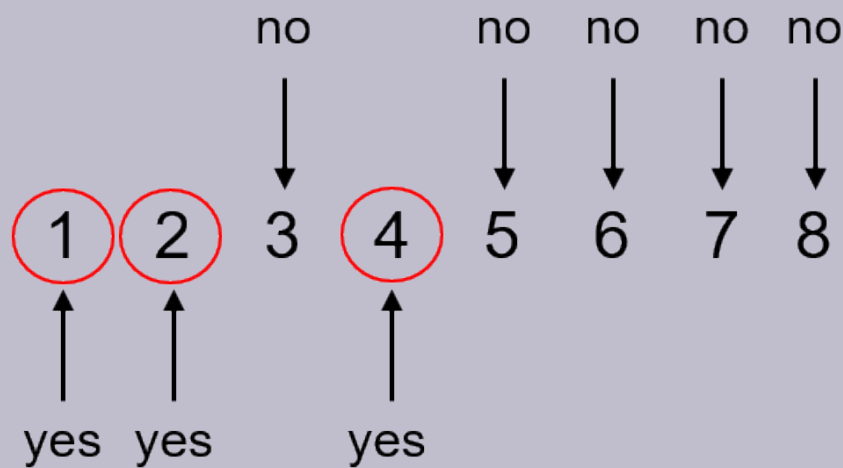
Although this still works mathematically, we often want to get a fraction into its lowest terms, meaning to a point where it can no longer be reduced. The fraction 2 over 4 could be reduced even further to 1 over 2, so there is further work we could do, if we chose to.

Example

Let's go through the thought pattern when reducing fractions. Take the following fraction and reduce it to its lowest terms:

$$\frac{8}{12}$$

**Step 1:** What we want to do here is take a look at both the numerator and the denominator and determine if there is a number that can go into both of them. It might be easier if you write down numbers starting from 1 and then decide which numbers can go into both 8 and 12.



From this, we can conclude that the largest number that can go into both 8 and 12 is 4.

**Step 2:** Divide both the numerator and the denominator by 4.

$$\frac{8}{12} \div 4 = \frac{2}{3}$$

There you have it: the fraction has now been reduced to its lowest terms. Always take a look at the answer when you are done, just to make sure that there definitely isn't another number that can go into the numerator and denominator, as this would mean the fraction could be reduced even further.

The example shown above is fairly straightforward. Once there are larger numbers involved, it is sometimes

easier to work through the question in a couple of steps to slowly reduce the fraction. Take a look at the following example to see what I mean.

### Example

Reduce the following fraction into its lowest terms:

$$\frac{24}{168}$$

**Step 1:** Determine if there is a number that can go into both the numerator and the denominator. If there is more than one number, then use the larger number.

This is a bit tougher than the first question, as the numbers are a lot larger and harder to work with. Going back to our times tables, we can see that 6, 8, and 12 all go into 24. We could also say that 24 goes into 24. But what about 168? What goes into that?

One thing we know for sure is that 2 goes into both, so why don't we start by taking each part of the fraction and dividing it by 2. If you have trouble dividing 168 by 2 in your head, go ahead and use your calculator.

$$\frac{24 \div 2}{168 \div 2} = \frac{12}{84}$$

**Step 2:** Determine if the fraction can be reduced any further. We can see that, once again, we can divide both numbers by 2.

$$\frac{12 \div 2}{84 \div 2} = \frac{6}{42}$$

**Step 3:** Repeat step 2 and determine if the fraction can be reduced any further. What we note this

time is that 6 can go into both 6 and 42, so we divide both the numerator and the denominator by 6.

$$\frac{6}{42} \div 6 = \frac{1}{7}$$

There you have it: we have reduced that large fraction into its lowest terms in just a few steps. I will admit that, if we had used a calculator for this whole procedure, we could have come up with the answer with less work, but that is not the point. Doing it the long way starts to train your brain in the relationships between numbers. As you get more familiar with numbers, you will be able to pick apart patterns and understand the relationships formed in math. Although it might be a little more time-consuming in the beginning, the payoff as time goes by is great.

## Practice Questions

Try a couple questions for yourself and check the video answers to see how you did.

Reduce the following fractions into their lowest terms.

Question 1

$$\frac{15}{18}$$

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## Question 2

$$\frac{24}{36}$$

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8.

## ADDING AND SUBTRACTING FRACTIONS

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### Adding Fractions with Common Denominators



Abigail, Hanna, and Naomi are studying for their midterm exam. The material they are required to study consists of 16 chapters of reading. The three of them realize that 16 chapters is a lot of reading for each of them to do, so they decide to study in a more efficient manner. They come up with a plan in which each of them reads a certain number of chapters and then summarizes it for the other two. They will share notes, and each will find online videos corresponding to their particular set of chapters.

Now, the chapters are not created equally. Some are quite easy, while others are much tougher. Their goal is to spread the workload evenly between the three of them. Remember that there are 16 chapters.

Abigail has the highest number of chapters to go through with 6. Hanna has 5, while Naomi has only 4. If you were to add those up, you would notice that that only comes to 15 chapters. The last chapter in the book is about troubleshooting electrical systems, and the apprentices decide that they will go through that one together.

We can represent each of their workloads as a fraction of a whole:

Abigail has  $\frac{6}{16}$

Hanna has  $\frac{5}{16}$

Naomi has  $\frac{4}{16}$

What if we were to add those fractions? It would look something like this:

$$\frac{6}{16} + \frac{5}{16} + \frac{4}{16} = ?$$

What you'll note is that the numerators are all different, while the denominators are all the same (16). When

adding or subtracting fractions, the denominators must be the same. We refer to this as having a common denominator.

So, in order to get the answer to the above question, you just add all the numerators. Adding fractions is very simple in this respect.

Abigail		Hanna		Naomi		
$\frac{6}{16}$	+	$\frac{5}{16}$	+	$\frac{4}{16}$	=	$\frac{15}{16}$
↙		↑		↘		
common denominator						

Notice that the denominator in the final answer is the same as that in the fractions being added. By the end, the apprentices will have gone through 15 of the 16 chapters separately, and then they will go through the last chapter together.

The concept of adding fractions with common denominators is easy enough, and we did enough adding whole numbers that going through examples at this point might not be worth it (but if you need a review, see [Adding Whole Numbers](#)). What we will do instead is write down some examples of adding fractions so you can see the idea.

$$\frac{1}{8} + \frac{2}{8} = \frac{3}{8}$$

$$\frac{5}{16} + \frac{6}{16} = \frac{11}{16}$$

$$\frac{13}{32} + \frac{11}{32} = \frac{24}{32}$$

Do you notice anything about the answer to the last one? It can be reduced.

$$\frac{24}{32} \longrightarrow \frac{2}{3}$$



Before we get going any further with work on fractions, this might be a good time to state that, when working with fractions, we generally want to put the answer in lowest terms.

## Subtracting Fractions with Common Denominators

What about subtracting fractions? Well, it follows the same principle: you must have a common denominator, and then you subtract the numerators. Here are some examples of subtracting fractions:

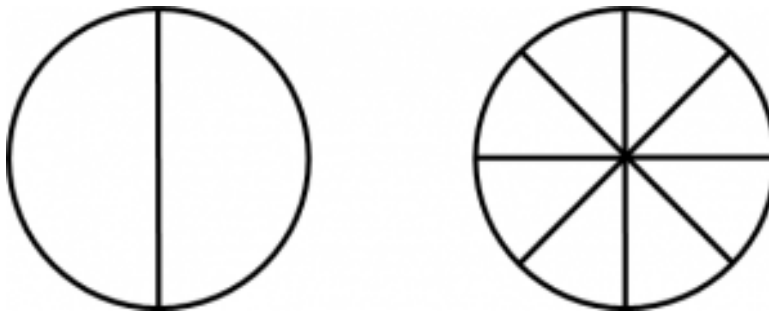
$$\begin{array}{r} \frac{5}{8} - \frac{2}{8} = \frac{3}{8} \\ \frac{9}{16} - \frac{5}{16} = \frac{4}{16} \longrightarrow \frac{1}{4} \\ \frac{27}{32} - \frac{14}{32} = \frac{13}{32} \end{array}$$

## Adding and Subtracting Fractions with Uncommon Denominators

We're going to step it up a bit now. Our examples of adding and subtracting fractions are fairly straightforward, due to the fact that the denominators are the same. A more difficult situation involves adding or subtracting fractions that have different denominators. Take a look at the following example:

$$\frac{1}{2} + \frac{3}{8} = ?$$

We can't simply add up the numerators and the denominators, as it just won't work. Take a look at the two circles drawn below. One is split into 2 parts, and one is split into 8 parts. Do you notice anything about the sizes of the parts?







You'll note that the parts in the 2-part circle are much larger than those in the 8-part circle. If we were to add up the parts in each of the circles, it would be like adding apples and oranges.

So the idea becomes making it so that the parts we are adding are the same size. If we can somehow get to that point, then we are good to go, and we can add the two fractions. This is referred to as finding a common denominator, and most often, we try and find the **lowest common denominator**.

**Lowest common denominator:** The lowest number that two denominators can evenly go into.

Take a look at the equation below. One of the denominators is 2, while the other is 8.

$$\frac{1}{2} + \frac{3}{8} = ?$$

The process here is similar to when we were putting fractions into their lowest terms in the last section, only this time, we will be increasing at least one of the denominators, and sometimes, we will be increasing both until we find one that is common. What we are looking for is a number that both denominators can go into evenly. In this example, we see that 2 can go into 8 and 8 can go into 8. This leaves us with a common denominator of 8.

$$\left. \begin{array}{l} 2 \text{ goes into } 8 \text{ four times} \\ 8 \text{ goes into } 8 \text{ one time} \end{array} \right\} 8 = \text{common denominator}$$

We have determined that 8 is going to be our common denominator, which means that one of the fractions is already good to go.

$$\frac{3}{8} \quad \checkmark$$

But what about 1 over 2, or one-half? We have to turn the one-half into a fraction with 8 as the denominator.

$$\frac{1}{2} \longrightarrow \frac{x}{8}$$

As we calculated above, 2 goes into 8 four times.

$$2 \times 4 = 8$$

That's good for the denominator, but what about the numerator? Well, whatever we do to the one part of the fraction, we must do the same to the other part. This leaves the fraction with the same value. We then have to also multiply the 1 by 4.

$$1 \times 4 = 4$$

If we wanted to do it all in one step, it would look something like this:

$$\frac{1 \times 4}{2 \times 4} = \frac{4}{8}$$

Now we have something we can work with. Go back to the original equation and replace the  $\frac{1}{2}$  with  $\frac{4}{8}$ .

$$\frac{4}{8} + \frac{3}{8} = \frac{7}{8}$$

Okay, so that works for adding fractions, but what about subtracting fractions? Well, subtracting fractions follows the same principle: if the denominators are not the same, then we have to find a common denominator first before subtracting the two fractions.

Example

Calculate the following:

$$\frac{7}{8} - \frac{13}{16} =$$

**Step 1:** Find the common denominator. This can get a little tricky when the numbers begin to get larger. As you get more familiar with the patterns in numbers, the answers will come easier. The question we are asking right now is, “What number can both 8 and 16 go into evenly?”

We might even start by seeing if the smaller denominator can go into the larger denominator. In this case, it does.

$$\left. \begin{array}{l} 8 \text{ goes into } 16 \text{ two times} \\ 16 \text{ goes into } 16 \text{ one time} \end{array} \right\} 16 = \text{common denominator}$$

The fraction with the common denominator of 16 is already good to go, but we have to work with the fraction with a denominator of 8.

**Step 2:** Multiply both the numerator and the denominator of  $\frac{7}{8}$  by 2 to give the fraction the common denominator of 16.

$$\frac{7}{8} \times \frac{2}{2} = \frac{14}{16}$$

**Step 3:** Subtract the new versions of the fractions.

$$\frac{14}{16} - \frac{13}{16} = \frac{1}{16}$$

## Practice Questions

Answer the following practice questions and check for the video answers. Make sure to put each answer into its lowest terms or a mixed number, if necessary.

### Question 1

$$\frac{3}{16} + \frac{5}{8} =$$

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### Question 2

$$\frac{5}{8} - \frac{5}{16} =$$

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### Question 3

$$\frac{1}{2} + \frac{7}{8} =$$

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#### Question 4

$$2\frac{1}{2} + 1\frac{7}{8} =$$

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Wait a minute! That last question stepped it up a notch by adding mixed numbers. I know you've already taken a look at the video answer, but let's take a step back and go through the motions of adding and subtracting mixed numbers. We'll start with a short explanation.

## Adding or Subtracting Mixed Numbers

The issue we run into when adding or subtracting mixed numbers is that a mixed number is composed of two separate parts: there is the whole number, and then there is the fraction. When adding the numbers, this can be straightforward, like the following:

$$4\frac{3}{8} + 3\frac{2}{8} = 7\frac{5}{8}$$

Pretty straightforward, right? You simply add the two whole numbers, and then add the fractions. It works out quite nicely. But what about a situation like the next example?

$$4\frac{5}{8} + 3\frac{4}{8} = ?$$

Do you see the issue?

The issue (it's not really an issue) is that, when we add the fractions, we end up with a bigger number in the numerator than in the denominator.

$$4\frac{5}{8} + 3\frac{4}{8} = 7\frac{9}{8}$$

The solution is to change the improper fraction part of the answer into a mixed number, and then add that to the whole number part of the answer.

$$\frac{9}{8} \longrightarrow 1\frac{1}{8}$$

Take the 7 and add it to the mixed number to get our final answer.

$$7 + 1\frac{1}{8} = 8\frac{1}{8}$$

Okay, that seemed pretty straightforward, but what about subtraction? Well, we follow the same rules. Take a look at the following example:

Example

$$8\frac{7}{8} - 6\frac{3}{8} = ?$$

The procedure is similar to that for adding fractions, but instead of adding, we are subtracting. We can break it down into two parts. We start by subtracting the whole numbers, and then follow that up by subtracting the fractions portion.

**Step 1:** Subtract the whole numbers.

$$8 - 6 = 2$$

**Step 2:** Subtract the fractions portion of the equation.

$$\frac{7}{8} - \frac{3}{8} = \frac{4}{8} \rightarrow \frac{1}{2}$$

**Step 3:** Put it all together.

$$8\frac{7}{8} - 6\frac{3}{8} = 2\frac{4}{8} \rightarrow 2\frac{1}{2}$$

Okay, not too hard, right? But take a look at the next example and see if you can figure out the problem that we are going to have as we run through it.

Example

$$5\frac{2}{8} - 3\frac{7}{8} = ?$$

The problem emerges not when you are subtracting the whole numbers, but when you are subtracting the fractions.

$$\frac{2}{8} - \frac{7}{8} = ?$$

We would end up with an answer less than zero. This is not going to work for us. So how do we solve the problem? Well, the answer lies in borrowing, and what we are borrowing from is the whole number, 5. Let's just say we borrow 1 from the 5. This would leave us with 4, and then what? Take a look at the following logic.

$$\begin{aligned} 5 &= 4 + 1 \\ 1 &= \frac{8}{8} \end{aligned}$$

If we go ahead and break the 5 down into 4 and 1, and then split that 1 down into parts of 8, we have a lot more eighths to work with. We can now put everything together to get the following:

$$5\frac{2}{8} = 4 + \frac{8}{8} + \frac{2}{8} = 4\frac{10}{8}$$

We now have numbers we can work with in our original question.

$$4\frac{10}{8} - 3\frac{7}{8} = ?$$

We now follow the same steps as before.

**Step 1:** Subtract the whole numbers.

$$4 - 3 = 1$$

**Step 2:** Subtract the fractions portion of the equation.

$$\frac{10}{8} - \frac{7}{8} = \frac{3}{8}$$

**Step 3:** Put it all together.

$$4\frac{10}{8} - 3\frac{7}{8} = 1\frac{3}{8}$$

## Practice Questions

Add or subtract the following mixed numbers, making sure to put your answer in lowest terms. Check the video answers at the end to see how you did.

Question 1



$$7\frac{3}{16} + 4\frac{5}{16} =$$

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Question 2

$$2\frac{7}{16} + 3\frac{7}{8} =$$

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Question 3

$$8\frac{27}{32} - 1\frac{15}{32} =$$

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Question 4

$$6\frac{5}{16} - 5\frac{5}{8} =$$

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9.

## MULTIPLYING FRACTIONS

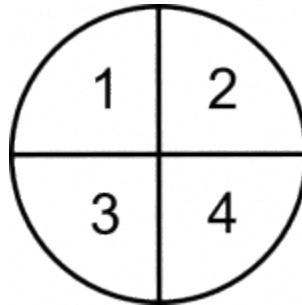
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$$\frac{x}{y} \times \frac{x}{y}$$

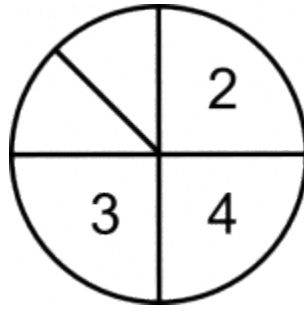
The following equation is an example of multiplying fractions. At first glance, it might look harder than either adding or subtracting fractions, but in reality, it's much easier. What might be tougher to understand is the answer that you get when you multiply fractions.

$$\frac{1}{4} \times \frac{1}{2} = ?$$

We'll take a look at this visually, using a circle cut into parts to work this out. To start, we'll divide the circle into 4 equal parts. One of those parts would equal one-quarter of the circle.



If we were to multiply that  $\frac{1}{4}$  by  $\frac{1}{2}$ , what we would be doing mathematically is taking  $\frac{1}{2}$  of the  $\frac{1}{4}$  piece, or essentially splitting that  $\frac{1}{4}$  into two equal parts. That would end up representing  $\frac{1}{8}$  of the circle.



Mathematically, it's done like this:

**Multiply the numerators together**

$$1 \times 1 = 1$$

**AND**

**Multiply the denominators together**

$$2 \times 4 = 8$$

What we end up with is this:

$$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$$

### Example

Let's go back to Abigail, Hanna, and Naomi. They've now completed another level of their schooling and are getting to the end of their apprenticeships. All three are working on the same job, which is a three-storey wood-frame building, and each is responsible for roughing-in 30 suites. They are required to wire  $\frac{1}{6}$  of those suites every week. One week, Hanna had to miss two days. Therefore, she only worked 3 out of the 5 days, or  $\frac{3}{5}$  of the time. What fraction of suites would she have been able to rough-in that week, taking into consideration her time away?

Start by writing down the fractions we are going to work with in this situation.

$\frac{1}{6}$  The amount of suites needed to be completed during a 5-day work week.

$\frac{3}{5}$  The fraction of time worked during the week, 3 out of 5 days.

Then multiply the two fractions together, sticking to our formula of multiplying the numerators together and then multiplying the denominators together.

$$\begin{array}{l} \text{numerators } 1 \times 3 = 3 \\ \text{denominators } 6 \times 5 = 30 \end{array}$$

And that makes the answer:

$$\frac{1}{6} \times \frac{3}{5} = \frac{3}{30}$$

Which can then be reduced to its lowest terms:

$$\frac{3}{30} \rightarrow \frac{1}{10}$$

Here is another example. Let's go through the steps in this one.

Example

$$\frac{5}{8} \times \frac{3}{4} = ?$$

**Step 1:** Multiply the numerators together.

$$5 \times 3 = 15$$

**Step 2:** Multiply the denominators together.

$$8 \times 4 = 32$$

**Step 3:** Put each of the answers in the appropriate place in the fraction.

$$\frac{5}{8} \times \frac{3}{4} = \frac{15}{32}$$

**Step 4:** Put the answer in lowest terms, if necessary, and change to a mixed number, if necessary. In this question, we are good on both accounts.

$$\text{Final answer} = \frac{15}{32}$$

Up to this point, you may have been thinking that you got this and this is easy, but now, let's step up the difficulty level a bit.

### Example

$$4\frac{2}{5} \times 2\frac{1}{4} = ?$$

Before you start, do you see a problem? The problem is that you are now trying to multiply two mixed numbers together. How does that work? Can you just go ahead and try to multiply them as they are? The answer is NO, but the solution to the problem is not that difficult: you just have to take one extra step before going through the process.

The first thing you have to do is change each of the mixed numbers into improper fractions. From that point on, the process is the same.

**Step 1:** Change each of the mixed numbers into improper fractions. This is the only way to answer this question. You cannot multiply the numbers in the state they are in.

$$4\frac{2}{5} = \frac{22}{5}$$

( $5 \times 4 + 2 = 22$ )

$$2\frac{1}{4} = \frac{9}{4}$$

( $4 \times 2 + 1 = 9$ )

**Step 2:** Multiply the numerators together.

$$22 \times 9 = 198$$

**Step 3:** Multiply the denominators together.

$$5 \times 4 = 20$$

**Step 4:** Put each of the answers in the appropriate place in the fraction.

$$\frac{22}{5} \times \frac{9}{4} = \frac{198}{20}$$

**Step 5:** Put the answer in lowest terms, if necessary, and change to a mixed number, if necessary. In this case, we have to do both. We'll start by putting the fraction into lowest terms.

$$\frac{198 \div 2}{20 \div 2} = \frac{99}{10} \quad \text{Lowest terms}$$

Then take that and put it into a mixed number.

$$\frac{99}{10} = 9\frac{9}{10} \quad \text{Mixed number}$$

## Practice Questions

Try a couple questions yourself. Make sure to put your answer into lowest terms and, if necessary, turn it back into a mixed number. Check the video answers when you are done to see if you are on the right track.

## Question 1

$$\frac{4}{7} \times \frac{3}{8} =$$

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## Question 2

$$\frac{6}{11} \times \frac{5}{9} =$$

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## Question 3

$$5\frac{1}{2} \times 6\frac{3}{8} =$$

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## Question 4

$$7\frac{5}{9} \times 8\frac{5}{7} =$$

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10.

## DIVIDING FRACTIONS

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$$\frac{x}{y} \div \frac{x}{y}$$

Dividing fractions follows similar logic to multiplying fractions. It involves working with the numerators and denominators separately. Once you have done your initial calculations, you put it all together to get your answer. There is a little twist, though, that we'll have to examine as we work through some questions.

Before we get to all that, let's start by revisiting dividing whole numbers, and then we'll work our way up to fractions.

Start with 20 screwdrivers:



Now divide those 20 screwdrivers by 10 (or into groups of 10).

$$20 \div 10 = 2$$

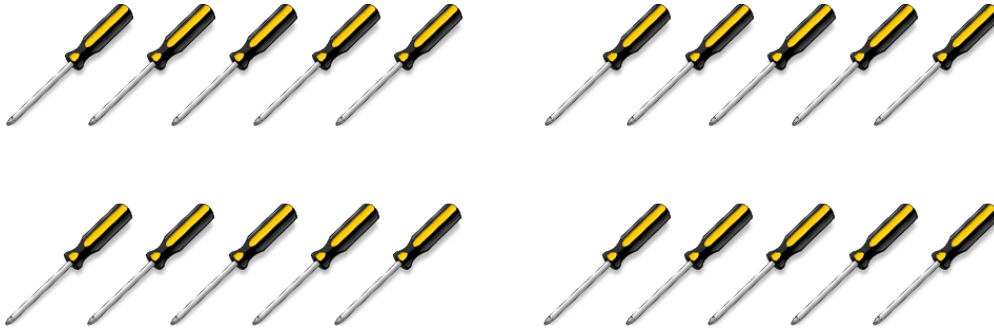
You end up with 2 groups of 10.



Now divide those 20 screwdrivers by 5 (or into groups of 5).

$$20 \div 5 = 4$$

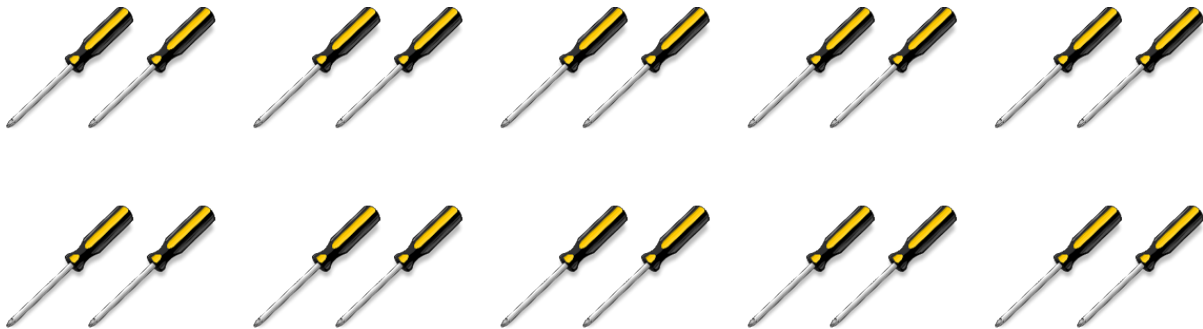
You end up with 4 groups of 5.



Now divide those 20 screwdrivers by 2 (or into groups of 2).

$$20 \div 2 = 10$$

You end up with 10 groups of 2.



Take a look at the math here. Do you see a pattern? What did you come up with? Did you notice that, when you take your original amount (in this case, 20) and divide it by a number that continues to get smaller (10, then 5, then 2), we end up with an answer that gets larger.

$$\begin{aligned} 20 \div 10 &= 2 \\ 20 \div 5 &= 4 \\ 20 \div 2 &= 10 \end{aligned}$$

Follow this logic into fractions, keeping in mind that fractions are not only less than 10, 5, and 2, but 1. Using this pattern, we determine that dividing the 20 screwdrivers by a number less than 1 would get us a larger answer than if we divided the 20 by 10, 5, or 2.

Try this. Take the 20 screwdrivers, and divide them by  $\frac{1}{2}$ . What do you think your answer will be?

$$20 \div \frac{1}{2} = ?$$

Following our logic, the answer should be more than 10, and in fact, it is.

$$20 \div \frac{1}{2} = 40$$

This does not mean that we end up with 40 screwdrivers, though. What it means is that we end up with 40 parts of screwdrivers. You have to imagine that each of the screwdrivers were split into 2. Twenty screwdrivers split in half would give us 40 pieces in the end. The question now becomes, how do we do this mathematically? The answer lies in using what is known as a reciprocal. Here is the definition.

**Reciprocal:** A number that has a relationship with another number such that their product is 1.

This means that, when you take a number such as 5 and then multiply it by its reciprocal, you will end up with an answer of 1. We'll start with the whole number 5. We could also write the number 5 as a fraction.

$$5 = \frac{5}{1}$$

Using our definition of reciprocal, we need to find a number that, when multiplied by  $\frac{5}{1}$ , gives us an answer of 1.

$$\frac{5}{1} \times ? = 1$$

To find the answer, we have to go back to multiplying fractions. Remember that, when we multiply fractions, we just multiply the numerators together and then multiply the denominators together. From this, we can conclude that:

$$\frac{5}{1} \times \frac{1}{5} = 1$$

In the end, to find the reciprocal of a fraction, we simply take the numerator and make it the denominator and take the denominator and make it the numerator. Essentially, we are just flipping the fraction around. Here are some more examples of reciprocals.

$$\frac{3}{8} \text{ and } \frac{8}{3}$$

$$\frac{2}{9} \text{ and } \frac{9}{2}$$

$$\frac{24}{17} \text{ and } \frac{17}{24}$$

Okay, so now that we have the reciprocal issue out of the way, the question then becomes, why do we need reciprocals in the first place? Well, the answer lies in the rule for dividing fractions.



The rule for dividing fractions is you take the first fraction and multiply it by the reciprocal of the second fraction. Yes, you heard that right: to divide, you end up multiplying, but only after first flipping the second fraction around.

Flipping the second fraction around (finding its reciprocal) changes the value of the equation. In order to keep the equation mathematically the same, we have to change the division question into a multiplication question. Take a look at the following example to see how this is done.

### Example

$$\frac{1}{2} \div \frac{3}{8} = ?$$

**Step 1:** Put the question into a form that you can work with. This involves finding the reciprocal of the second fraction and then multiplying it by the first.

The reciprocal of  $\frac{3}{8}$  is  $\frac{8}{3}$

$$\text{Check: } \frac{3}{8} \times \frac{8}{3} = \frac{24}{24} = 1$$

So we end up with:

$$\frac{1}{2} \div \frac{3}{8} \text{ becomes } \frac{1}{2} \times \frac{8}{3} = ?$$

**Step 2:** Follow the same procedure as we did when multiplying fractions. Multiply the numerators together and then multiply the denominators together.

**Multiply the numerators together**

$$1 \times 8 = 8$$

**Multiply the denominators together**

$$2 \times 3 = 6$$

**Step 3:** Take these answers and put them back into a fraction.

$$\frac{1}{2} \times \frac{8}{3} = \frac{8}{6}$$

**Step 4:** Put the answer into lowest terms and then into a mixed number, if necessary.

$$\text{Lowest terms } \frac{8 \div 2}{6 \div 2} = \frac{4}{3}$$

$$\frac{4}{3} = 1\frac{1}{3} \text{ Mixed number}$$

Final answer:

$$\frac{1}{2} \div \frac{3}{8} = 1\frac{1}{3}$$

Example

$$\frac{5}{9} \div \frac{7}{4} = ?$$

**Step 1:** Put the question into a form that you can work with. This involves finding the reciprocal of the second fraction and then multiplying it by the first.

$$\frac{5}{9} \div \frac{7}{4} \text{ becomes } \frac{5}{9} \times \frac{4}{7}$$

**Step 2:** Multiply the numerators together and then multiply the denominators together.

**Multiply numerators together**

$$5 \times 4 = 20$$

**Multiply denominators together**

$$9 \times 7 = 63$$

**Step 3:** Take these answers and put them back into a fraction.

$$\frac{5}{9} \times \frac{4}{7} = \frac{20}{63}$$

**Step 4:** Put the answer into lowest terms and then into a mixed number, if necessary. In this case, the answer is both in lowest terms and is a proper fraction already, so we are done.

$$\frac{5}{9} \div \frac{7}{4} = \frac{20}{63}$$

## Practice Questions

Try these practice questions and take a look at the video answers to see how you did.

Question 1

$$\frac{7}{8} \div \frac{7}{16} =$$

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## Question 2

$$4\frac{7}{8} \div 2\frac{3}{4} =$$

Note: This one is slightly different from what we have been doing, as it involves dividing mixed numbers. What do you think you are going to have to do when dealing with this?

**ANSWER:** You have to change the mixed number into an improper fraction first. Then you can work through the question the same way we have been doing before.

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11.

## FRACTIONS QUIZ

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*An interactive or media element has been excluded from this version of the text. You can view it online here:*

<https://pressbooks.nsc.ca/mathtrades/?p=160>

If using the print, PDF, or eBook copy of this book, navigate to the above link to complete the quiz. However, the quiz questions are also provided in [Appendix C](#) at the end of the book for offline use.



## PART IV

# DECIMALS



### Outcomes

- Understand decimal notation.
- Compare and round decimals.
- Convert fractions to decimals.
- Convert decimals to fractions.



# UNDERSTANDING DECIMAL NOTATION

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We started by going through whole numbers, then we moved onto fractions, and now we will look at decimals.



We start this story with Chad. Now, Chad is not an apprentice, like the people in the first two stories, but rather someone who received his Red Seal Electrical Ticket a number of years ago. He then went on to start his own electrical company, which he still runs today.

After owning his own business for several years, Chad recognized the fact that apprentices coming through the system could use some advice about starting and running a company, especially when it came to the money portion. As Chad has some experience in this, he decided to start a course for apprentices wishing to start their own businesses.

When dealing with numbers, we don't always have the luxury of working with just whole numbers. If we referred back to the last chapter and looked at fractions, we would see that a fraction represents a part of a whole number. Decimals are the same in that they represent a part of a whole number. The difference is that decimals are written with a different notation than fractions. Decimal notation allows us to visually quantify how much of a whole number we are dealing with.



In his first class, Chad starts off by talking about the price of tools and materials. He starts with a hammer priced at \$12.47. Note that the cost of the hammer is not exactly \$12, but a little more than \$12. This is where decimals come into play. The numbers after the point ( $.$ ), also known as the decimal point, are called decimals.

Each dollar contains 100 cents. In fact, the word “cent” means 100. Think of a century: it has 100 years in it. All decimals are based on the number 10 and can be referred to as base 10. These numbers include 10, 100, 1000, 10000, and so on. If we were to relate this to fractions, we would be dealing with fractions that have a denominator of 10, 100, 1000, 10000, and so on.

Although we only have two decimal places to worry about when dealing with money, we can continue a decimal on for as long as we want. For instance, the number pi ( $3.14$  or  $\pi$ ) actually continues on forever and has no ending. Check it out:

3.141592653589793238462643383279502  
 88419716939937510582097494459230781  
 64062862089986280348253421170679821  
 48086513282306647093844609550582231  
 72535940812848111745028410270193852

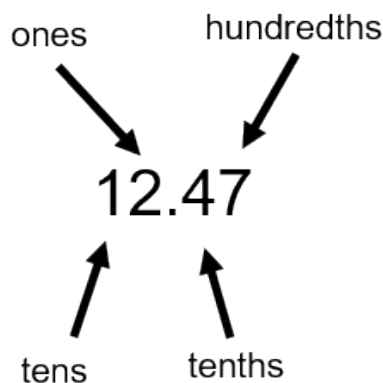
This just shows pi to the first few dozen decimal places, but there are lots more.

What we need to do is get an understanding of the decimal system and how it works. Take a look at the following number.

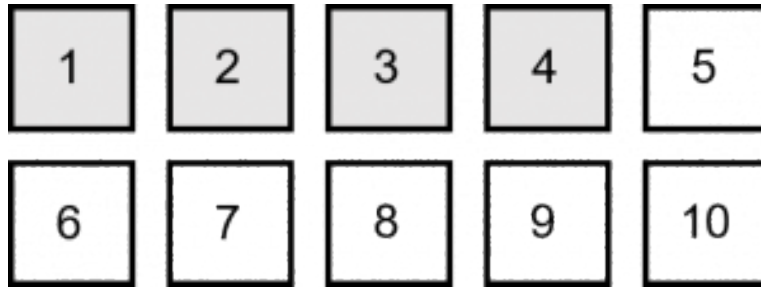


Back in Chapter 1, we dealt with the place value system for whole numbers (see [The Place Value System](#)). Decimals follow a similar pattern that starts with the first number to the right of the decimal. In our example of 5.9, the pattern would begin with the 9.

Back to Chad. In Chad’s lecture, he stresses the importance of detailed accounting when dealing with your own business. Now, we all know that there are 100 cents in a dollar, and making sure that you get the cents correct when accounting is important. Rounding up or down to the nearest dollar and being “close enough” won’t cut it. Exact dollar and cent amounts are important, so Chad deals with numbers that go to the second decimal place. Take a look at the number below (which we could assume represents dollars and cents) to see what places the digits are in before and after the decimal place. Let’s use the hammer example of \$12.47.



Think of the 0.47 as it relates to money. It would represent 47 cents out of a possible 100 cents. What does the 4 represent by itself? Well, in terms of a dollar, it refers to 40 cents, or 40 out of 100. This is all good, except for the fact that the 4 is actually in what is referred to as the tenths place. What the number is indicating is that we have 4 out of a possible 10 parts. The tenths place can be looked at as being split up into 10 parts, and we have 4 full parts.



The next step here is to visualize what the 7 is all about. The 7 is in the hundredths place, which means that we have 7 out of a possible 100.

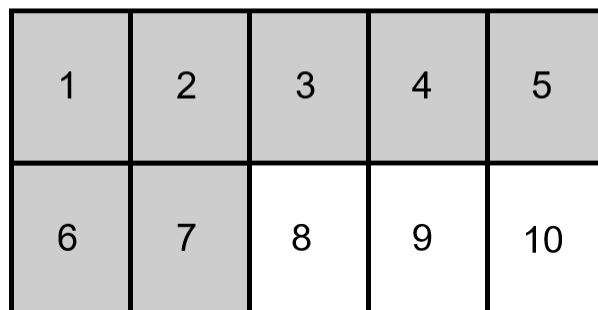
The idea here is to take each box and break it into even smaller parts. The first four boxes are already shaded grey because they represent 4 complete boxes or 40 cents. What we do next is take the fifth box and break it into 10 parts.

**Question:** If we were to break all 10 boxes into 10 small parts, how many would we have?

$$10 \times 10 = 100$$

We would end up with 100 parts, or we could look at it like 100 cents. Now, take the fifth box out, divide that box into 10 parts, and fill up 7 of them.

### Box 5



Imagine that each of the original 10 boxes are split into 10 even parts. What gets crazier is that we could then take each of those 100 boxes and split them into another 10 parts each. What this would leave us with is a number that takes us to the thousandths place, because we would be splitting things up into 1000 parts. What we are really doing with decimals is taking a whole number and breaking it down into parts, similar to fractions. In fact, if we were to look at decimals like fractions, this is how we would think about it:

$$\frac{1}{10} = 0.1$$

$$\frac{1}{100} = 0.01$$

$$\frac{1}{1000} = 0.001$$

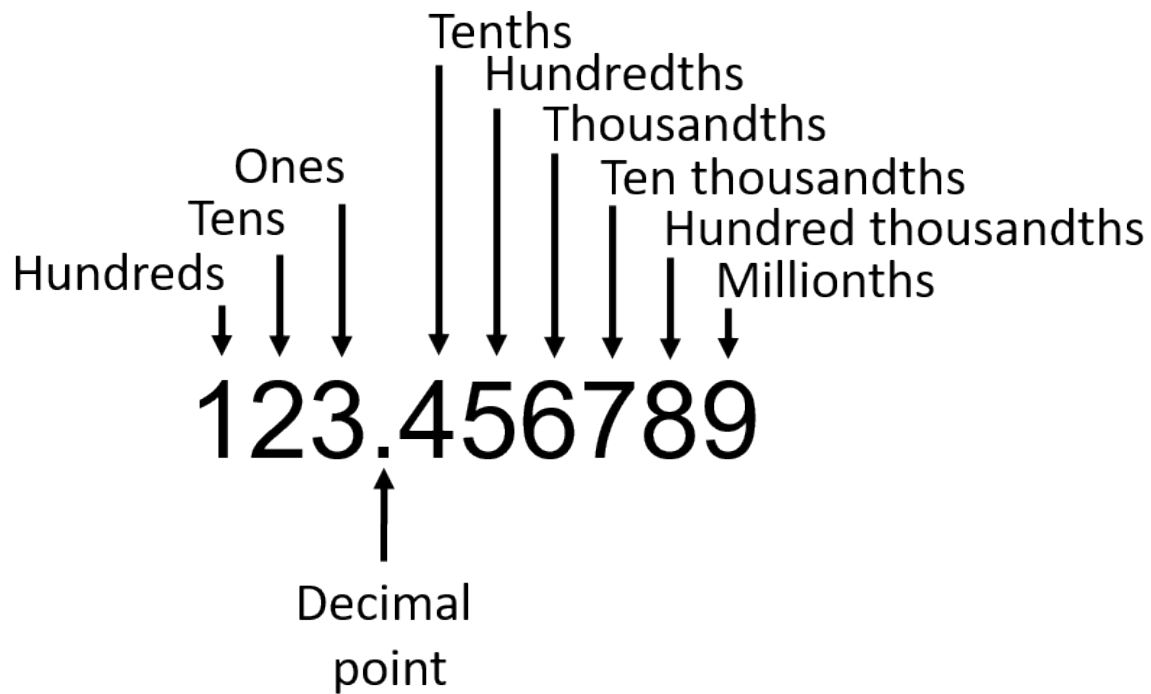
Now that we have the first three digits after the decimal point figured out, let's work our way even further. Take a look at the number below. To the left of the decimal point is the whole number, and to the right is the decimal or part of a whole number.

**123.456789**

Note: 1, 2, and 3 are all part of the whole number, and 4, 5, 6, 7, 8, and 9 are all part of the decimal. Using the place value system for both whole numbers and decimals, we would end up with the following:

- 1 = hundreds place
- 2 = tens place
- 3 = ones place
- (.) = decimal
- 4 = tenths place
- 5 = hundredths place
- 6 = thousandths place
- 7 = ten thousandths place
- 8 = hundred thousandths place
- 9 = millionths place





A number with all its digits categorized according to the place value system.

Let's go through a couple quick examples, just to make sure we are all on the same page.

### Example

In the number below, indicate which digit is in the thousandths place.

**57.29652**

The easiest way to do this is to write out the number, and then, starting from the left, indicate the place value of each of the digits.

- 5 = tens place
- 7 = ones place
- 2 = tenths place
- 9 = hundredths place
- **6 = thousandths place**

- 5 = ten thousandths place
- 2 = hundred thousandths place

The 6 is in the thousandths place.

Try another example.

### Example

In the number below, indicate which digit is in the hundred thousandths place.

**369.246813**

Once again, to find the answer, write down the number, and then indicate the place value of each of the digits.

- 3 = hundreds place
- 6 = tens place
- 9 = ones place
- 2 = tenths place
- 4 = hundredths place
- 6 = thousandths place
- 8 = ten thousandths place
- **1 = hundred thousandths place**
- 3 = millionths place

The 1 is in the hundred thousandths place.

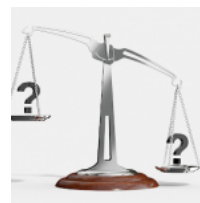
13.

## COMPARE AND ROUND DECIMALS

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### Compare Decimals

Chad wants to make sure that all his students understand the value of money, and an important aspect of that is knowing how to accurately determine the sizes of decimals. Take a look at the following four decimals.



- a. 8.16
- b. 8.151
- c. 8.1513
- d. 8.15138

Which of these is the largest? To answer the question, you would have to compare decimals.

At first glance, you might say the largest number is 8.15138, as it has the most digits and may look like the largest. But if we referred back to the chapter on [Understanding Decimal Notation](#), we would remember that, as you go further to the right from the decimal point (.), the numbers become smaller. So in the example 8.15138, the 3 near the end is worth far less than the 5 two spots to the left of the decimal point.

8.15138



Worth more  
than the 3

If we followed this logic, then the largest number of the four would be 8.16, even though it has the fewest digits. When comparing decimals, we really have to start with the whole number. In this situation, it's 8 in all four answers, so it doesn't come into play.

Once we have decided that the whole number is equal in all four cases, the next thing we have to look at is the number directly to the right of the decimal. This is the tenths place and is worth the most in decimals. Once again, all four answers give us a value of 1 in the tenths column, so this will not be the determining factor.

Move one space to the right again and you get the hundredths place. All the numbers contain a 5, except the first number, where we have a 6. Therefore, this is the defining place value.

All four whole  
numbers are equal



8.16

8.151

8.1513

8.15138



All four values in  
the tenths place are  
equal

The deciding factor  
becomes the  
hundredths place.

I'll go through another example, and then you can try a couple on your own.

### Example

Which of the following is the largest number?

- a. 7.34
- b. 7.332
- c. 7.323
- d. 7.3234

Following the same pattern as before, we would go through the following steps.

**Step 1:** Start with the whole number. Which number has the largest whole number? As they are all equal, this will not be the determining factor.

**The whole number in all four cases is a 7. This is not the deciding factor.**

**Step 2:** Go to the first digit to the right of the decimal. It works out that the digit is a 3 in all cases. Therefore, this is not the deciding factor, either.

**The number in the tenths column in all four cases is a 3. This is not the deciding factor.**

**Step 3:** Go one place to the right again. This would be the hundredths place. Here, we find the first difference. Answer a gives us the largest number in this column. Therefore, answer a is the largest number.

**The digit in the hundredths column is different for each number. In this case, the number with the highest value in the hundredths column is the largest number.**

Therefore, the largest number of the four is:

**7.34**

## Practice Questions

Now try a couple for yourself and check out the video answers.

Find the largest number.

### Question 1

- a. 27.5
- b. 27.54
- c. 27.539
- d. 27.5392

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## Question 2

- a. 14.001
- b. 13.099
- c. 13.999
- d. 13.001

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**BRAIN BREAK**

**Take a couple moments to review what you just learned before moving on.**

## Rounding Decimals



During the course, Chad talks about the importance of money and how to work effectively with it, especially when it comes to bookkeeping. Making sure that you track business transactions down to the cent (or decimal). As we have 100 cents in a dollar, if we were to round accounting totals, we would be rounding to the nearest 10 cents (tenths) or even to the nearest dollar (ones).

Before we get to rounding decimals, we should start off with rounding in general. The students are asked to count fittings on a table.



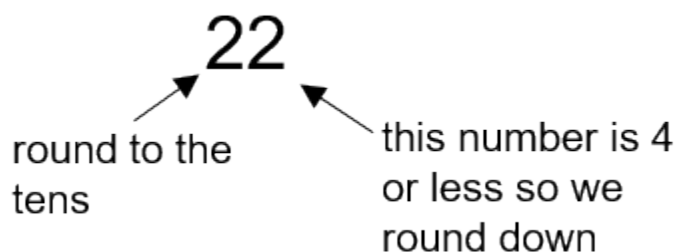
**How many did you count?**

There are a total of 22 fittings on the table, and the students are now asked to round the number of fittings to the nearest 10. This would mean rounding up or down to 10, 20, 30, 40, and so on. This example may seem straightforward, as we are much closer to 20 than to 30, and if you answered 20, then you would be correct. But what is the rule that we follow?



**RULE:** Find the place value we are rounding (in this case, the tens position). Look at the digit to the right of that number (in this case, the ones place). If that number is 4 or less, we round down. If that number is 5 or greater, we round up.

In our example with the 22 fittings, we have a 2 in the ones column, so we end up rounding down to 20. If the number in the ones column were 5 or greater, we would round up to 30.

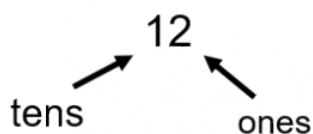


Let's go through a couple of quick example questions before we move on.

Round the following numbers to the nearest 10. Remember your place value system here.

**Questions**

Remember



- a. 12
- b. 27
- c. 45
- d. 98

## Answers

- a. 12 rounds down to 10
- b. 27 rounds up to 30
- c. 45 rounds up to 50
- d. 98 rounds up to 100

Check out the answer for d. When rounding the tens column, we end up rounding up. We run out of room in the tens column, as the highest round number we can make with it is 90. This forces us to move into the hundreds column and start with 1, making the answer 100.

Now that we have rounding whole numbers solved, let's move onto rounding decimals. Rounding decimals follows the same principles. It's important here to remember the place value system, not only for whole numbers, but also for decimals. Note the following rule when dealing with numbers:

43729.725429  
←  
getting larger

As you go from right to left in a number, including the decimals, the value of the digits becomes greater. When dealing with decimals, this principle means that, as you go further to the right from the decimal, you are dealing with smaller and smaller portions of a whole number.

Start with an example.

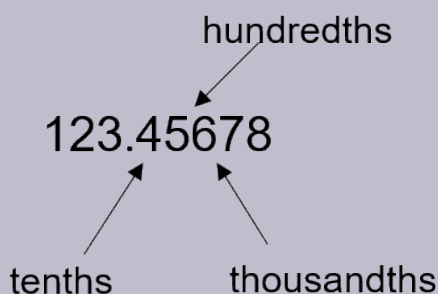
Example

Round the following number to the nearest hundredth.

123.45678

**Step 1:** Identify which number is in the hundredths column.





**Step 2:** Look at the number directly to the right of the hundredths column. In this case, it would be the digit in the thousandths column, which is 6.

**Step 3:** Follow the rules of rounding.

**RULE:** If the number is 4 or less, round down. If the number is 5 or greater, round up.

As we have 6 in the thousandths column, we round up.

Therefore, 123.4567 rounded to the nearest hundredth is:

**123.46**

## Practice Questions

Round the following decimals. Make sure to heed the place value system when answering the questions. Check the video answers to see if you are correct.

### Question 1

Round to the nearest tenth.

**27.71536**

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### Question 2

Round to the nearest thousandth.

**14.75638**

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### Question 3

Round to the nearest hundredth. This is a new concept. Before checking the video answer, give this one a try and see how you do.

**75.9999**

[https://video.bccampus.ca/id/0\\_77elprhp?width=608&height=402&playerId=23448552](https://video.bccampus.ca/id/0_77elprhp?width=608&height=402&playerId=23448552)

14.

## CONVERT DECIMALS TO FRACTIONS

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Chad has decided he wants to work with money in fraction form and not in decimal form. Why, you ask? I don't know. Work with me here, though, because it's been hard to get Chad into the decimal story. Maybe he just wants his students to know a little more math. No harm there. What we're looking at here is the following:

$$0.5 \text{ to } \frac{1}{2}$$

It's important to know how to do this in order to work with different number forms in the workplace. For instance, David, a student in Chad's class, is a machinist. Machinists produce metal parts with a precision of up to one one-thousandth of an inch. If you are continuously working in decimals, you might forget how to work with fractions, so David wants to know how to change those decimals into fractions. The best way to illustrate how to do this is simply with an example.

### Example

We'll start with an easy decimal like 0.25 and work towards changing this into a fraction.

**Step 1:** Put the question into fraction form such that the decimal is over 1.

$$\frac{0.25}{1}$$

**Step 2:** Take both the numerator and the denominator and multiply each by 10 for every digit to the right of the decimal point. In this case, we have 2 digits to the right. Therefore, we multiply each by 100 ( $10 \times 10$ ).

$$\frac{0.25}{1} \times \frac{100}{100} = \frac{25}{100}$$

**Step 3:** Reduce the fraction to its lowest terms, and we end up with the final answer.

$$\frac{25}{100} \div \frac{25}{25} = \frac{1}{4}$$

### Example

Change the decimal 0.729 into a fraction.

**Step 1:** Put the equation into fraction form such that the decimal is over 1.

$$\frac{0.729}{1}$$

**Step 2:** Take both the numerator and the denominator and multiply each by 10 for every digit to the right of the decimal point. In this case, we have 3 digits to the right. Therefore, we multiply each by 1000 ( $10 \times 10 \times 10$ ).

$$\frac{0.729}{1} \times \frac{1000}{1000} = \frac{729}{1000}$$

**Step 3:** Reduce the fraction to get our final answer.

This is a tough one to reduce. My suggestion would be to do this in steps. Start with small numbers like 2 and 3. Does the number 2 go into both the numerator and the denominator? The answer would be no. How about 3? Also no. Continue this process until you find a number that works.

It is possible that you will not find a number and we are already in lowest terms. This is the case here. Our final answer is simply:

$$\frac{729}{1000}$$

## Practice Questions

Try a couple practice questions and check the video answers.

Change the following decimals into fractions. Put your answer in lowest terms.

Question 1

**0.362**

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Question 2

**0.963**

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## BRAIN BREAK

Take a couple moments to review what you just learned before moving on.



Now it's time to move to a situation we might find on a job site. Have you ever been on a job site, and your boss asked you to cut a piece of pipe, or maybe a piece of wood? What was the measurement? Maybe it was something like  $8 \frac{3}{8}$  inches. Maybe it was 2 feet,  $2 \frac{1}{4}$  inches. Those are pretty standard.

But what if you were asked to cut a piece of material 2.9384 feet? For David, the machinist, this might be exactly the type of measurement that he would get while creating metal products. But what would you do in some of the other trades?

This is the reason we need to be able to change decimals into fractions. The whole number in this example stays the same, so we don't really need to deal with that. What we need to work with is the 0.9384.

The goal is to change 2.9384 into feet, inches, and fractions of an inch. We'll go through an example to show you how that is done.

### Example

**Step 1:** First, note that the number we are dealing with is in feet. We have 2.9384 feet. The 2 whole feet will not require any changing and is good the way it is. What we have to do first is change 0.9384 into inches before moving onto fractions of an inch. Start with how many inches there are in a foot.

$$1 \text{ foot} = 12 \text{ inches}$$

Take the decimal of a foot and multiply it by 12.

$$0.9384 \times 12 = 11.2608 \text{ inches}$$

**Step 2:** We are left with 11 inches and decimals of an inch. The 11 inches is good the way it is, but

we need to change the decimal of an inch into fractions of an inch. The question becomes, which fraction of an inch should we change it into?

This is up to you. This could be fourths, eighths, sixteenths, or thirty-seconds. The most common way would be to change the decimal of an inch into sixteenths. This is done by multiplying the decimal of an inch by the number in the denominator of whichever fraction you are working towards. For our example, we are looking to convert the decimal into sixteenths, so we multiply it by 16.

$$0.2608 \times 16 = 4.1728$$

When we are looking at 4.1728, what we are actually looking at is:

$$\frac{4.1728}{16}$$

If we had taken the 0.2608 inches and multiplied it by 8, our answer would be in eighths of an inch. If we had multiplied by 4, our answer would be in fourths (or quarters) of an inch.

**Step 3:** Round the answer to the nearest fraction of an inch, and then reduce the fraction if necessary.

$$\frac{4.1728}{16} \rightarrow \frac{4}{16} \rightarrow \frac{1}{4}$$

Our final answer then becomes:

$$2' \ 4 \frac{1}{4}''$$

Example

Change the following feet and decimals of a foot into feet, inches, and sixteenths of an inch.

**7.6939 feet**

**Step 1:** Change the decimal of a foot into inches.

$$0.6939 \times 12 = 8.3268 \text{ inches}$$

**Step 2:** We are left with 8 inches and decimals of an inch. The 8 inches is good the way it is, but we need to change the decimal of an inch into fractions of an inch. We need to change the decimal of an inch into sixteenths, so multiply it by 16.

$$0.3268 \times 16 = 5.2288$$

**Step 3:** Round the answer to the nearest fraction of an inch, and then reduce the fraction if necessary.

$$\frac{5.2288}{16} \rightarrow \frac{5}{16}$$

Our final answer becomes:

**7' 8 5/16"**

## Practice Questions

Change the following feet and decimals of a foot into feet, inches, and sixteenths of an inch. Put your answer in lowest terms and check out the videos answers when you are done.

Question 1



**9.1234 feet**

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Question 2

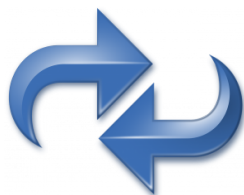
**0.058 feet**

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15.

## CONVERT FRACTIONS TO DECIMALS

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We just learned how to change fractions to decimals, and now it's time to do the opposite. How do you think this is done? If you guessed that it's the reverse of what we just did, then you are right. We'll start by going through an example dealing with fractions to decimals, and then move onto changing feet, inches, and fractions of an inch to feet and decimals of a foot.

What we are looking to do is go from:

$$\frac{1}{2} \text{ to } 0.5$$

Once again, the quick and easy way to do this is to go through a few examples. Start with a plain old fraction and work towards changing this into a decimal.

$$\frac{3}{8}$$

This can be done a number of ways, with the simplest being:

$$3 \div 8 = 0.375$$

You could plug the numbers into your calculator, or you could use long division to come up with the answer.

That's it. You're done!



Okay, that was pretty easy, but now let's move on to an example that is a bit more difficult. We are going to change a number written in feet, inches, and fractions of an inch into a decimal.

## Example

Change this number into a decimal:

Seven feet nine and five sixteenths inches

OR

$$7' 9\frac{5}{16}''$$

**Step 1:** Identify any part of the number that is already in a form that works. In this case, the 7 feet is good to go and nothing needs to be done with it.

7'



**Step 2:** Take the fraction of an inch and turn it into a decimal.

$$\frac{5}{16} \text{ OR } 5 \div 16 = 0.3125$$

Pause here for a moment and ask yourself what unit the 0.3125 is in. All we have done is change the fraction of an inch into decimals of an inch. We now have turned the 9 inches and fractions of an inch into 9 inches and decimals of an inch.

## $9\frac{5}{16}$ inches into 9.3125 inches

**Step 3:** Change the inches and decimals of an inch into decimals of a foot using the fact that 1 foot = 12 inches.



In the last section, when we went from decimals to fractions, we multiplied by 12 to get inches. As we are doing the reverse calculation here by going from fractions to decimals, we end up dividing by 12 inches to get decimals of a foot.

$$9.3125 \div 12 = 0.776$$

Once again, ask yourself what units you end up with here. When we take the inches and divide it by 12, we end up with feet or, in this case, decimals of a foot. So our final answer becomes:

$$7' 9\frac{5}{16}'' = 7.776'$$

We'll go through another example, and then let you try a couple on your own after that.

### Example

Change the following number written in feet, inches, and fractions of an inch into feet and decimals of a foot.

$$3' 6\frac{3}{8}''$$

**Step 1:** Identify any part of the number that is already in a form that works. In this case, the 3 feet is good to go and nothing needs to be done with it.

3'



**Step 2:** Take the fraction of an inch and turn it into a decimal.

$$\frac{3}{8} \text{ OR } 3 \div 8 = 0.375$$

Put the inches and decimals of an inch together.

$$6\frac{3}{8} \text{ inches into } 6.375 \text{ inches}$$

**Step 3:** Change the inches and decimals of an inch into decimals of a foot using the fact that 1 foot = 12 inches.

$$6.375 \div 12 = 0.531$$

So our final answer becomes:

$$3' 6\frac{3}{8}'' = 3.531'$$

## Practice Questions

Change the following feet, inches, and fractions of an inch into feet and decimals of a foot. Check the video answers to make sure you are on the right track.

### Question 1

$$2' 2 \frac{17''}{32}$$

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### Question 2

$$1' 11 \frac{15''}{16}$$

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16.

## DECIMALS QUIZ

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*An interactive or media element has been excluded from this version of the text. You can view it online here:*

<https://pressbooks.nsc.ca/mathtrades/?p=206>

If using the print, PDF, or eBook copy of this book, navigate to the above link to complete the quiz. However, the quiz questions are also provided in [Appendix C](#) at the end of the book for offline use.





## PART V

# PERCENTAGES



### Outcomes

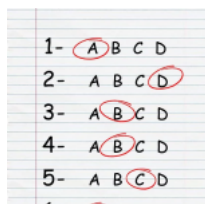
- Define percent
- Understand the similarities between percent, ratios and fractions
- Calculate the percent of a number
- Calculate change as a percent and percent change



17.

## WHAT IS A PERCENT ANYWAY?

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Remember the last time you wrote a test? What percentage did you receive? If you got 30 out of 35, could you calculate the percentage? How about finding out the percentage of students who like math? How would you calculate that number?

Percentages are used in many different areas of our lives. For example, we hear about the interest rate or unemployment rate, the percentage of people who prefer hockey over football or the percentage of people who prefer science over math. Percentages are a useful way of making comparisons. This chapter will deal with percentages: how we get them, and how we can use them.

## What Exactly is a Percent?



The word “per cent” means “out of 100.” In math language, when we say “out of” we are really saying “divided by.” Therefore, when we are dealing with percent, the basis of it is to divide by 100. The word “cent” on its own refers to 100. The word cent is actually Latin for 100. This relates to many things. The easiest might be the word century, which means 100 years.

You can also think of words like centimetre. There are 100 centimetres (cm) in 1 metre. Or, how about the word cent when it comes to money? In money, 100 cents equals 1 dollar.

Are you beginning to see the trend here?

Now, think about the last time you wrote a test out of 100. Let’s say you scored very well and scored 92 out of 100. That would be the same as saying you received 92 percent.

Written mathematically, it looks like this:

92%





The symbol % means percent and can be used in place of writing the actual word percent. You can find this symbol on your computer keyboard, generally on the same key as one of the numbers. On my keyboard, the percent symbol is on the same key as the number 5. To get the % symbol, you would press the [Shift] and [5] keys at the same time.

The following is an example of percent.

According to a recent survey by BCIT Piping Foundation Instructors, 25% of students coming into the Piping Foundation Program would like to get into the plumbing trade, 20% would like to get into the steamfitting trade, and 10% would like to get into the gasfitting trade. The remaining 45% of students are undecided when they begin the program. If you add up those percentages, they equal 100.

**Percentage of students indicating which trades they would like to pursue**

<b>Piping Trade</b>	<b>Percentage</b>
Plumbing	25%
Gasfitting	10%
Undecided	45%
Total	100%

What the numbers reveal is that if you took 100 students and asked them which piping trade they were looking to get into, on average, 25 would say plumbing, 20 would say steamfitting, 10 would say gasfitting, and 45 would say that they are undecided. **(Note: These are generalizations, not actual facts.)**



Stop here for one second. Does that mean that every time you have a survey, and you are looking to find percentages, you need to have 100 participants?

No, it doesn't.

We'll go through that in just a bit, but for now just know that percentages are based on 100 but don't necessarily reflect that we have 100 people, items, or questions, etc.

18.

## RATIOS AND FRACTIONS AND HOW THEY RELATE TO PERCENTAGE

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What does a ratio have to do with percentage? What does a fraction have to do with percentage? What is a ratio? These are all good questions, and we'll answer them here. Percentages, ratios, and fractions are all very similar and can be used to represent numbers in different ways, but with similar outcomes.

We already went through fractions in a past chapter, so we know what those are all about. We should take a bit of time to talk about ratios and what they are.

Mathematically speaking, a ratio is a relationship between two numbers. If we were to once again order a pizza and eat 3 of the 8 slices, we could look at that as a ratio. We ate three-eighths of the pizza. Written as a ratio, this looks like:

$$3 : 8$$

Notice how the ratio is written. It has its own style, just like a fraction does. A ratio of 3:8 means that we have eaten 3 of the 8 pieces in the pizza. We could also write a ratio that identified how many pieces of the pizza were eaten and how many were not eaten. The ratio for that would look like:

$$3 : 5$$

In this case, 3 pieces of the pizza were eaten while 5 pieces were not. Regardless of which way we describe our pizza eating, what we are dealing with is the relationship between two numbers.

Here is another example: let's say we are on a job site, and we have to install some pipe. We have 42 feet of plastic pipe and 79 feet of steel pipe to install. What is the ratio of plastic pipe to steel pipe that we need to install?

The answer:

$$42 : 79$$

Another question might be how much of the pipe is plastic in relation to how much total pipe we have. In this case, the ratio would look like:

$$42 : 121$$

In this case, the 121 is derived from adding the plastic pipe and the steel pipe lengths together.

$$42 + 79 = 121$$

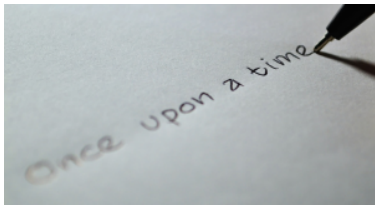
At this point, you might be thinking that this looks and sounds familiar, and you would be correct. Ratios are similar to fractions, and each ratio can be written as a fraction.

$$\frac{42}{121}$$

We would say that 42 feet out of a total of 121 feet of pipe is plastic.

All right: we've added a few new things to our math library here, but you might be asking right about now, "Aren't we dealing with percentages in this chapter? How does all this ratio and fraction stuff relate to percentages?"

The idea is we can take these ratios or fractions and turn them into percentages by making the ratio or fraction out of 100. In truth, we don't even need to make a ratio or fraction out of 100 to do this, but it's a good place to start our understanding of the math behind the whole process.



We are at a point where we can finally introduce you to some new people in the story. For the story of percentage, we're going to use apprentices from carpentry, electrical, and plumbing, in addition to statistics.

**Note: all the statistics are just made up.**

A trades college teaches many trades, including carpentry, electrical, and plumbing. There are currently 97 carpentry apprentices, 123 electrical apprentices, and 80 plumbing apprentices at the college.

The college likes to keep statistics on apprentices, including what percentage of all the apprentices each of the three trades holds. The question is, how do we go about this?

The first thing we should do is add up the total number of apprentices in each of the three trades.

$$97 + 123 + 80 = 300$$

What we can do now is put each of the apprentice totals into both a ratio and a fraction of the whole.



Carpentry

97:300

$\frac{97}{300}$

Ratio

Fraction



## Electrical

123:300

$$\frac{123}{300}$$

Ratio

Fraction



## Plumbing

80:300

$$\frac{80}{300}$$

Ratio

Fraction

If percentages are based on 100, then we have to translate the ratio and the fraction into forms based on 100. In other words, we have to make it so that the number on the right in the ratio is 100 and the denominator in the fraction is 100. For this, we have to go back to our work with fractions.

We'll start with the 97 carpentry apprentices. We have 97 apprentices out of a total of 300. Our goal here is to get this fraction down to a point where the denominator is 100.

$$\frac{97}{300} \longrightarrow \frac{?}{100}$$

We are essentially reducing the fraction. Luckily for us, going from 300 down to 100 is quite easy. We just divide the denominator by 3, and then also divide the numerator by 3.

$$\frac{97}{300} \div 3 = \frac{32.33}{100}$$

We could now write this number as a ratio.

$$32.33 : 100$$

What we end up with is the fact that 32.33% of the trades apprentices are carpentry apprentices. Once we find out the number of carpentry students per 100 students, we automatically have our percentage. In just a bit, we'll talk about working with numbers that don't give us a nice even number of 100 and simply translate to percentage in one step. But for now, let's continue on with our apprentices.

### Example

Find the percentage of electrical students.

**Step 1:** Put the numbers into an equation that we can work with. In this case, put the numbers into a fraction.

$$\frac{123}{300}$$

**Step 2:** Turn the fraction into one with a denominator of 100.

$$\frac{123}{300} \div 3 = \frac{41}{100}$$

As a ratio, we would have:

$$41 : 100$$

And finally, as a percentage, we would get:

$$41$$



## Practice Question

Now calculate the percentage of plumbing apprentices. Check the video to see if your answer is correct.

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**BRAIN BREAK**

**Take a couple moments to review what you just learned before moving on.**



The next step actually makes the whole process of finding percentages easier. Remember that “percent” really means out of 100, and in the last section, we took our numbers and made them into fractions with denominators of 100. In this section, we skip that whole procedure and just take the numbers we have and work with them as they are.

We’ll go back to our apprentices for this. Out of the total of 97 carpentry apprentices, there are 57 with a grade of “A” or “B.” What percentage of them have an “A” or “B”? We could go back to our old ways and get to a fraction with a denominator of 100, or we could simply divide the 57 by the 97.

Mathematically, it looks like this:

$$57 \div 97 = 0.59$$

Now take the 0.59 and turn that into a percentage by multiplying the 0.59 by 100. This moves the decimal point 2 spots to the right and leaves us with a whole number. It’s as simple as that.

$$0.59 \times 100 = 59$$

## Example

Using the same method we just used with the carpentry apprentices, find the percentage of plumbing students in the trades college.

**Step 1:** Write down the equation in a format we can use.

$$80 \div 300 = ?$$

**Step 2:** Work this out in your head. Just kidding: grab your calculator and plug the numbers in.

$$80 \div 300 = 0.267$$

**Step 3:** Multiply the answer by 100 to put it into a percentage.

$$0.267 \times 100 = 26.7$$

## Practice Questions

Try these practice questions. You can either change each number into a fraction with a denominator of 100, or you can divide one number by the other. The choice is yours, but the video answers will go through both ways.

### Question 1

The electrical students are wiring an electrical project as part of their practical mark. The students must receive a minimum of 35 out of a possible 50 marks. What percentage is that?

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### Question 2

The plumbing students have been asked to move a bunch of pipe from one side of the shop to the other. There are 279 pieces of cast iron pipe that have to be moved. At the end of the day, they have moved 222 pieces. What percentage of the pipe have they moved?

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### Question 3

Referring to question 2, what percentage of the pipe do the students still have to move in relation to the original amount?

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19.

## FINDING THE PERCENTAGE OF A GIVEN NUMBER

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It's Christmas, and the apprentices have a chance to win the jar of candy on the left. The only requirement is that they guess the number of candies in the jar.

**Correct answer = 117**

The winner is Bryce, a level 4 plumbing student. He credits his success in the candy counting game to the fact that his family owned a candy store when he was a kid, and it's finally paid off.

Bryce decides to give some of the candy away. In fact, he decides to give 12% to his friend Patrick, 25% to his friend Matt, and 17% to his instructor.

The question becomes, how much do each of these percentages represent out of the total number? And maybe in the end, we can find out how much candy Bryce will keep for himself.

We'll start with Patrick, who will get 12% of the candy. The first thing to do is turn 12% into a fraction.

$$\frac{12}{100}$$

What this will eventually tell us is that, for every 100 candies there are in the jar, Patrick will get 12 of them. Now turn that fraction into a decimal by dividing 12 by 100.

$$12 \div 100 = 0.12$$



Have you noticed a pattern when it comes to decimals? Here is the relationship showing a few different ways in which we can end up with 12%:

$$12\% = \frac{12}{100} = 0.12$$

All three of these numbers represent the same amount. Learning to work between them is important in math, and it's also important to begin to see the relationships between numbers. Okay, back to Patrick.

The final step in this situation is to take the 0.12, which is really 12%, and multiply it by the number of candies in the jar.

$$0.12 \times 117 = 14.04$$

Now, slicing off 0.04 of a candy is hard to do, so we'll use our rounding skills and round down to 14 candies.

### Example

Now move on to Bryce's next friend, Matt. Matt gets 25% of the candy.

**Step 1:** Turn the percentage into a fraction with a denominator of 100.

$$\frac{25}{100}$$

**Step 2:** Take the 25 and divide it by the 100.

$$25 \div 100 = 0.25$$

**Step 3:** Take the 0.25 and multiply it by the number of candies in the jar.

$$0.25 \times 117 = 29.25$$

We would again round down to get our final answer of 29.

### Example

We won't worry about how many candies the instructor gets, but what we will calculate is how many candies Bryce keeps for himself. We want to calculate the percentage of candies Bryce ends up with, and then calculate how many candies that is.

**Step 1:** Add up the percentage of the candies that the other three people have, and then subtract the sum from 100.

- Patrick gets 12%
- Matt gets 25%
- Instructor gets 17%

$$12\% + 25\% + 17\% = 54\%$$

$$\text{Bryce gets } 100\% - 54\% = 46\%$$

**Step 2:** At this point, we've done a couple of examples in which we first took the percentage and turned it into a fraction over 100. Then, we took the percentage and divided it by 100. What if we just skipped that step? What we end up doing for a percentage is just taking the decimal point and moving it over 2 places to the left (this represents dividing by 100). Try that.

$$46\% = 0.46$$

**Step 3:** Take the 0.46 and multiply it by the number of candies in the jar.

$$0.46 \times 117 = 53.82$$

This can be rounded up to 54. Bryce still gets to eat a lot of candy, which will satisfy his sweet tooth, but he won't eat so much that he'll end up getting sick.

## Practice Questions

Try a couple of practice questions. Check the video answer to see how you did.

Question 1

Find 37% of 229

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Question 2

Find 78% of 1928

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# CHANGE AS A PERCENT AND PERCENT CHANGE

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## Expressing Change as a Percent

Melissa is a carpentry foundation student who has just finished her six-month foundation program. This is a program for people who are looking to get into the trade but are not yet apprentices.



It's now time to make some money after spending the previous six months in school. After Melissa graduated from the program, she applied for three jobs and decided to take the one that offered the best value. Her starting wage is \$14 per hour, and after six months, it will move up to \$16 per hour.

This is a \$2 per hour pay raise, but does this represent a good percentage increase? If she started at \$25 per hour and went up to \$27 per hour, would this be the same

percentage increase?

The answer lies in the following formula:

$$\text{Percentage change} = \frac{\text{Actual increase or decrease}}{\text{Original amount}} \times 100$$

There are a couple of things to note here. One is that this formula works if you have either an increase or a decrease from the original amount. If Melissa's wage went down, you could also express that as a percentage. The second thing to note is that there is the number 100 at the end of the formula. This is due to the fact that the calculation would give us a decimal, and multiplying this number by 100 makes it a percentage.

So, to follow through on our calculation, let's put the numbers in and see what we get.

$$\begin{aligned} \text{Percentage change} &= \frac{\$2}{\$14} \times 100 \\ \text{Percentage change} &= 0.1429 \times 100 \\ \text{Percentage change} &= 14.29\% \end{aligned}$$

This is telling us that Melissa will get a 14.29% increase in her wage after the first six months. Not bad!



## Example



During her foundation program, Melissa had to keep her job working at a coffee shop in order to pay the bills. On her first day of work, she served 78 cups of coffee. During her six months on the job, the most cups of coffee she made in one day was 201. How can the difference between these amounts be expressed as a percentage increase?

**Step 1:** Calculate the increase in cups served.

$$201 - 78 = 123 \text{ more cups of coffee}$$

**Step 2:** Write down the formula you are going to work with.

$$\text{Percentage change} = \frac{\text{Actual increase or decrease}}{\text{Original amount}} \times 100$$

**Step 3:** Plug the numbers into the formula.

$$\text{Percentage change} = \frac{123}{78} \times 100$$

**Step 4:** Work through the answer.

$$\begin{aligned} \text{Percentage change} &= 1.58 \times 100 \\ \text{Percentage change} &= 158\% \end{aligned}$$

This answer tells us that, from the time Melissa started to the time she had her most productive day, she had a one-day increase of 158%. If she were to increase her output by 100%, that would mean mathematically that she would be making twice as much coffee as before. Increasing by 158% indicates that she is making MORE than twice the amount of coffee as when she first started.

## Practice Questions

Answer the questions below and then check the video answers to see how you did.

### Question 1

In its first year of operation, a trade school has 57 students. In its second year of operation, its student body increased to 104 students. What percent increase is this?

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### Question 2

Referring back to the trade school in question 1, their third year of operation did not go so well. In their third year, the number of students went down to 86. What percent decrease is this from year two?

[https://video.bccampus.ca/id/0\\_4ow2w0um?width=608&height=402&playerId=23448552](https://video.bccampus.ca/id/0_4ow2w0um?width=608&height=402&playerId=23448552)

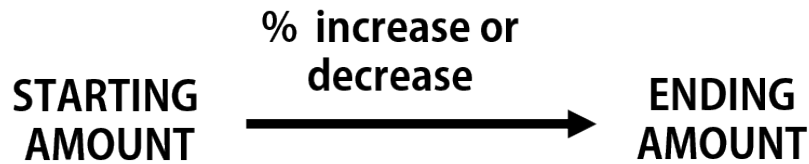
**BRAIN BREAK**

**Take a couple moments to review what you just learned before moving on.**

## Calculating Percentage Changes

The company Melissa works for currently has 27 apprentices working, and they want to increase that number in the coming year. In the previous section, we would have asked how many more apprentices they were looking to hire. But what if the goal was to increase the number of apprentices by 20%? How would you go about doing that?

So what we have is this:



We have to incorporate percent increase to find the number of apprentices the company wants to have in the end. The company currently has 27 apprentices and is looking to increase that number by 20% in the next year. The first thing we need to find out is what 20% of 27 is. Remember that 20% is like 20 out of 100. We can use this concept to find our answer.

$$\frac{20}{100} = 0.2$$

Multiply 27 by 0.2 to get our increase.

$$27 \times 0.2 = 5.4$$

What this answer indicates is that the company wants to increase their number of apprentices by 5.4. Now, we can't have 0.4 of an apprentice, so we have to round up or down. Following the rules we established before, we round down to 5.

In the end, the company wants to increase the number of apprentices by 5. But we are not quite done yet: we still have to calculate how many apprentices they want to have in the future. To do this, we simply add the 5 extra students to our starting number, and we end up with 32 students.

$$27 + 5 = 32$$

### Example



We're going to change it up here and use sports as our example.

A professional hockey team scored 219 goals in the 2018–2019 season and finished in 26th place in the league. In the 2019–2020 season, they want to improve on that total and are looking for a

25% increase in goal production. How many goals would they have to score to achieve their goal?

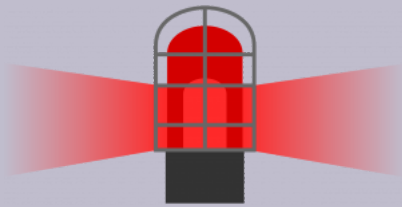
**Step 1:** Calculate 25% of 219.

$$\frac{25}{100} = 0.25, \text{ so } 219 \times 0.25 = 54.75$$

As we can't score 0.75 of a goal, we need to round this up to 1 complete goal and make it 55 more goals.

**Step 2:** Add 55 to our original number, and we get:

$$219 + 55 = 274 \text{ goals}$$



In the end, to achieve the 25% increase in goal production that they are looking for, the team would have to score 274 goals. If they scored that many in the 2018–2019 season, they would end up in the top 5 or 6 teams for goal production.

## Practice Questions

Answer the following questions and check the video answers to see how you did.

### Question 1

Melissa, the carpentry apprentice, likes to garden during the spring and summer. This year, she planted 42 plants. Next year, she wants to increase the total number of plants by 15%. How many plants will she need to plant in order to achieve her goal?

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### Question 2

Let's go back to sports for this one. A quarterback threw 47 touchdown passes in one year. The next year, the number of touchdown passes he threw dropped by 35%. How many touchdown passes did he throw that year?

[https://video.bccampus.ca/id/O\\_z2oxbpct?width=608&height=402&playerId=23448552](https://video.bccampus.ca/id/O_z2oxbpct?width=608&height=402&playerId=23448552)

21.

## PERCENTAGE QUIZ

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*An interactive or media element has been excluded from this version of the text. You can view it online here:*

<https://pressbooks.nsc.ca/mathtrades/?p=239>

If using the print, PDF, or eBook copy of this book, navigate to the above link to complete the quiz. However, the quiz questions are also provided in [Appendix C](#) at the end of the book for offline use.

## PART VI

# PRACTICE TEST



*An interactive or media element has been excluded from this version of the text. You can view it online here:*

<https://pressbooks.nsc.ca/mathtrades/?p=241>





# APPENDIX A: TIMES TABLES

---

## One times

$1 \times 1 =$	1
$1 \times 2 =$	2
$1 \times 3 =$	3
$1 \times 4 =$	4
$1 \times 5 =$	5
$1 \times 6 =$	6
$1 \times 7 =$	7
$1 \times 8 =$	8
$1 \times 9 =$	9
$1 \times 10 =$	10
$1 \times 11 =$	11
$1 \times 12 =$	12

## Three times

$3 \times 1 =$	3
$3 \times 2 =$	6
$3 \times 3 =$	9
$3 \times 4 =$	12
$3 \times 5 =$	15
$3 \times 6 =$	18
$3 \times 7 =$	21
$3 \times 8 =$	24
$3 \times 9 =$	27
$3 \times 10 =$	30
$3 \times 11 =$	33
$3 \times 12 =$	36

## Two times

$2 \times 1 =$	2
$2 \times 2 =$	4
$2 \times 3 =$	6
$2 \times 4 =$	8
$2 \times 5 =$	10
$2 \times 6 =$	12
$2 \times 7 =$	14
$2 \times 8 =$	16
$2 \times 9 =$	18
$2 \times 10 =$	20
$2 \times 11 =$	22
$2 \times 12 =$	24

## Four times

$4 \times 1 =$	4
$4 \times 2 =$	8
$4 \times 3 =$	12
$4 \times 4 =$	16
$4 \times 5 =$	20
$4 \times 6 =$	24
$4 \times 7 =$	28
$4 \times 8 =$	32
$4 \times 9 =$	36
$4 \times 10 =$	40
$4 \times 11 =$	44
$4 \times 12 =$	48

**Five times**

$5 \times 1 =$	5
$5 \times 2 =$	10
$5 \times 3 =$	15
$5 \times 4 =$	20
$5 \times 5 =$	25
$5 \times 6 =$	30
$5 \times 7 =$	35
$5 \times 8 =$	40
$5 \times 9 =$	45
$5 \times 10 =$	50
$5 \times 11 =$	55
$5 \times 12 =$	60

**Seven times**

$7 \times 1 =$	7
$7 \times 2 =$	14
$7 \times 3 =$	21
$7 \times 4 =$	28
$7 \times 5 =$	35
$7 \times 6 =$	42
$7 \times 7 =$	49
$7 \times 8 =$	56
$7 \times 9 =$	63
$7 \times 10 =$	70
$7 \times 11 =$	77
$7 \times 12 =$	

**Six times**

$6 \times 1 =$	6
$6 \times 2 =$	12
$6 \times 3 =$	18
$6 \times 4 =$	24
$6 \times 5 =$	30
$6 \times 6 =$	36
$6 \times 7 =$	42
$6 \times 8 =$	48
$6 \times 9 =$	54
$6 \times 10 =$	60
$6 \times 11 =$	66
$6 \times 12 =$	72

**Eight times**

$8 \times 1 =$	8
$8 \times 2 =$	16
$8 \times 3 =$	24
$8 \times 4 =$	32
$8 \times 5 =$	40
$8 \times 6 =$	48
$8 \times 7 =$	56
$8 \times 8 =$	64
$8 \times 9 =$	72
$8 \times 10 =$	80
$8 \times 11 =$	88
$8 \times 12 =$	96

**Nine times**


---

$9 \times 1 =$	9
$9 \times 2 =$	18
$9 \times 3 =$	27
$9 \times 4 =$	36
$9 \times 5 =$	45
$9 \times 6 =$	54
$9 \times 7 =$	63
$9 \times 8 =$	72
$9 \times 9 =$	81
$9 \times 10 =$	90
$9 \times 11 =$	99
$9 \times 12 =$	108

---

**Eleven times**


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$11 \times 1 =$	11
$11 \times 2 =$	22
$11 \times 3 =$	33
$11 \times 4 =$	44
$11 \times 5 =$	55
$11 \times 6 =$	66
$11 \times 7 =$	77
$11 \times 8 =$	88
$11 \times 9 =$	99
$11 \times 10 =$	110
$11 \times 11 =$	121
$11 \times 12 =$	132

---

**Ten times**


---

$10 \times 1 =$	10
$10 \times 2 =$	20
$10 \times 3 =$	30
$10 \times 4 =$	40
$10 \times 5 =$	50
$10 \times 6 =$	60
$10 \times 7 =$	70
$10 \times 8 =$	80
$10 \times 9 =$	90
$10 \times 10 =$	100
$10 \times 11 =$	110
$10 \times 12 =$	120

---

**Twelve times**


---

$12 \times 1 =$	12
$12 \times 2 =$	24
$12 \times 3 =$	36
$12 \times 4 =$	48
$12 \times 5 =$	60
$12 \times 6 =$	72
$12 \times 7 =$	84
$12 \times 8 =$	96
$12 \times 9 =$	108
$12 \times 10 =$	120
$12 \times 11 =$	132
$12 \times 12 =$	144

---

# Multiplication Chart

×	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	3	6	9	12	15	18	21	24	27	30	33	36
4	0	4	8	12	16	20	24	28	32	36	40	44	48
5	0	5	10	15	20	25	30	35	40	45	50	55	60
6	0	6	12	18	24	30	36	42	48	54	60	66	72
7	0	7	14	21	28	35	42	49	56	63	70	77	84
8	0	8	16	24	32	40	48	56	64	72	80	88	96
9	0	9	18	27	36	45	54	63	72	81	90	99	108
10	0	10	20	30	40	50	60	70	80	90	100	110	120
11	0	11	22	33	44	55	66	77	88	99	110	121	132
12	0	12	24	36	48	60	72	84	96	108	120	132	144

# APPENDIX B: ANSWERS TO PRACTICE QUESTIONS

---

The answers to a practice questions that are provided in video walk-throughs have been duplicated here for easy reference.

## Whole Numbers

### Chapter 1: The Place Value System

#### Question 1

3 = hundred millions

8 = ten millions

5 = millions

9 = hundred thousands

2 = ten thousands

2 = thousands

1 = hundreds

0 = tens

2 = ones

#### Question 2 ">

2 = ten billions

0 = billions

4 = hundred millions

3 = ten millions

3 = millions

8 = hundred thousands

7 = ten thousands

6 = thousands

0 = hundreds

1 = tens

1 = ones

## Chapter 2: Adding Whole Numbers

### Question 1

$$97 + 74 = 171$$

### Question 2

$$128 + 456 = 584$$

## Chapter 3: Subtracting Whole Numbers

### Question 1

$$94 - 62 = 32$$

### Question 2

$$434 - 78 = 356$$

## Chapter 4: Multiplying Whole Numbers

### Question 1

$$522 \times 196 = 102,312$$

### Question 2

$$647 \times 381 = 246,507$$

## Chapter 5: Dividing Whole Numbers

Question 1

$$130 \div 8 = 16\frac{1}{4}$$

Question 2

$$684 \div 12 = 57$$

## Fractions

### Chapter 7: The Basics of Fractions

Practice Question A

$$\frac{27}{8}$$

Practice Question B

$$5\frac{2}{3}$$

Question 1

$$\frac{5}{6}$$

Question 2

$$\frac{2}{3}$$

## Chapter 8: Adding and Subtracting Fractions

Question 1

$$\frac{13}{16}$$

Question 2

$$\frac{5}{16}$$

Question 3

$$1\frac{3}{8}$$

Question 4

$$4\frac{3}{8}$$

Question 1 (Mixed numbers)

$$11\frac{1}{2}$$

Question 2 (Mixed numbers)

$$6\frac{5}{16}$$

Question 3 (Mixed numbers)

$$7\frac{3}{8}$$



## Question 4 (Mixed numbers)

$$\frac{11}{16}$$

## Chapter 9: Multiplying Fractions

### Question 1

$$\frac{3}{14}$$

### Question 2

$$\frac{10}{33}$$

### Question 3

$$35\frac{1}{16}$$

### Question 4

$$65\frac{53}{63}$$

## Chapter 10: Dividing Fractions

### Question 1

$$2$$

### Question 2

$$1\frac{17}{22}$$

## Decimals

### Chapter 13: Compare and Round Decimals

Question 1

b. 27.54

Question 2

a. 14.001

Question 1 (Rounding)

27.72

Question 2 (Rounding)

14.756

### Chapter 14: Convert Decimals to Fractions

Question 1

$$\frac{181}{500}$$

Question 2

$$\frac{963}{1000}$$

Question 1 (Convert feet)

$$9.1234' \text{ or } 9' 1\frac{1}{2}''$$

Question 2 (Convert feet)

$$\frac{11}{16} \text{ inches}$$

## Chapter 15: Convert Fractions to Decimals

Question 1

$$2.21'$$

Question 2

$$2'$$

## Percentages

Practice Question

$$26.7\%$$

Question 1

$$70\%$$

Question 2

$$80\%$$

Question 3

$$20\%$$

## Chapter 19: Finding the percentage of a Given Number

### Question 1

**84.73**

### Question 2

**1503.84**

## Chapter 20: Change as a Percent and Percent Change

### Question 1 (Change as a percent)

**83%**

### Question 2 (Change as a percent)

**17.3%**

### Question 1 (Percent change)

**48 plants**

### Question 2 (Percent change)

**30 touchdowns**

# APPENDIX C: OFFLINE COPIES OF CHAPTER QUIZZES

---

## Whole Numbers Quiz

Find the missing value.

1. An electrician used switch outlet boxes on eight different jobs. The number of boxes used on each job is 56, 9, 36, 93, 105, 42 and 56. The total number of outlet boxes used is  boxes.
2. For a residential job, a reel containing 1050 feet of cable is delivered. Three 45 foot lengths and three 65 foot lengths are used. There are  feet left.
3. Two thousand five hundred feet of plastic tubing are ordered. The tubing is shipped in 250 foot coils. There are  coils shipped.
4. The materials charged to a wiring job are as follows: 100 amp distribution panel, \$36; meter switch, \$8; conduit, \$28; number 2 wire, \$43; BX cable, \$25; conduit fittings, \$9; outlet boxes, \$92; switches, \$35; fixtures, \$65; and \$37 for wire nuts, grounding clips, staples and pipe clamps. The total amount charged for these materials is  dollars.
5. An HVAC company has a payroll of 27 people. Seven people earn \$8/hr, eleven people earn \$10/hr, and nine people earn \$6/hr. If all the employees work 40 hours during the week, the total amount of money earned in one week is  dollars.
6. A large room contains forty fluorescent lamps. Twenty three of the lamps are 40 watt lamps, and the remaining number are 60 watt lamps. The lamps use a total of  Watts.
7. On a certain job, a sum of \$438 is spent for materials. Of this amount, \$76 is spent for 1 fittings, and \$105 is spent for miscellaneous materials.  dollars is spent on other materials.
8. Three plumbers install 180 fixtures in 4 working days. The average number of fixtures installed by each plumber in one day is  fixtures.
9. On a 124 foot length of pipe, 32 supports are used. The supports are equally spaced. If one support is placed at the beginning and one at the end of the pipe, there will be a support every  feet.
10. A 7 floor apartment building has an average of 7 electrical circuits per apartment, and there are 8 apartments per floor. There are  circuits in the building.

## Answers

1. 483
2. 720
3. 10
4. 378
5. 8800
6. 1940
7. 257
8. 15
9. 4
10. 392

## Fractions Quiz

1.  $11.25 - 3.875 =$ 
  - A. 7.15
  - B. 7.875
  - C. 7.375
  - D. 15.125
2. Reduce  $24/64$  to lowest terms.
  - A.  $12/32$
  - B.  $6/16$
  - C.  $3/8$
  - D.  $1.5/4$
3. Reduce  $150/275$  to lowest terms.
  - A.  $6/11$
  - B.  $3/5.5$
  - C.  $3/11$
  - D.  $12/22$
4. Change  $34/8$  into a mixed number.
  - A.  $3 \frac{4}{8}$
  - B.  $4 \frac{1}{4}$
  - C.  $4 \frac{1}{8}$
  - D.  $4 \frac{1}{16}$
5. Express  $5 \frac{7}{16}$  as an improper fraction.

A.  $103/16$

B.  $57/16$

C.  $87/16$

D.  $35/16$

6.  $1/4 + 5/16 + 7/32 =$

A.  $7/16$

B.  $13/32$

C.  $25/32$

D.  $13/16$

7.  $5/8 + 9/32 - 1/16 =$

A.  $7/8$

B.  $27/32$

C.  $13/16$

D.  $7/16$

8.  $7/8 \times 5 =$

A.  $4\ 3/4$

B.  $3\ 5/40$

C.  $6\ 3/8$

D.  $4\ 3/8$

9.  $3/16 \times 5/8 =$

A.  $9/32$

B.  $9/16$

C.  $80/24$

D.  $15/128$

10.  $4\ 1/2 \times 4 =$

A. 18

B.  $8\ 1/2$

C.  $16\ 1/2$

D.  $16\ 1/8$

11.  $3/8 \div 1/16 =$

A. 6

B. 12

C.  $3/16$

D.  $3/128$

12.  $5/16 \div 4 =$

A.  $20/16$

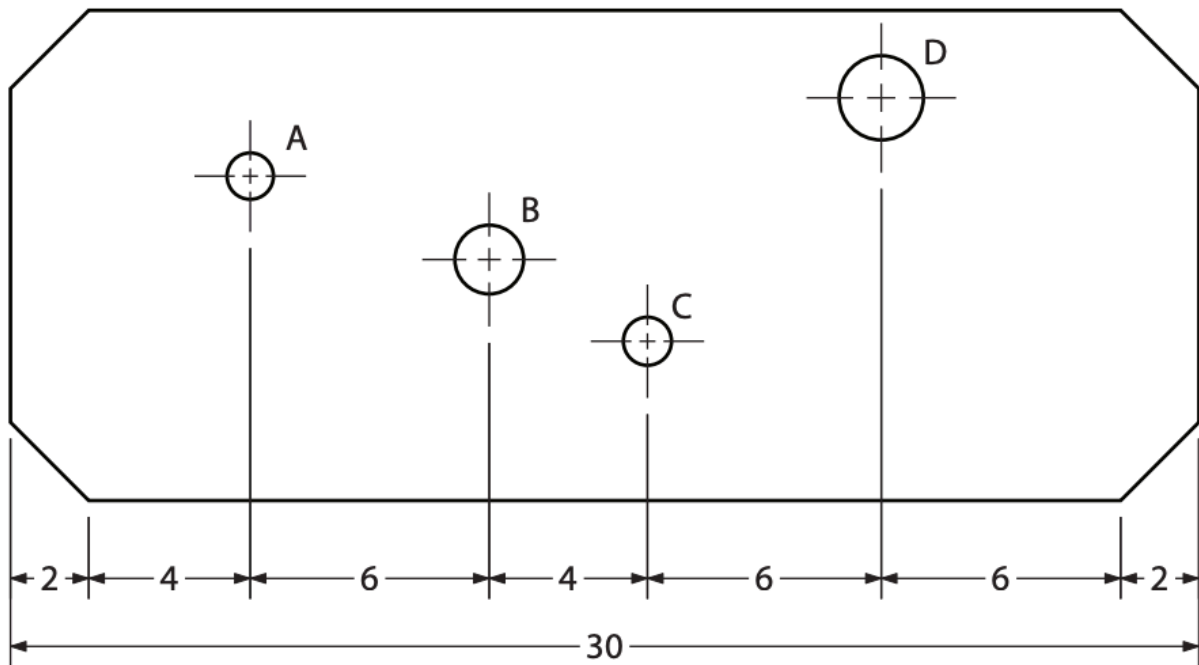
B.  $1\ 1/4$

C.  $5/16$ D.  $5/64$ 

13.  $17/32 + 1/2 \times 1/4 =$

A.  $8/32$ B.  $21/32$ C.  $23/32$ D.  $19/32$ 

14. On the sheet metal layout in the drawing, the distance between the centres of hole A and hole D, measuring lengthwise, is 16 centimetres. All dimensions are in centimetres. True or false?



15.

A total of 256 bronze castings were machined. Each casting had a mass of 793 g, and 158 g were removed from each by machining. The total mass of bronze removed by machining was 162,560 g. True or false?

16. A counter top is made of  $3/4$ " particleboard and is covered with  $1/32$ " plastic laminate. The width of metal edging required to finish off the edge is  $27/32$ ". True or false?

17. In one minute, a large reduction gear turns  $61/3$  revolutions. How many revolutions will it turn in  $27\frac{1}{2}$  minutes?

A.  $174\frac{1}{6}$ B.  $162\frac{1}{6}$ C.  $168\frac{1}{3}$ D.  $170\frac{1}{3}$ 

18. 12 billets of cold drawn steel  $11\frac{1}{2}$ " long can be cut from a bar 20" long, if  $1/8$ " is allowed for each saw cut. True or false?

19. A  $3/16$  sheet of metal is thicker than a  $13/64$ " fastener. True or false?



20. Eight pins, each  $\frac{43}{8}$ " long, have to be cut from a piece of drill rod. There is a  $\frac{3}{32}$ " waste for each cut. What is the minimum length of rod you will need?
- A. 35"
  - B.  $35\frac{3}{4}$ "
  - C.  $35\frac{21}{32}$
  - D.  $35\frac{43}{64}$ "

## Answers

- 1. C
- 2. C
- 3. A
- 4. B
- 5. C
- 6. C
- 7. A
- 8. D
- 9. D
- 10. A
- 11. A
- 12. D
- 13. B
- 14. True
- 15. False
- 16. False
- 17. A
- 18. True
- 19. False
- 20. C

## Decimals Quiz

- 1. Round 5237.02046 to the nearest thousandth.
  - A. 5000
  - B. 5237.02
  - C. 5237.021

- D. 5237.02106
2. Round 12465.078 to the nearest thousand.
- A. 12465
  - B. 12000
  - C. 10000
  - D. 12465.078
3.  $13.963 + 335.021 + 2267.123 =$
- A. 2616.17
  - B. 2616.107
  - C. 2616.296
  - D. 2616.182
4. Add 2.72, 0.6, 110, and 17.225.
- A. 130.545
  - B. 135.945
  - C. 136.545
  - D. 131.545
5.  $11.25 - 3.875 =$
- A. 7.15
  - B. 7.875
  - C. 7.375
  - D. 15.125
6. Multiply 19.5 by 7.65.
- A. 14917.5
  - B. 14.9175
  - C. 1491.75
  - D. 149.175
7.  $18.753 \div 0.87 =$
- A. 210.55
  - B. 21.555
  - C. 2.1555
  - D. 0.21555
8. Divide 0.0028 by 0.95. Express your answer to the nearest thousandth.
- A. 0.003
  - B. 0.002
  - C. 0.0029
  - D. 0.00295
9.  $178 + 18 \times 6 =$

- A. 286
  - B. 295
  - C. 1176
  - D. 19224
10.  $158.58 - 33 \times 4 =$
- A.  $-26.58$
  - B. 502.32
  - C. 26.58
  - D.  $-502.32$
11.  $1890.33 - 543.48 + 101 =$
- A. 1447.52
  - B. 2534.81
  - C. 2534.85
  - D. 1447.85

## Answers

- 1. C
- 2. B
- 3. B
- 4. A
- 5. C
- 6. D
- 7. B
- 8. A
- 9. A
- 10. C
- 11. D

## Percentages Quiz

To complete the quiz on the computer, go to the link on the previous page. When completing on the computer, questions will be randomized. The same questions are provided in print below.

- 1. 40% of 24 is what?
  - 1. 14.4%

2. 9.6%
  3. 8.4%
  4. 10.2%
2. What percent of 1.32 is 0.6?
    1. 46%
    2. 55%
    3. 45%
    4. 10%
  3. 3 is 15% of 22. True or false?
  4. One part acid and four parts water are mixed as an electrolyte for a storage battery. What percent of the electrolyte is acid?
    1. 25%
    2. 20%
    3. 75%
    4. 80%
  5. The efficiency of a motor is 90%. If the motor delivers 13.5 horsepower, what is the input?  
( $\text{efficiency} = \text{output} / \text{input}$ )
    1. 15 hp
    2. 15.85 hp
    3. 12.15 hp
    4. 15.28 hp
  6. What percent is wasted when 3.6 of every 120 sheets of metal are spoiled?
    1. 3%
    2. 0.3%
    3. 30%
    4. 300%
  7. A floor joist is allowed to vary 5% of its stated width of 184 mm. The maximum width would be what?
    1. 200 mm
    2. 188.4 mm
    3. 193.2 mm
    4. 197.9 mm
  8. A contractor wants to make a profit of 12% on a job. If the break-even cost is \$2345, what should the total job price be, including the profit?
    1. \$2626
    2. \$2600.40
    3. \$2600
    4. \$2626.40

9. The minimum idle speed for a particular engine is 630 rpm. This is 94% of the maximum idle speed of 668 rpm. True or false?
10. The operating spindle speed of a lathe spindle that rotates freely is 354 rpm. When 16% is lost through slippage and cutting pressure, the new speed is 297 rpm. True or false?

## Answers

1. B
2. C
3. False
4. B
5. A
6. A
7. C
8. D
9. False
10. True

# VERSIONING HISTORY

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This page provides a record of edits and changes made to this book since its initial publication in the [B.C. Open Textbook Collection](#). Whenever edits or updates are made in the text, we provide a record and description of those changes here. If the change is minor, the version number increases by 0.01. If the edits involve substantial updates, the version number increases to the next full number.

The files posted by this book always reflect the most recent version. If you find an error in this book, please fill out the [Report an Open Textbook Error form](#).

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Version	Date	Change	Details
1.00	DATE	Added to the B.C. Open Textbook Collection.	

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